HURLSTONE AGRICULTURAL HIGH SCHOOL

PRO PATRIA

MATHEMATICS – EXTENSION TWO

2009 HSC

ASSESSMENT TASK 1

Examiners ~ G Huxley, G Rawson

GENERAL INSTRUCTIONS

- Reading Time – 3 minutes.
- Working Time – 40 MINUTES.
- Attempt **all** questions.
- **All** necessary working should be shown in every question.
- This paper contains two (2) questions.
- Marks may not be awarded for careless or badly arranged work.
- Board approved calculators may be used.
- **Each question is to be started on a new piece of paper.**
- This examination paper must **NOT** be removed from the examination room.

STUDENT NAME: ______________________________

TEACHER: ______________________________

QUESTION ONE **18 marks** *Start a SEPARATE sheet*

(a) Let $z = 1 + 2i$ and $\omega = 1 + i$. Find, in the form $x + iy$,

(i) $z\,\omega$ **1**

(ii) $z\,\overline{\omega}$ **1**

(iii) $\dfrac{1}{\omega}$ **1**

(b) z has modulus r and argument θ. Find in terms of r and θ the modulus and argument of:

(i) z^2 **1**

(ii) $\dfrac{1}{z}$ **1**

(iii) iz **1**

(c) Find all pairs of integers x and y that satisfy $(x+iy)^2 = 24+10i$ **3**

(d) Consider the equation $z^2 + az + (1+i) = 0$
Find the value of a, given that i is a root of the equation. **2**

(e) It is given that two of the roots of $P(z) = z^3 + pz^2 + qz + 32$ are -2 and $4i$.
If it is known that p and q are real numbers, state the third root of $P(z)$,
giving a reason for your answer **1**

(f) Let $z = -2 + 3i$

(i) Evaluate $\bar{z}$. Verify that $z\,\bar{z}$ is real. **2**

(ii) Use $\dfrac{1}{z} = \dfrac{\bar{z}}{z\bar{z}}$ to find $\dfrac{1}{z}$ in the form a + bi, where a and b are real. **2**

(g) Simplify $\dfrac{\left(\cos\dfrac{5\pi}{12} + i\sin\dfrac{5\pi}{12}\right)\left(\cos\dfrac{3\pi}{4} + i\sin\dfrac{3\pi}{4}\right)}{\left(\cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3}\right)}$ **2**

QUESTION TWO **18 marks** *Start a SEPARATE sheet*

(a) By considering the complex number $z = x + iy$ in the Argand plane, sketch the locus Of the following on separate Argand diagrams.

(i) $\arg z = \frac{\pi}{3}$ **1**

(ii) $\arg \bar{z} = \frac{\pi}{3}$ **1**

(iii) $\arg(-z) = \frac{\pi}{3}$ **1**

(b) Let $w = r(\cos\phi + i\sin\phi)$ where ϕ is an acute angle.
With the aid of a suitable diagram, or otherwise:

(i) Show that the distance between w and $\bar{w}$ in the complex plane is $2r\sin\phi$ **2**

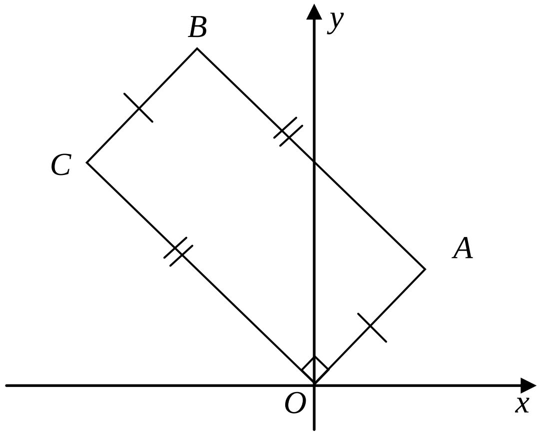
(ii) Find $|w + \bar{w}|$ in terms of r and ϕ. **1**

(c) Sketch the region of the complex plane for which the complex number $z = x + iy$ has a positive real part and $|z + 3i| \le 2$. **2**

(d) Sketch the region defined by $1 < \left|z - \left(1 + i\sqrt{3}\right)\right| < 2$ and $0 \le \arg z \le \frac{\pi}{3}$ **3**

Question 2 continued on next page…

(e) In the Argand diagram, $OABC$ is a rectangle, where $OC = 2OA$.
The vertex A corresponds to the complex number ω.



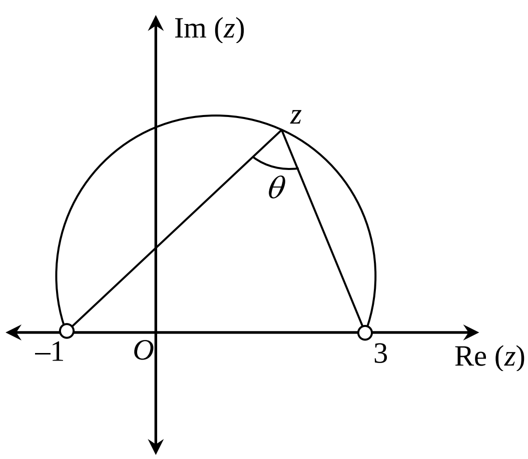
(i) What complex number corresponds to the vertex C? **1**

(ii) What complex number corresponds to the point of intersection D of the diagonals OB and AC? **2**

(f) The diagram below shows the locus of points z in the complex plane such that

$$\arg(z-3)-\arg(z+1)=\frac{\pi}{3}.$$

This locus is part of a circle. The angle between the lines from –1 to z and from 3 to z is θ, as shown.



Copy this diagram into your Writing Booklet

(i) Explain why $\theta = \dfrac{\pi}{3}$ **2**

(ii) Find the centre of the circle. **2**

Year 12	Mathematics Extension 2	HSC Task 1 Dec 2008

Question No. 1 Solutions and Marking Guidelines

Outcomes Addressed in this Question

E3 - uses the relationship between algebraic and geometric representations of complex numbers and of conic sections

Outcome	Solutions	Marking Guidelines
E3 **(a)**	(i) $z\omega = (1+2i)(1+i)$ $= -1+3i$	**1 mark:** Correct answer
	(ii) $z\overline{\omega} = (1+2i)(1-i)$ $= 3+i$	**1 mark:** Correct answer
	(iii) $\frac{1}{\omega} = \frac{1}{1+i} \times \frac{1-i}{1-i}$ $= \frac{1-i}{2}$	**1 mark:** Correct answer
(b)	(i) mod (z^2) = (mod z)^2 = r^2 arg (z^2) = 2 arg (z) = 2θ	**1 mark:** Both mod and arg correct.
	(ii) $\text{mod}\left(\frac{1}{z}\right) = \frac{1}{\text{mod}(z)} = \frac{1}{r}$ $\arg\left(\frac{1}{z}\right) = -\arg(z) = -\theta$	**1 mark:** Both mod and arg correct.
	(iii) mod (i z) = mod (i) mod (z) = r arg (iz) = arg (i) + arg (z) = $\frac{\pi}{2} + \theta$	**1 mark:** Both mod and arg correct.
(c)	If $(x+iy)^2 = 24+10i$ Then: $x^2 - y^2 = 24$ ....(1) $2xy = 10$ ....(2) Solving simultaneously for real values of x and y gives: $(x,y) = (5,1)\ or\ (-5,-1)$ i.e: $x+iy = \pm(5+i)$	**3 marks:** Correct solution. **2 marks:** Solution with one error when calculating the pairs required. **1 mark:** Correct pair of simultaneous equations.
(d)	$P(i) = i^2 + ai + (1+i) = 0$ $i(a+1) = 0$ $\therefore a = -1$	**2 marks:** Correct solution. **1 mark:** Substitution of i followed by some correct simplification.

(e)	3^{rd} root = $4i$ Complex roots are in conjugate pairs, since all coefficients are real.	**<u>1 mark:</u>** Need to name <u>all three</u> components of the solution shown.
(f)	$z\bar{z} = (-2+3i)(-2-3i)$ (i) $= (-2)^2 - (3i)^2$ $= 13$ which is real.	**<u>2 marks:</u>** Correct solution. **<u>1 mark:</u>** Correct conjugate and significant progress towards solution.
	(ii) $\frac{\bar{z}}{z\bar{z}} = \frac{-2-3i}{13}$ $\therefore a = \frac{-2}{13}$, $b = \frac{-3}{13}$	**<u>2 marks:</u>** Correct values for *a* and *b*. (Correct from answer to (i)) **<u>1 mark:</u>** 1 of the 2 values correct.
(g)	$\frac{\left(cis\frac{5\pi}{12}\right)\left(cis\frac{3\pi}{12}\right)}{\left(cis\frac{2\pi}{3}\right)} = cis\left(\frac{5\pi}{12} + \frac{3\pi}{4} - \frac{2\pi}{3}\right)$ $= cis\left(\frac{\pi}{2}\right)$ $= i$	**<u>2 marks:</u>** Correct answer, fully simplified. **<u>1 mark:</u>** Correct application of sum and difference of arguments. **or:** correct simplification after 1 error in initial calculation.