2012 Mathematics Extension 2 Term 4 Name ______________________

Question 1. **Marks**

(a) Find the modulus of $7 + 24i$ **1**

(b) Expand and simplify $(2 - 3i)\overline{(4 + 3i)}$ **2**

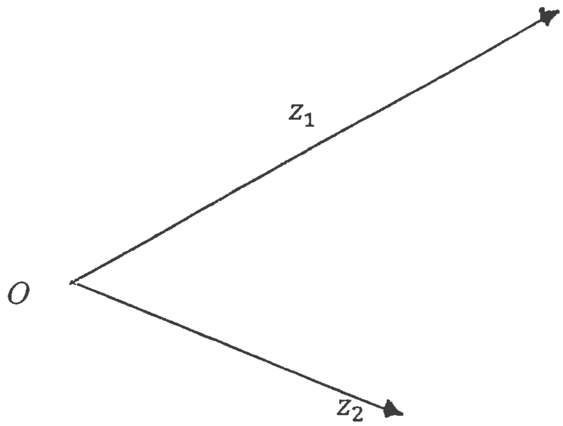
(c) Write in the form $a+ib$ $(a, b \in R)$: $\frac{3-2i}{1+4i}$ **2**

(d) Expand and simplify $(z - 1 - 3i)(z + 5 - i)$ **2**

(e) A square $OABC$ has $\overrightarrow{OA} = 3 + 4i$. **4**
Find the co-ordinates of B and C.

(f) Describe the transformation when z is multiplied by $-\sqrt{3} - i$ **2**

(g) **Construct** the vector $z_1 + z_2$ in the space below. **2**
Clearly show the vector $z_2 - z_1$.



Question 2. (Start A New Page)

(a) Write in the form $a+ib$, where a,b are integers : $(2 - 3i)^3$ **2**

(b) (i) Shade the region defined by : $3 \leq |z + 2| \leq 4$ and $\frac{\pi}{6} \leq Arg(z + 2) \leq \frac{\pi}{2}$. **2**

(ii) Find the area of the shaded region. **2**

(c) Describe the locus of z : $z\bar{z} - 2z - 2\bar{z} - 15 = 0$ **2**

(d) (i) Find the values of z such that $z^2 = 5 - 12i$ **2**

(ii) Solve the equation $z^2 + (4i - 7)z + 7 - 11i = 0$ **2**

(iii) Solve and describe the solutions of : **3**
$|z^2 + (4i - 7)z + 7 - 11i| = |z - 2 + i|$

Question 3. (Start A New Page) **Marks**

(a) Given : $\alpha = 3\left(cos\frac{\pi}{6} + isin\frac{\pi}{6}\right)$, $\beta = 2\left(cos\frac{\pi}{4} - isin\frac{\pi}{4}\right)$ and $\gamma = -1 + i$

(i) Express α^4 in the form $a+ib$ $(a, b \in R)$ **2**

(ii) Express $\alpha\beta$ in Modulus / Argument form. **2**

(iii) Express $\alpha\gamma$ in Modulus / Argument form. **2**

(b) (i) Solve over the set of Complex numbers the equation $z^3 = 1$. **2**
Write the solutions in the form $a+ib$ where $a, b \in R$.

(ii) Plot the solutions of $z^3 = 1$ on the Argand Diagram. (Use 1 unit = 2cm) **2**

(iii) Show that the solutions of $z^3 = 1$ can be represented by $1, \omega, \omega^2$, where ω is a non real solution. **2**

(iv) Hence solve in terms of ω the equation : $(1 + z)^3 = (1 - z)^3$ **3**

Question 4. (Start A New Page)

(a) A triangle *ABC* is represented by the position vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ by $1 + 5i, 3 + 6i$ and $4 - i$ respectively.

(i) Draw the triangle *ABC* on the Argand Diagram. (Use 1unit = 1cm) **1**

(ii) Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. **2**

(iii) Find the Complex number, *M*, representing the median from $C\ in\ \Delta ABC$, hence find the length of the median. **2**

(b) A polynomial *P(z)* is defined by : $P(z) = z^6 + 4z^4 + 16z^2 + 64$.

(i) Find *P(2i)* **1**

(ii) Deduce a quadratic factor of *P(z)* giving reasons. **1**

(iii) Factorise the polynomial *P(z)* over the set of real numbers *R*. **3**

(c) Find the Cartesian equation of the locus of: $\left|\frac{z-3}{z+3}\right| = 2$. **3**
Describe the locus .

(d) A function *P(z)* is always a non real number for all values of *z*. **2**
If $P(z)P(\bar{z}) = 16$ find the value of $|P(z)|$ giving reasons.

End of Exam

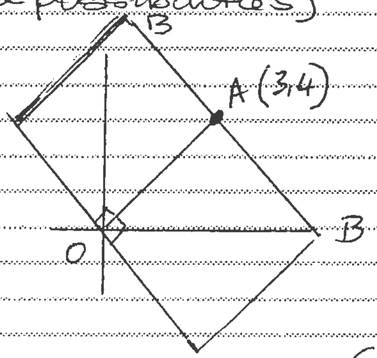
***** PLEASE ATTACH THE EXAM PAPER TO QUESTION 1.*****

MATHS EXT 2 ASSESSMENT TEST 1
TERM 4, 2012

MATHEMATICS Extension 2: Question 1

Suggested Solutions	Marks	Marker's Comments
(a) $\lvert 7+24i \rvert = \sqrt{7^2+24^2} = 25$	1	
(b) $(2-3i)\overline{(4+3i)} = (2-3i)(4-3i) = 8-9-18i = -1-18i$	2	1 + 1 each term
(c) $\dfrac{3-2i}{1+4i} = \dfrac{3-2i}{1+4i} \times \dfrac{1-4i}{1-4i} = \dfrac{3-12i-2i-8}{1+16} = \dfrac{-5}{17} - \dfrac{14i}{17}$	2	1; $\frac{1}{2}$ + $\frac{1}{2}$; must write in $a+ib$ form ($\frac{1}{2}$)
(d) $(z-1-3i)(z+5-i)$ $= z^2+5z-iz-z-5+i-3iz-15i-3$ $= z^2+4z-4iz-8-14i$	2	
(e) Rotation could be anticlockwise or clockwise (2 possibilities)	4	1 set of: 1 C coordinates, 2 B coordinates; 1 other set of coordinates; $-\frac{1}{2}$ if coordinates not written

(e)



anticlockwise $(\times i)$

$\overrightarrow{OC} = (3+4i)\times i$
$= 3i-4$

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$
$= \overrightarrow{OA} + \overrightarrow{OC}$
$= (3+4i) + (-4+3i)$
$= (-1+7i)$

Clockwise $(\times -i)$

C $\overrightarrow{OC} = (3+4i)\times -i$
$= (-3i+4)$

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OA} + \overrightarrow{OC}$
$= (3+4i) + (4-3i) = (7+i)$

Coordinates: $C(-4,3)$ $B(-1,7)$ OR
$C(4,-3)$ $B(7,1)$

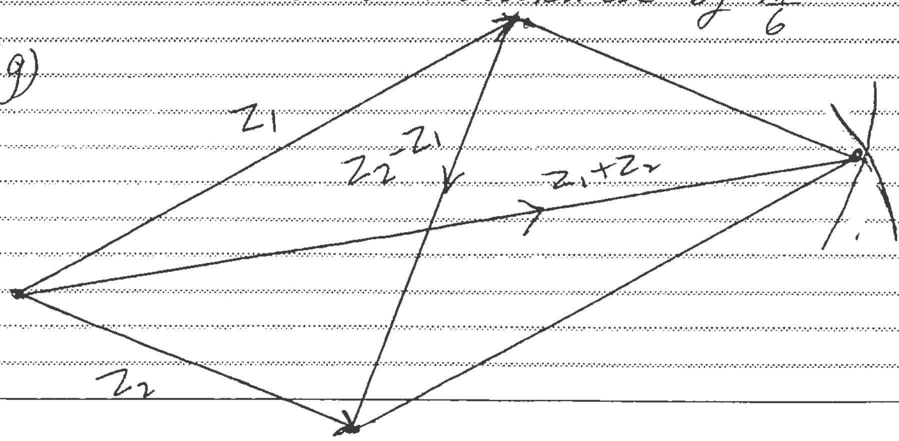
(f)

$(-\sqrt{3}-1) = 2\operatorname{cis}\left(\frac{-5\pi}{6}\right)$ or $2\operatorname{cis}\left(\frac{7\pi}{6}\right)$

$\therefore$ Z is enlarged by a factor of 2 and rotated clockwise about O by $\frac{5\pi}{6}$ or anticlockwise by $\frac{7\pi}{6}$

Marker's comments (f): 1 factor of 2 enlargement; 1 correct rotation

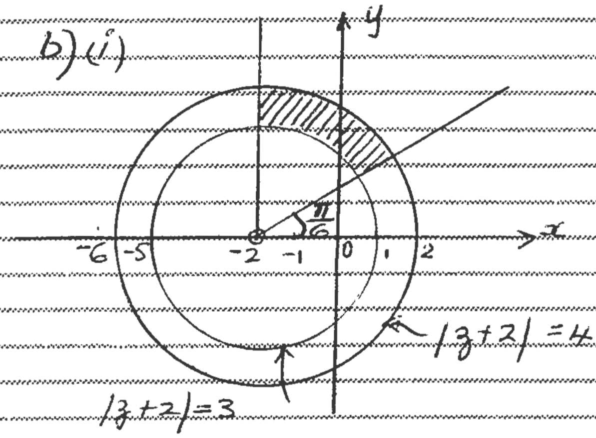
(g)



Marker's comments (g): $\frac{1}{2}$ Must show construction arcs used. $\frac{1}{2}$ (z_1+z_2); $\frac{1}{2}$ accuracy; $\frac{1}{2}$ $z_2 - z_1$ (accuracy + direction).

\\mercury\staffhome$\WOH\Admin_M Fac\Assessment info\Suggested Mk solns template_V3.doc

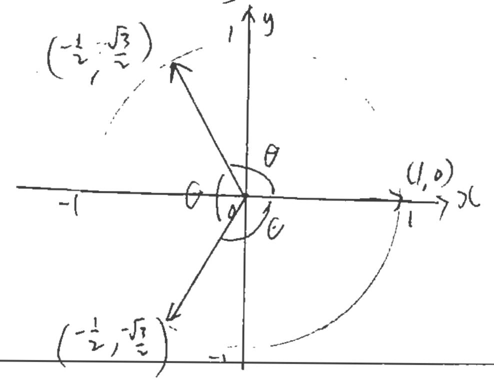
MATHEMATICS Extension 2: Question 2

Suggested Solutions	Marks	Marker's Comments
2a) $(2-3i)^3 = 2^3 + 3(2)^3(-3i) + 3(2)(-3i)^2 + (-3i)^3$ $= 8 - 36i - 54 + 27i$ $= -46 - 9i$ Alternatively, $(2-3i)^3 = (4-12i-9)(2-3i)$ $= (-5-12i)(2-3i)$ $= -10+15i-24i+36i^2$ $= -10-9i-36$ $= -46-9i$.	2	Students that useded the binomial expansion did not understand the concept of coefficients & Pascal's Triangle
b)(i) 	2	This question was done well.
(ii) Area $= \frac{1}{2}(R^2 - r^2)\theta$ $= \frac{1}{2}(4^2-3^2)\left(\frac{\pi}{2}-\frac{\pi}{6}\right)$ $= \frac{7}{2} \times \frac{\pi}{3}$ $\therefore$ Area $= \frac{7\pi}{6}$ units squared.		Poorly done Students confused the Year 8 & Yr 11 Formulae $A = \pi r^2 \times \frac{\theta^\circ}{360^\circ}$ $A = \frac{1}{2} r^2 \theta^c$
c) Describe the locus z: $z\bar{z} - 2z - 2\bar{z} - 15 = 0$ Let $z = x+iy$ then $\bar{z} = x - iy$ $z\bar{z} - 2z - 2\bar{z} - 15 = 0$ $x^2+y^2 - 2(2x) - 15 = 0$ $x^2+y^2-4x-15=0$ $(x-2)^2+y^2 = 15+4$ $(x-2)^2+y^2=19$ The locus is a circle, centre $(2,0)$, radius $\sqrt{19}$ units		

MATHEMATICS Extension 2: Question 2

Suggested Solutions	Marks	Marker's Comments
2 d) i) Find the values of z, such that $z^2 = 5-12i$ Let $z = x+iy$, then $(x+iy)^2 = 5-12i$ $x^2-y^2+2xyi = 5-12i$ $x^2-y^2 = 5$ —① on equating real & imaginary parts $2xy = -12$ —② From ② $y = \frac{-6}{x}$, substitute into ① $x^2 - \frac{36}{x^2} = 5$ $x^4-5x^2-36 = 0$ $(x^2-9)(x^2+4) = 0$ $x^2 = 9$ or $x^2 = -4$ $\therefore x = \pm 3$ $\Rightarrow$ no real solutions $\therefore y = \mp 2$ $\therefore z = \pm(3-2i)$	2	This question was well done apart from careless errors
ii) Solve $z^2+(4i-7)z+7-11i = 0$ $z = \frac{7-4i \pm \sqrt{(4i-7)^2-4(7-11i)}}{2}$ $= \frac{7-4i \pm \sqrt{-16+49-56i-28+44i}}{2}$ $= \frac{7-4i \pm \sqrt{5-12i}}{2}$ $= \frac{7-4i \pm (3-2i)}{2}$ from (i) $= \frac{10-6i}{2}$ or $\frac{4-2i}{2}$ $= 5-3i$ or $2-i$	2	This question was well done apart from careless errors.
(iii) $\lvert z^2+(4i-7)z+7-11i \rvert = \lvert z-2+i \rvert$ $\lvert (z-(5-3i))(z-(2-i)) \rvert = \lvert z-(2-i) \rvert$ $\lvert z-5+3i \rvert \lvert z-2+i \rvert - \lvert z-2+i \rvert = 0$ $\lvert z-2+i \rvert \left[\lvert z-5+3i \rvert - 1 \right] = 0$ $\therefore z = 2-i$ The point $(2,-1)$ OR $\lvert z-(5-3i) \rvert = 1$, the circle $(5,-3)$, radius 1 unit.	3	Students got to this line but then claimed that if $\lvert x \rvert \lvert y \rvert = \lvert p \rvert$ then $\lvert x \rvert = \lvert p \rvert$ or $\lvert y \rvert = \lvert p \rvert$ which is incorrect assumption.

Yr 11 2012 T4 **MATHEMATICS Extension 1: Question** 3 P.1/2

Suggested Solutions	Marks	Marker's Comments
a i) $\alpha^4 = \left(3\,\text{cis}\,\frac{\pi}{6}\right)^4 = 81\,\text{cis}\,\frac{2\pi}{3}$	1	
$= 81\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ $= -\frac{81}{2} + i\,\frac{81}{2}\sqrt{3}$ #	1	Some get the sign wrong -1m
ii) $\alpha\beta = 3\,\text{cis}\,\frac{\pi}{6}\,.\,2\,\text{cis}\left(-\frac{\pi}{4}\right)$	1	Many students write $\beta = \text{cis}\,\frac{\pi}{4}$ max 1 m.
$= 6\,\text{cis}\left(\frac{\pi}{6} - \frac{\pi}{4}\right)$ $= 6\,\text{cis}\left(-\frac{\pi}{12}\right)$ $= 6\left(\cos -\frac{\pi}{12} + i\sin -\frac{\pi}{12}\right)$ #	1	Need to write out the expanded form $-\frac{1}{2}$m
iii) $\alpha\gamma = 3\,\text{cis}\,\frac{\pi}{6}\,.\,\sqrt{2}\,\text{cis}\,\frac{3\pi}{4}$	1	Many students write $\gamma = \sqrt{2}\,\text{cis}\,\frac{-\pi}{4}$ max 1 m
$= 3\sqrt{2}\,\text{cis}\left(\frac{\pi}{6} + \frac{3\pi}{4}\right)$ $= 3\sqrt{2}\,\text{cis}\left(\frac{11\pi}{12}\right) = 3\sqrt{2}\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)$	1	Some do not know Mod-Arg. FORM eg $6\left(\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}\right)$ is Not.
b i) Let $z = r\,\text{cis}\,\theta$ $z^3 = r^3\,\text{cis}\,3\theta = 1$ $r^3\,\text{cis}\,3\theta = 1^3\,\text{cis}(0 + 2n\pi)$ $n \in J$ or $n = 0, 1, 2$		Need to define n $-\frac{1}{2}$m
$\therefore r = 1,\quad \theta = \frac{2n\pi}{3}$	1	Generally well done
$n=0$: $z = \text{cis}\,0 = 1 + 0i$ $n=1$: or $= \text{cis}\,\frac{2\pi}{3} = \frac{1}{2}\left[-1 + i\sqrt{3}\right]$ $n=2$: or $= \text{cis}\,\frac{4\pi}{3} = \frac{1}{2}\left[-1 - i\sqrt{3}\right]$	1	Need to answer in $a+bi$ form otherwise max 1m
ii) 	2	$\frac{1}{2}$m measuring + label angle $\frac{1}{2}$m radius (2cm) + intercepts 1 m for circle with labelled z_1, z_2, z_3

Suggested Solutions	Marks	Marker's Comments

iii) FROM (ii)

$z = 1, \ \text{cis}\frac{2\pi}{3}$ or $\text{cis}\frac{4\pi}{3}$

Let $w = \text{cis}\frac{2\pi}{3}$

$w^2 = \left(\text{cis}\frac{2\pi}{3}\right)^2 = \text{cis}\left(\frac{2\pi}{3}\times 2\right) = \text{cis}\frac{4\pi}{3}$

$\therefore 1, w, w^2$ are roots of $z^3 = 1$

Alternatively Let $w = \frac{1}{2}[-1 + i\sqrt{3}]$

$w^2 = \left[\frac{1}{2}(-1 + i\sqrt{3})\right]^2 = \frac{1}{4}(1 - 3 - 2i\sqrt{3})$

$w^2 = \frac{1}{4}(-2 - 2\sqrt{3}i) = \frac{1}{2}(-1 - i\sqrt{3})$

$\therefore 1, w, w^2$ are roots of $z^3 = 1$

or $z^3 = 1$ and w is a non real root

$\therefore w^3 = 1$

$(w^3)^2 = (w^2)^3 = 1$

and w, w^2 are different in part i

$\therefore w^2$ is also a root

iv) $(1+z)^3 = (1-z)^3$

$\left(\frac{1+z}{1-z}\right)^3 = 1 \qquad z \neq 1$

FROM ii) $\frac{1+z}{1-z} = 1, \ w$ or w^2

$\frac{1+z}{1-z} = w^n \quad n = 0, 1, 2$

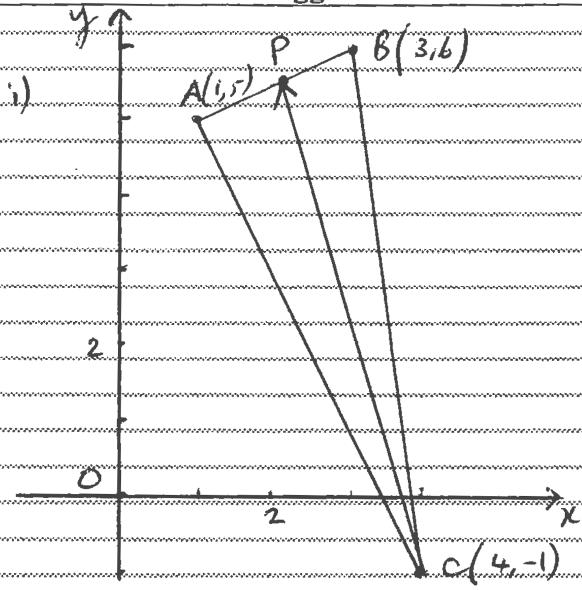
$1 + z = w^n - z w^n$

$z = \frac{w^n - 1}{w^n + 1}$

$z = 0, \ \frac{w-1}{w+1}$ or $\frac{w^2-1}{w^2+1}$

2012 TERM 4 EXT. 2

MATHEMATICS Extension 2 : Question 4

Suggested Solutions	Marks	Marker's Comments
a) i) 	1	Diagrams generally fine (surprising what guidance with scales will do)
ii) $\overrightarrow{AB}\ (=\overrightarrow{OB}-\overrightarrow{OA})$ represents $(3-1)+i(6-5)$ $= \underline{\underline{2+i}}$	1	Occasional issue with signs but generally fine
$\overrightarrow{AC}\ (=\overrightarrow{OC}-\overrightarrow{OA})$ represents $(4-1)+i(-1-5)$ $= \underline{\underline{3-6i}}$	1	
iii) The vector $\overrightarrow{CP}$ as shown represents the complex number M. P is the point $(2, 5\tfrac{1}{2})$ $\therefore \overrightarrow{CP}\ (=\overrightarrow{OP}-\overrightarrow{OC})$ represents $(2-4)+i(5\tfrac{1}{2}-{}^-1)$ $\therefore M = \underline{\underline{-2+\dfrac{13i}{2}}}$	1	Many people mixed up M with the midpoint of AB. Although they got the length right (with correct working)
Hence median length $= \lvert M\rvert$ $= \sqrt{4+\dfrac{169}{4}} = \underline{\underline{\dfrac{\sqrt{185}}{2}}}$ units	1	
b) i) $P(2i) = -64+4(16)+16(-4)+64$ $= \underline{0}$	1	
ii) $(z-2i)$ is a factor by Remainder/Factor theorem. Since coefficients are real, complex roots come in conjugate pairs		

MATHEMATICS Extension 1 : Question 4

Suggested Solutions	Marks	Marker's Comments
$\therefore (z+2i)$ is also a factor.		½ mark for reason
$\therefore (z-2i)(z+2i) = \underline{\underline{(z^2+4)}}$ is a real quadratic factor.	1	
iii) $P(z) = (z^2+4)(z^4+16)$ by long division or inspection.	½	Only ½ mark for this which is where many people stopped.
Method 1 $\quad (z^4+16) = (z^4+8z^2+16) - 8z^2$		
$= (z^2+4)^2 - (2\sqrt{2}z)^2$		
$= (z^2+4-2\sqrt{2}z)(z^2+4+2\sqrt{2}z)$		
$\therefore P(z) = \underline{\underline{(z^2+4)(z^2-2\sqrt{2}z+4)(z^2+2\sqrt{2}z+4)}}$	2½	Method 1 is clearly quicker but method 2 is standard book-work and should have been accessible to all.
Method 2 $\quad$ Solve $z^4 = -16$		
$= 2^4(\text{cis } \pi + 2n\pi) \quad n \in \mathbb{Z}$		
$\therefore z = 2\text{ cis}\left(\frac{\pi}{4} + \frac{n\pi}{2}\right)$ (de Moivre)		
$= 2\text{ cis}\left(\frac{-3\pi}{4}\right), 2\text{ cis}\left(\frac{-\pi}{4}\right), 2\text{ cis}\left(\frac{\pi}{4}\right), 2\text{ cis}\left(\frac{3\pi}{4}\right)$		
Combine pairs of linear factors		
$\left(z - 2\text{ cis}\frac{\pi}{4}\right)\left(z-2\text{ cis}\left(\frac{-\pi}{4}\right)\right) = z^2 - 4\cos\frac{\pi}{4}z + 4$		
$= z^2 - 2\sqrt{2}z + 4$		
Similarly $\left(z-2\text{ cis}\frac{3\pi}{4}\right)\left(z-2\text{ cis}\left(-\frac{3\pi}{4}\right)\right) = z^2+2\sqrt{2}z+4$		
$\therefore P(z) = \underline{\underline{(z^2+4)(z^2-2\sqrt{2}z+4)(z^2+2\sqrt{2}z+4)}}$	2½	
c) Let $z = x+iy \quad x, y \in \mathbb{R}$		
$\|z-3\| = 2\|z+3\| \quad (z \neq 3)$		
$\|(x-3)+iy\| = 2\|(x+3)+iy\|$		
$\therefore (x-3)^2 + y^2 = 4\{(x+3)^2+y^2\}$		

Suggested Solutions	Marks	Marker's Comments
c) (continued)		
$x^2-6x+9+y^2=4x^2+24x+36+4y^2$		Most people got this. Some small errors.
$3x^2+3y^2+30x+27=0$		(½ off if add/sub)
$x^2+10x+y^2=-9$		(1 off if $2^2=2$!!)
$(x+5)^2+y^2=16$	2	
This is a circle, centre $(-5,0)$, radius 4	1	
d) (Original question invalid)		