JAMES RUSE AGRICULTURAL HIGH SCHOOL

Year 11 MATHEMATICS EXTENSION 2

TERM 4 - 2000

Instructions:
Time allowed: 85 minutes
Attempt all questions
Marks may not be awarded for poorly arranged work
Show all working
Approved silent calculators may be used
Return your answers in 3 separate bundles

SECTION A (START A NEW PAGE)

Question 1:

Given $z = 4 - 3i$, write down the values of

(i) $-z$ (ii) $\bar{z}$ (iii) z^2 (iv) $\dfrac{1}{z}$

Question 2:

On the Argand diagram provided, the points A and B represent the complex numbers α and β. The points P, Q, R and S represent the complex numbers $-\beta$, $2i\beta$, $\alpha + \beta$ and $\alpha - \beta$ respectively. Use compasses, protractor and rule to show the positions of points P, Q, R and S. Show all construction lines and relevant angles.

Question 3:

The number z has modulus $\sqrt{2}$ and argument $\dfrac{3\pi}{5}$ and $w = 2 + 2i$. Express the following in mod/arg form.

(i) w (ii) wz (iii) $\dfrac{w}{z}$ (iv) z^4

Question 4:

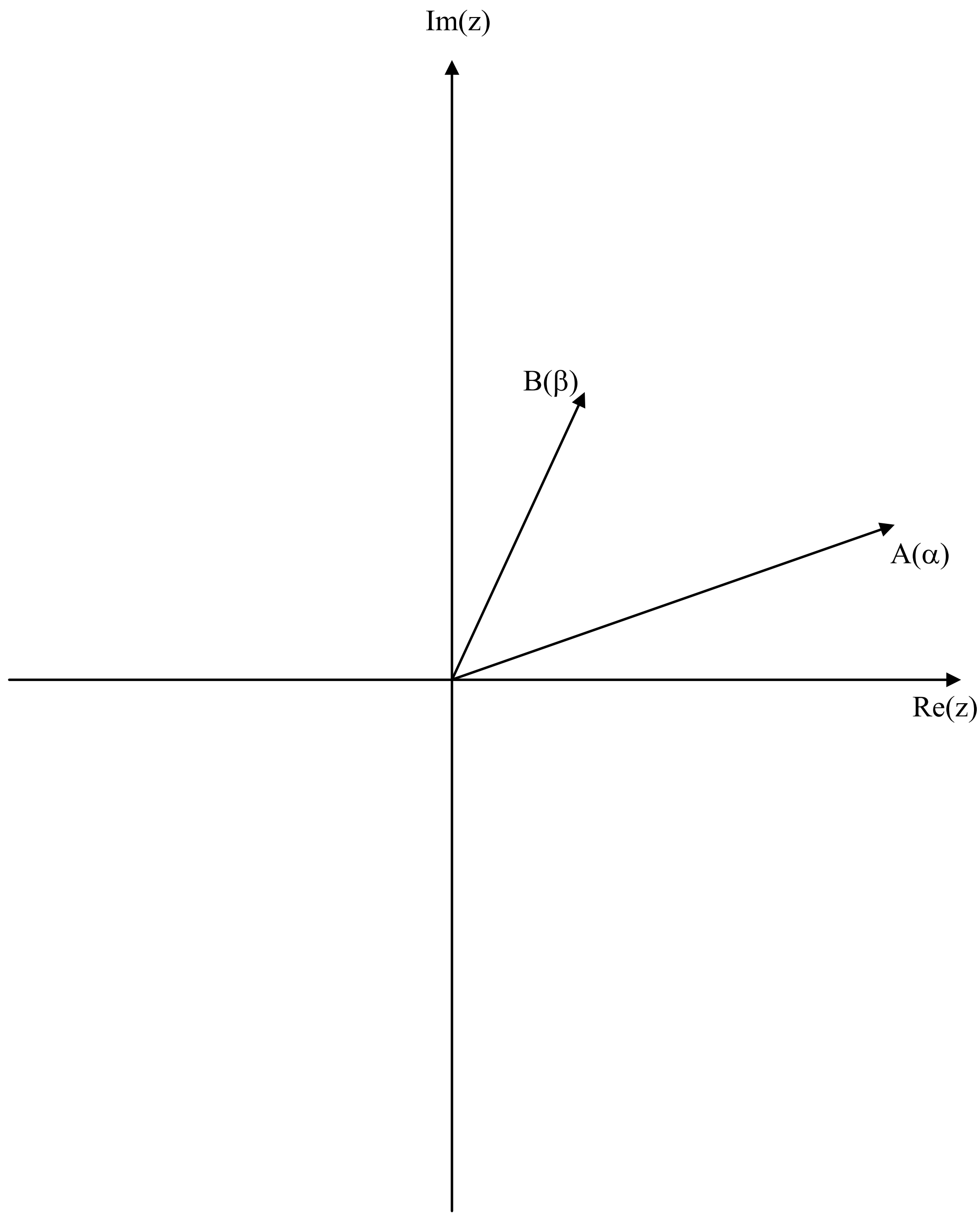
If $z = \cos\theta + i\sin\theta$

(i) write z^2 in mod/arg form

(ii) prove that $\dfrac{2}{1+z^2} = 1 - i\tan\theta$, $(z \neq i)$

Argand diagram for SECTION A – Question 2 NAME:……………………

Attach to your answers.



SECTION B (START A NEW PAGE)

Question 1:

Given $z_1 = r_1(\cos\theta + i\sin\theta)$ and $z_2 = r_2(\cos\phi + i\sin\phi)$ prove that

(i) $|z_1 z_2| = |z_1||z_2|$

(ii) $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$

Question 2:

(i) Express both complex roots of $z^2 = -3 + 4i$ in the form $a + ib$ where a and b are real.

(ii) Hence solve $z^2 - (4 - 2i)z + (6 - 8i) = 0$ expressing your answer in the form $p + iq$ where p and q are real.

Question 3:

(i) Describe algebraically and sketch the locus defined by $\arg(z - 2 + i) = -\frac{\pi}{4}$.

(ii) Describe algebraically and sketch the locus defined by $\text{Im}\left(z + \frac{1}{z}\right) = 0$.

(iii) Shade the region defined by $|z| \le 2$ and $0 \le \arg z \le \frac{\pi}{3}$.

SECTION C (START A NEW PAGE)

Question 1:

(i) Describe algebraically the loci of the complex number w and z if $|z-i|=3$ and $|w+1-i|=|w-7-7i|$.

(ii) Sketch both loci on the same set of axes.

(iii) Hence find the least value of $|w-z|$.

Question 2:

(i) Show on an Argand diagram the points A, B and P representing the complex numbers z, w and $z+w$ respectively. (Assume that $0<\arg(w)<\arg(z)<\frac{\pi}{2}$)

(ii) Given that O is the origin and $|z|=|w|$, what type of quadrilateral is OAPB? (Give a reason)

(iii) With the aid of the above diagram find $\arg\left(\frac{z+w}{z-w}\right)$. (Give a reason)

Question 3:

Given that w is a complex root of unity,

(i) Prove that $1+w+w^2=0$.

(ii) Prove that $1+w^2=\frac{1}{1+w}$.

(iii) Prove that $1+w$ is a complex cubic root of -1.

(iv) Express the other complex cubic root of -1 as a quadratic expression in w.

END of EXAMINATION PAPER

Question 4:

(i) Express $-4+4i\sqrt{3}$ in mod/arg form.

(ii) Hence find the three complex roots of $z^3 = -4+4i\sqrt{3}$. Leave your answer in mod/arg form.

(iii) Show the three roots on an Argand diagram.

Question 2:

If $|z|=1$, prove that $\frac{1+z}{1+\bar{z}} = z$.

Question 5:

If $z = \cos\theta + i\sin\theta$

(iii) write z^2 in mod/arg form

(iv) prove that $\frac{2}{1+z^2} = 1 - i\tan\theta$.