JAMES RUSE AGRICULTURAL HIGH SCHOOL
YEAR 11 MATHEMATICS EXTENSION 2
ASSESSMENT TASK 1, TERM 4 2003

Time allowed: 85 minutes + 5 minutes (reading time)
Instructions:

- Approved calculators may be used.
- All questions may be attempted.
- All questions are of equal value.
- In every question, show all working
- Marks may not be awarded for careless or badly arranged setting out.
- Start a new page for each NEW question.
- *Return Question 1, 2, 3 & 4 as separate bundles, showing your candidate number clearly in top right hand corner.*

Question 1. **Marks**

(a) Let $z = 3 - 4i$ and $w = 2 + 5i$. Express each of the following in the form $a + ib$, where a and b are real numbers.

(i) $w - z$. **1**

(ii) z^2. **2**

(iii) $\dfrac{z}{w}$. **2**

(b) Find a and b, where a and b are real numbers if $(a+ib)^2 = 21 - 20i$. **3**

(c) Let $z = 3(\cos\theta + i\sin\theta)$.

(i) Find $\overline{1-z}$. **1**

(ii) Show that the real part of $\dfrac{1}{1-z}$ is $\dfrac{1-3\cos\theta}{10-6\cos\theta}$. **2**

(iii) Express the imaginary part of $\dfrac{1}{1-z}$ in terms of θ. **1**

(d) Given $\psi = -\dfrac{1}{2} - i\dfrac{\sqrt{3}}{2}$, calculate

(i) $|\psi|$. **1**

(ii) $Arg(\psi)$. **1**

(iii) ψ^{-10}. **2**

Question 2. [START A NEW PAGE] **Marks**

(a) If $z_1 = 2i$ and $z_2 = 1 + 3i$ are two complex numbers, describe the loci of z such that: $z = z_1 + k(z_2 - z_1)$, when

(i) $k = 0$ and $k = 1$. **1**

(ii) $0 < k < 1$. **1**

(iii) k is any real number. **1**

(b) Sketch the region for z in the Argand plane defined by: **3**

$$|z - 1 + i| < 2 \text{ and } -\frac{\pi}{4} \le \arg(z - 1 + i) \le \frac{5\pi}{4}.$$

(c) Find the locus of z when

(i) $|z - 4i| = |z - 6 - 2i|$. **2**

(ii) $\arg(z^2) = \frac{\pi}{2}$. **2**

(iii) $\text{Re}(z - \frac{9}{z}) = 0$. **2**

(d) Find the new complex number when the complex number $3 + i$ is rotated 45^0 anticlockwise about the origin in the Argand plane. **2**

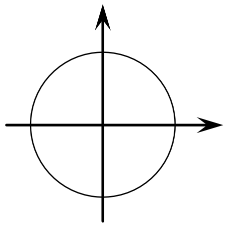
(e) Evalaute: i^{2003}. **1**

Question 3. [START A NEW PAGE] **Marks**

(a) (i) Define the argument of complex number z, that is $\arg(z)$. **1**

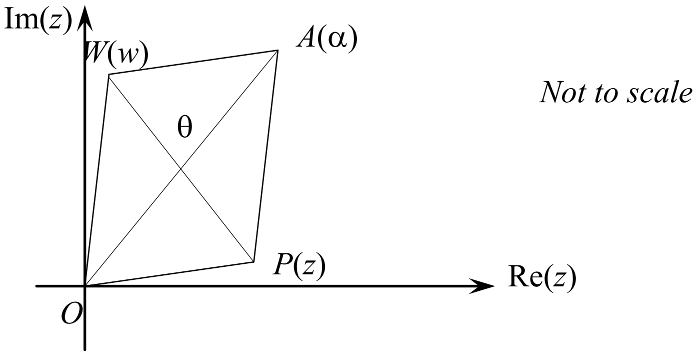
(ii) If z is a complex number such that **4**

$$|z| = a \text{ and } \arg(z) = \theta, \text{ for } 0 < \theta < \frac{\pi}{2}.$$



Prove that $\arg(z^2 - a) = \theta + \frac{\pi}{2}$.

(b) The diagram shows a parallelogram $OPAW$ in the Argand plane.
The diagonals intersect with an angle of θ as shown.



Let z, α and w be the complex numbers represented by the points P, A and W respectively, where z, α and $w \neq 0$.

(i) Given that $2\,\text{Re}(\varphi) = \varphi + \overline{\varphi}$, where φ is a complex number, hence, or otherwise show that $2\,\text{Re}(w\overline{z}) = w\overline{z} + \overline{w}z$. **2**

(ii) Suppose $w\overline{z} + \overline{w}z = 0$, deduce that $\text{Re}(\frac{w}{z}) = 0$. **2**

(iii) Hence show that $OPAW$ is a rectangle. **2**

(iv) Consider now that $\frac{w}{z} = ki$, where k is a real number,

(α) Express $\frac{w - z}{w + z}$ in the form $a + ib$, where a and b are real numbers. **2**

(β) Hence find the expression for $\tan\theta$, where θ is the acute angle between the diagonals of $OPAW$ (assuming $k \neq 1$). **2**

Question 4. [START A NEW PAGE] **Marks**

(a) Given $z = \cos\theta + i\sin\theta$, show that $(\bar{z})^n = \overline{(z^n)}$. **2**

(b) It is known that $3 - i$ is a zero of $f(z) = z^3 + pz^2 + qz + 20$, where p and q are real numbers.

(i) Explain why $3 + i$ is also a zero of $f(z)$. **1**

(ii) Factorise $f(z)$ over the real numbers. **2**

(c) Let $w = \cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}$.

(i) Show that w^k, where k is an integer, is a solution of $z^7 - 1 = 0$. **2**

(ii) Show that $w^3 + w^2 + w + 1 + w^{-1} + w^{-2} + w^{-3} = 0$. **2**

(iii) State the expression for $w^k + w^{-k}$, and hence show that

(α) $\cos\frac{\pi}{7} - \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} = \frac{1}{2}$. **3**

(β) $\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{3\pi}{7} = \frac{1}{8}$. **3**

END