QUESTION 1 (15 Marks)

a) Let z=5-i and *w*= 3+2i, Express the following in the form a+ib where a,b are real numbers.

(i) $z + \bar{w}$ 1

(ii) z +i*w* 1

b) Express $\frac{(1+3i)^2}{3+i}$ in the form a+ib where a and b are real numbers 2

c) (i) Find the square roots of 16-30i. Give your answers in the form a+ib 2

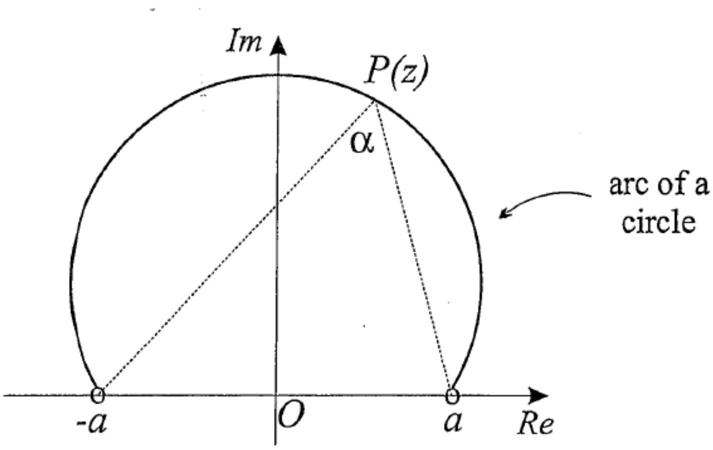
(ii) Hence solve the equation $z^2 - (3-i)z - 2 + 6i = 0$ 2

d) (i) Express $z = -3\sqrt{3} + 3i$ in mod-arg form 2

(ii) Hence find the smallest positive integer n such that z^n is real 1

e) Let P(z)= $z^3 + az^2 + bz + c$ where a,b and c are real.
Two of the roots of P(z)=0 are -2 and (-3+2i). Find the value of a,b and c. 3

f)



In the diagram above,the locus of the point P representing the complex number z is graphed. Write down a possible equation in terms of z,b and α for the locus of P. Note that constants b and α are real.

1

QUESTION 2 (15 marks)

(a) On an Argand diagram sketch the locus of z satisfying

(i) $|z| = |z - 6 - 3i|$ 2

(ii) $\frac{1}{2}(z + \bar{z}) = |z| - 2$ 3

(b) The complex number z lies on the locus $\arg(z+i) = \frac{\pi}{4}$.

(i) Sketch the locus, showing any intercepts with the axes. 2

(ii) Find the least value of $|z|$ 2

(c) Let $z_1 = \frac{\sqrt{6} - i\sqrt{2}}{2}$ and $z_2 = 1 - i$

(i) Write z_1 and z_2 in mod–arg form 3

(ii) Show that $\frac{z_1}{z_2} = \cos\frac{\pi}{12} + i\sin\frac{\pi}{12}$ 1

(iii) Find the value of $\frac{z_1}{z_2}$ in the form a+ib where a and b are in surd form.

Hence or otherwise find the exact surdic expression for $\cos\frac{\pi}{12}$ 2

QUESTION 3 (15 Marks)

(a)

(i) Expand and simplify $(\cos\theta + i\sin\theta)^5$

2

(ii) Hence prove that $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ 2

(iii) Deduce that $\cos\frac{\pi}{10}\cos\frac{3\pi}{10}\cos\frac{7\pi}{10}\cos\frac{9\pi}{10} = \frac{5}{16}$ 2

(b) (i) Use De Moivres' Theorem to prove that,if $2\cos\theta = x + \frac{1}{x}$ then

$2\cos n\theta = x^n + \frac{1}{x^n}$ 1

(ii) Hence or otherwise ,solve the equation

$5x^4 - 11x^3 + 16x^2 - 11x + 5 = 0$ 4

(c) (i) Given that ω is one of the non-real roots of $z^3 = 1$ show that $1 + \omega + \omega^2 = 0$ 1

(ii) Using (i), or otherwise,show that

$$\left(\frac{\omega}{1+\omega}\right)^k + \left(\frac{\omega^2}{1+\omega^2}\right)^k = (-1)^k 2\cos\frac{2}{3}k\pi \text{ where } k \in Z$$

3

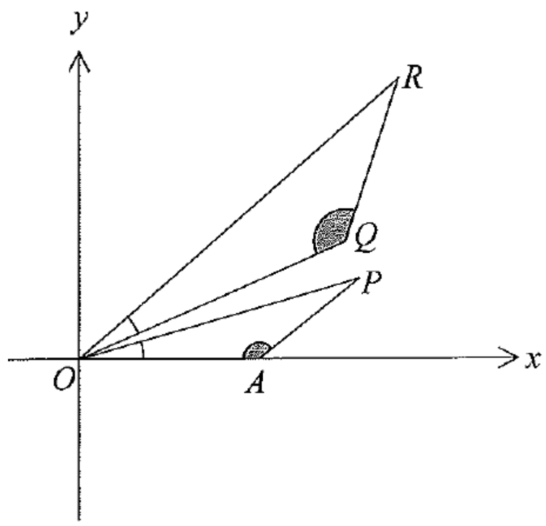
QUESTION 4 (15 Marks)

(a) Let z be a complex number of modulus 3 and *w* be a complex number of modulus 1

Show that $|z-\omega|^2 = 10 - (z\bar{w} + \bar{z}\omega)$ 2

(b) In the figure below, the points P,Q and A represent the complex numbers z_1, z_2 and 1 respectively. By construction, $\angle OAP = \angle OQR$ and $\angle AOP = \angle QOR$.

Explain why the point R represents the complex number $z_1 z_2$



3

(c) A and B are two points in an Argand diagram representing the complex numbers $z_1 = -1$ and $z_2 = \cos\theta + i\sin\theta$ respectively where $\frac{\pi}{2} < \theta < \pi$.

C is the point representing the complex number $z_3 = z_1 + z_2$

(i) Sketch the quadrilateral OACB on an Argand digram where O is the point representing the complex 0.
Then mark an angle on the diagram which is equal to θ

3

(ii) Let $z_4 = z_2 - z_1$

(α) Show that $\frac{z_4}{z_3} = i\left(\frac{\sin\theta}{\cos\theta - 1}\right)$, hence find $\arg\left(\frac{z_4}{z_3}\right)$ 4

(β) Using α show that the diagonals of the quadrilateral OACB are perpendicular to each other. 3

MATHEMATICS Extension 2: Question 1

Suggested Solutions	Marks	Marker's Comments
(a) (i) $5-i+3-2i=8-3i$	1	
(ii) $5-i+3i-2=3+2i$	1	
(b) $\frac{(1+3i)^2}{3+i}=\frac{1+6i-9}{3+i}$		
$=\frac{-8+6i}{3+i}\times\frac{3-i}{3-i}$	1	
$=\frac{-24+8i+18i+6}{9+1}$		
$=\frac{-18}{10}+\frac{26i}{10}$		
$=\frac{-9}{5}+\frac{13i}{5}$	1	
(c) (i) $(x+iy)^2=16-30i$ $x,y\in R$		
$x^2-y^2+2xy=16-30i$		
equate coefficients		
$x^2-y^2=16$ ---- (1)		Many methods
$2xy=-30$ ---- (2)		-1 for process
$y=-15/x$ sub into (1)		-1 for answer
$x^2-\frac{225}{x^2}=16$	1	
$x^4-16x^2-225=0$		
$(x^2-25)(x^2+9)=0$		
$x^2=25$ or $x^2=-9$		
$x=\pm5$ no real solution		
$\therefore y=-3$ or 3		1
square roots are $5-3i$, $-5+3i$		
(ii) $z=\frac{(3-i)\pm\sqrt{(3-i)^2-4(1)(-2+6i)}}{2}$		
$=\frac{(3-i)\pm\sqrt{9-6i-1+8-24i}}{2}$		1

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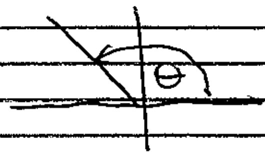
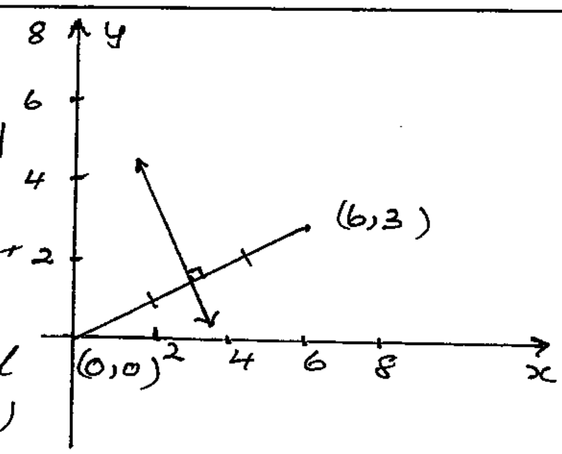
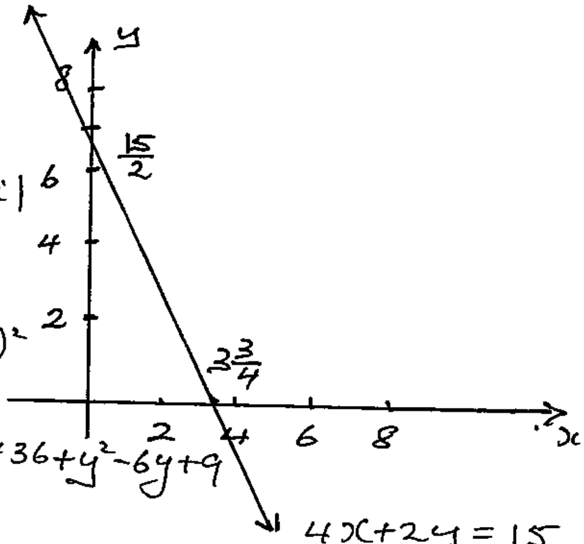
MATHEMATICS Extension 2: Question ..1..... cont

Suggested Solutions	Marks	Marker's Comments
$z=\dfrac{3-i\pm\sqrt{16-30i}}{2}$		
$=\dfrac{3-i\pm(5-3i)}{2}$ from (i)		
$=4-2i$ or $-1+i$	1	
(d) (i)  $r=\sqrt{(3\sqrt{3})^2+3^2}$		
$\therefore r=6$	1	If your final answer is $6\,\text{cis}\,\frac{5\pi}{6}$ maximum is one mark.
$\theta=\dfrac{5\pi}{6}$		
$-3\sqrt{3}+3i=6\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$	1	
(ii) $z^n=6^n\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)^n$		
$=6^n\left(\cos\frac{5\pi n}{6}+i\sin\frac{5\pi n}{6}\right)$ (De Moivres' Theorem)		
to be real then $\sin\left(\frac{5\pi n}{6}\right)=0$		
$\frac{5n\pi}{6}$ must be an integer n is a positive integer		
$\therefore n=6$	1	right or wrong
(e) since the coefficients are all real, the complex roots are conjugate pairs so roots are $-2, -3+2i$ and $-3-2i$	1	
$\therefore (z+2)(z+(3-2i))(z+(3+2i))$		many ways of doing this question
$=(z+2)(z^2+6z+13)$		
$=z^3+8z^2+25z+16$	1	
$\therefore a=8,\ b=25,\ c=26$	1	
(f) $\arg\left(\frac{z-b}{z+b}\right)=\alpha$		right or wrong
or// $\arg(z-b)-\arg(z+b)=\alpha$	1	
or// $\arg(z+b)-\arg(z-b)=-\alpha$		

Q2

a(i) $|z| = |z-6-3i|$ $|z| = |z-(6+3i)|$ Locus is the perpendicular bisector of the interval joining (0,0) and (6,3)  OR $|x+iy|$ $= |x+iy-6-3i|$ $\sqrt{x^2+y^2}$ $= \sqrt{(x-6)^2+(y-3)^2}$ $x^2+y^2 = x^2-12x+36+y^2-6y+9$ $12x+6y = 45$ $4x+2y = 15$ x intercept $3\frac{3}{4}$ y intercept $7\frac{1}{2}$ 		2 Marks for correct diagram showing the geometrical significance of the locus 2 Marks for the correct equation of the locus and a correct diagram showing the x and y intercepts 1 mark for the correct equation 1 mark for a geometrical interpretation of the locus on a diagram with only one of a) perpendicular OR b) bisector shown Note diagram drawn to scale

a)

(ii)

$$\frac{1}{2}(z+\bar{z}) = |z|-2$$

$$\frac{1}{2}(x+iy+x-iy) = \sqrt{x^2+y^2}-2$$

$$x = \sqrt{x^2+y^2}-2$$

$$x+2 = \sqrt{x^2+y^2}$$

$$(x+2)^2 = x^2+y^2$$

$$x^2+4x+4 = x^2+y^2$$

$$y^2 = 4x+4$$

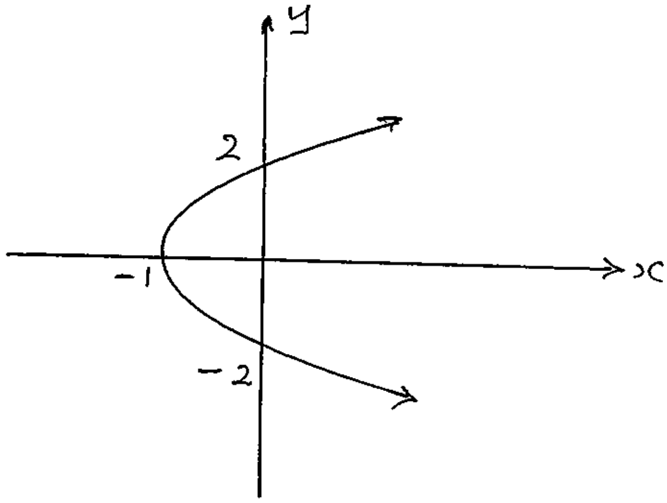
$$y^2 = 4(x+1)$$

Parabola

Vertex at $(-1,0)$

Axis of symmetry is the x axis

intercepts on y axis of ± 2



3 Marks for correct Cartesian equation of the locus with a sketch showing the intercepts on the axes

2 Marks for a correct equation

1 Mark for an incorrect equation resulting from $\frac{1}{2}(x+iy+x-iy) = x^2+y^2-2$

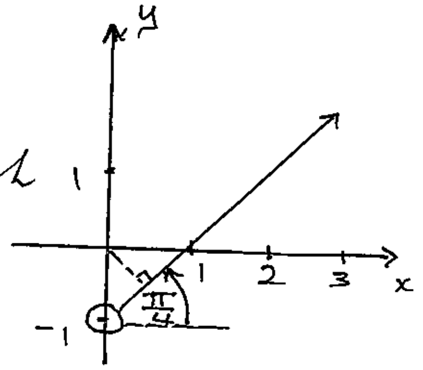
Some Candidates who wrote $y = \pm 2\sqrt{x+1}$ only drew the top half of the parabola.

b)

Equation of the line through $(0,-1)$ and $(1,0)$

is $\frac{x}{1}+\frac{y}{-1}=1$

i.e. $x-y-1=0$



least $|z| = \left|\frac{0-0-1}{\sqrt{1^2+(-1)^2}}\right|$

$= \frac{1}{\sqrt{2}}$

OR

$AB^2 = 1^2+1^2$

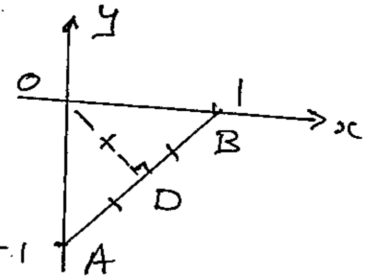
$= 2$

$AB = \sqrt{2}$

$OD = \frac{1}{2}AB$

$= \frac{1}{2}\times\sqrt{2}$

$= \frac{1}{\sqrt{2}}$



$\therefore$ least $|z| = \frac{1}{\sqrt{2}}$

2

2 Marks for correct sketch showing open circle at $(0,-1)$ and a ray through $(1,0)$

1 Mark for correct sketch without arrow on ray

1 Mark for correct sketch without open circle

Ray must be drawn using a ruler

Least value of $|z|$ was found using perpendicular distance from $(0,0)$ to $x-y-1=0$

OR

Using Pythagoras Theorem

c) (i) $\lvert z_1\rvert = \sqrt{\left(\frac{\sqrt6}{2}\right)^2+\left(\frac{-\sqrt2}{2}\right)^2}$ $\quad \lvert z_2\rvert=\sqrt{1^2+(-1)^2}$ $= \sqrt{\frac{6}{4}+\frac{2}{4}}$ $\quad = \sqrt2$ $= \sqrt2$ $\operatorname{Arg} z_1 = \tan^{-1}\left(-\frac{\sqrt2}{\sqrt6}\right)$ $\quad \operatorname{Arg} z_2 = \tan^{-1}\left(\frac{-1}{1}\right)$ $= \tan^{-1}\left(-\frac{1}{\sqrt3}\right)$ $\quad = \tan^{-1}(-1)$ $= -\frac{\pi}{6}$ $\quad = -\pi/4$ $z_1 = \sqrt2\left(\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right)$ $z_2 = \sqrt2\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)$	1 Mark for both $\lvert z_1\rvert$ and $\lvert z_2\rvert$ correct 1 Mark for $\operatorname{Arg} z_1$ 1 Mark for $\operatorname{Arg} z_2$ 3 Marks only if both z_1 and z_2 expressed in mod-arg form.
Let $\operatorname{Cis}\theta = \cos\theta + i\sin\theta$ $\frac{z_1}{z_2} = \frac{\sqrt2 \operatorname{Cis}\left(-\frac{\pi}{6}\right)}{\sqrt2 \operatorname{Cis}\left(-\frac{\pi}{4}\right)}$ $= \operatorname{Cis}\left(-\frac{\pi}{6}--\frac{\pi}{4}\right)$ $= \operatorname{Cis}\left(-\frac{2\pi}{12}+\frac{3\pi}{12}\right)$ $= \operatorname{Cis}\frac{\pi}{12}$ $= \cos\frac{\pi}{12}+i\sin\frac{\pi}{12}$	1 Mark for $\operatorname{Cis}\left(-\frac{\pi}{6}--\frac{\pi}{4}\right) = \operatorname{Cis}\left(\frac{\pi}{12}\right)$ 0 Marks for $\frac{\operatorname{Cis}-\frac{\pi}{6}}{\operatorname{Cis}-\frac{\pi}{4}} = \operatorname{Cis}\frac{\pi}{12}$ Must SHOW
$\frac{z_1}{z_2} = \frac{\frac{\sqrt6}{2}-i\frac{\sqrt2}{2}}{1-i}$ $= \frac{\sqrt6-i\sqrt2}{2(1-i)}\times\frac{(1+i)}{(1+i)}$ $= \frac{\sqrt6+\sqrt6 i-i\sqrt2-i^2\sqrt2}{2(1-i^2)}$ $= \frac{\sqrt6+\sqrt2}{4}+i\,\frac{\sqrt6-\sqrt2}{4}$ $= \cos\frac{\pi}{12}+i\sin\frac{\pi}{12}$ Equating Real Parts $\cos\frac{\pi}{12} = \frac{\sqrt6+\sqrt2}{4}$	Otherwise Approach $\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$ OR $\cos 2A = 2\cos^2 A - 1$ $\cos\frac{\pi}{6} = 2\cos^2\frac{\pi}{12}-1$

Suggested Solutions	Marks	Marker's Comments
a) i) Using Binomial Expansion: $(\cos\theta + i\sin\theta)^5 = \cos^5\theta + 5i\cos^4\theta\sin\theta + 10i^2\cos^3\theta\sin^2\theta + 10i^3\cos^2\theta\sin^3\theta + 5i^4\cos\theta\sin^4\theta + i^5\sin^5\theta$		
$= \underline{(\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta) + i(5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta)}$	1	
Also, by de Moivre's Theorem $(\cos\theta + i\sin\theta)^5 = \underline{\cos 5\theta + i\sin 5\theta}$	1	Words (de Moivre) necessary for mark.
ii) Equating real parts of each expansion $\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$	1	for this far and the wording
$= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1-\cos^2\theta)^2$		
$= \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta + 5\cos\theta - 10\cos^3\theta + 5\cos^5\theta$		
$= \underline{16\cos^5\theta - 20\cos^3\theta + 5\cos\theta}$	1	
iii) The 5 zeroes of this expression will be 5 distinct zeroes of $\cos 5\theta = 0$	1	
$\cos\theta$ when $5\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}$		
i.e. $\cos\theta$ when $\theta = \pi/10, 3\pi/10, \pi/2, 7\pi/10, 9\pi/10$		
Factoring out the zero $\cos\theta = 0$ leaves a polynomial of degree 4 with zeros $\cos\pi/10, \cos 3\pi/10, \cos 7\pi/10, \cos\frac{9\pi}{10}$		

Suggested Solutions	Marks	Marker's Comments

The product of the roots = "e/a" = $5/16$

$$\therefore \cos\frac{\pi}{10}\cos\frac{3\pi}{10}\cos\frac{7\pi}{10}\cos\frac{9\pi}{10} = 5/16$$

1

[An alternative is to consider the sum of roots taken 4 at a time from the degree 5 equation. As the root 0 occurs in all but one of the products, we arrive at the same result above.]

b) i) $2\cos\theta = x + \frac{1}{x}$

$$x^2 - 2x\cos\theta + 1 = 0$$

$$\therefore x = \frac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2}$$

$$\therefore x = \cos\theta \pm i\sin\theta$$

$$= \cos(\pm\theta) + i\sin(\pm\theta)$$

$$\therefore x^n = \cos(\pm n\theta) + i\sin(\pm n\theta) \quad \text{(de Moivre)}$$

$$x^{-n} = \cos(\mp n\theta) + i\sin(\mp n\theta) \quad (\text{" "})$$

$$x^n + \frac{1}{x^n} = 2\cos n\theta \quad \text{(Adding above lines)}$$

1

(Noting that $\cos(-x) = \cos x$, $\sin(-x) = -\sin x$)

Marker's comment: It was allowed (although not strictly correct) if the statement $x = \cos\theta + i\sin\theta$ was made without proof.

ii) Divide by x^2 $(x \neq 0)$

$$5\left(x^2 + \frac{1}{x^2}\right) - 11\left(x + \frac{1}{x}\right) + 16 = 0$$

1

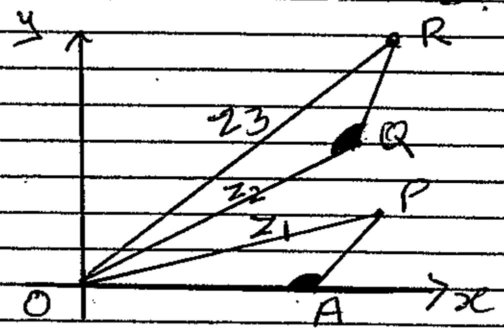
If $x = \cos\theta + i\sin\theta$, using part (i);

$$10\cos 2\theta - 22\cos\theta + 16 = 0$$

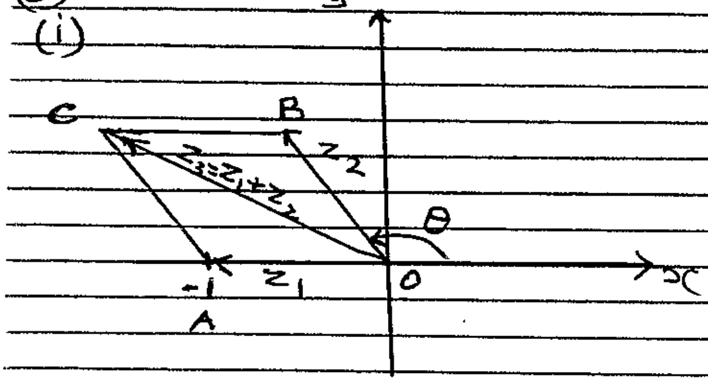
Suggested Solutions	Marks	Marker's Comments
Putting $\cos 2\theta = 2\cos^2\theta - 1$, $20\cos^2\theta - 22\cos\theta + 6 = 0$ $10\cos^2\theta - 11\cos\theta + 3 = 0$ $(5\cos\theta - 3)(2\cos\theta - 1) = 0$ $\cos\theta = 3/5$ or $1/2$ $\therefore \sin\theta = \pm 4/5$ or $\pm\sqrt{3}/2$ $x = \cos\theta + i\sin\theta$ $= \dfrac{3 \pm 4i}{5}$ or $\dfrac{1 \pm i\sqrt{3}}{2}$ [An alternative is to modify the original equation to $5\left(x+\frac{1}{x}\right)^2 - 11\left(x+\frac{1}{x}\right) + 6 = 0$ and solve for $\left(x+\frac{1}{x}\right)$. It is then necessary to solve a quadratic equation for each root, giving the same 4 roots of x.]	1 1	
c) i) If ω is a root of $z^3 = 1$, then $\omega^3 = 1$ i.e. $\omega^3 - 1 = 0$ $\therefore (\omega - 1)(\omega^2 + \omega + 1) = 0$ But $\omega \neq 1$ since ω is non real $\therefore \omega^2 + \omega + 1 = 0$	1	There was a lot of untidiness with z. Substitute ω early as shown here.

Suggested Solutions	Marks	Marker's Comments
c) i) (Alternative). If ω is a non real root, then ω^2 is also a root. $(\omega^2)^3 = (\omega^3)^2 = 1$. It is a different root: $\omega^2 - \omega = \omega(\omega - 1) \neq 0$. And 1 is clearly a real root. Sum of the roots $\underline{\underline{1 + \omega + \omega^2 = \text{"}-b/a\text{"} = 0}}$	1	
ii) $\therefore 1 + \omega = -\omega^2$ $\quad 1 + \omega^2 = -\omega$; $\therefore \frac{\omega}{1+\omega} = \frac{\omega}{-\omega^2} = -\frac{1}{\omega}$ $\quad \frac{\omega^2}{1+\omega^2} = \frac{\omega^2}{-\omega} = -\omega$; $\therefore \left(\frac{\omega}{1+\omega}\right)^k + \left(\frac{\omega^2}{1+\omega^2}\right)^k = \left(-\frac{1}{\omega}\right)^k + (-\omega)^k$ $\underline{\underline{= (-1)^k\left(\frac{1}{\omega^k} + \omega^k\right)}}$	1	
But from (b) i) since $\omega = \text{cis}\,\frac{2\pi}{3}$, $\omega + \frac{1}{\omega} = 2\cos\frac{2\pi}{3}$; $\therefore \omega^k + \frac{1}{\omega^k} = 2\cos\frac{2k\pi}{3}$	1	There were a lot of errors and poor explanations occuring when people solved $z^3 = 1$ and used k as a parameter.
$\therefore \left(\frac{\omega}{1+\omega}\right)^k + \left(\frac{\omega^2}{1+\omega^2}\right)^k \underline{\underline{= (-1)^k\, 2\cos\frac{2k\pi}{3}}}$	1	
[If $\omega = \text{cis}\,\frac{4\pi}{3}$ had been used, the same result would occur since $\cos 4k\pi/3 = \cos 2k\pi/3$ for any $k \in \mathbb{Z}$]		This was to be avoided at all costs.

MATHEMATICS Extension 2 : Question 4

Suggested Solutions	Marks	Marker's Comments
a) $\lvert z\rvert = 3 \quad \lvert w\rvert = 1$ $\lvert z-w\rvert^2 = (z-w)\overline{(z-w)}$	1	Accepted many methods involving manipulations of LHS and RHS.
$= (z-w)(\bar{z}-\bar{w})$ $= z\bar{z} - z\bar{w} - w\bar{z} + w\bar{w}$ $= \lvert z\rvert^2 + \lvert w\rvert^2 - (z\bar{w} + w\bar{z})$ $= 9 + 1 - (z\bar{w} + w\bar{z})$ $= 10 - (z\bar{w} + w\bar{z})$	1	Did not accept "special cases"
(b) Let R represent complex number z_3.  In Δ ORQ and Δ OPA $\angle OAP = \angle OQR$ (Given) $\angle POA = \angle ROQ$ (Given) $\therefore \Delta ORQ \;\vert\vert\vert\; \Delta OAP$ (equiangular)	1	This was a "show" question Must give complete proof for similar triangles ~~with~~ reasons
$\therefore \frac{OA}{OQ} = \frac{OP}{OR}$ corresponding sides in similar triangles are in the same ratio $\frac{1}{\lvert z_2\rvert} = \frac{\lvert z_1\rvert}{\lvert z_3\rvert}$ $\lvert z_3\rvert = \lvert z_1\rvert\lvert z_2\rvert$ $= \lvert z_1 \times z_2\rvert$	1	ratio of moduli with full reason
$\arg z_1 + \arg z_2 = \angle AOP + \angle AOQ$ $= \angle AOP + \angle AOP + \angle POQ$ $= \angle AOR$ $= \arg z_3$	1	Must show args in terms of angles.
$\lvert z_3\rvert = \lvert z_1 \times z_2\rvert$ $\text{Arg } z_3 = \arg z_1 + \arg z_2$ $= \arg(z_1 \cdot z_2)$ $\therefore z_3 = z_1 \cdot z_2$		No loss if conclusion not written

MATHEMATICS Extension 2 : Question 4

Suggested Solutions	Marks	Marker's Comments
(c) (i)	①	θ is obtuse
	①	$\|z_1\| = \|z_2\| = 1$
	①	Must show points A, B, C correctly.
(ii) (α) $z_4 = (\cos\theta + 1) + i\sin\theta$ $z_3 = -1 + \cos\theta + i\sin\theta$ $= (\cos\theta - 1) + i\sin\theta$ $\dfrac{z_4}{z_3} = \dfrac{(\cos\theta+1) + i\sin\theta}{(\cos\theta-1) + i\sin\theta} \times \dfrac{(\cos\theta - 1 - i\sin\theta)}{(\cos\theta - 1 - i\sin\theta)}$	①	realizing $\dfrac{z_4}{z_3}$ unsimplified
$= \dfrac{\cos^2\theta - 1 - i\sin\theta(\cos\theta+1) + i\sin\theta(\cos\theta-1) + \sin^2\theta}{(\cos\theta-1)^2 + \sin^2\theta}$ $= \dfrac{\cancel{1} - \cancel{1} - i\sin\theta\cos\theta - i\sin\theta + i\sin\theta\cos\theta - i\sin\theta}{\cos^2\theta - 2\cos\theta + 1 + \sin^2\theta}$		
$= \dfrac{-2i\sin\theta}{2 - 2\cos\theta}$ $= \dfrac{i\sin\theta}{\cos\theta - 1}$	①	correctly simplifying
$\dfrac{z_4}{z_3}$ is purely imaginery	①	purely imaginery
$\frac{\pi}{2} < \theta < \pi$ (2nd Quadrant) $\therefore \sin\theta > 0$ $\cos\theta - 1 < 0$ $\therefore \dfrac{\sin\theta}{\cos\theta - 1} < 0$	①	show for $\pi/2 < \theta < \pi$ $\dfrac{\sin\theta}{\cos\theta - 1} < 0$
$\therefore \arg\left[\dfrac{z_4}{z_3}\right] = -\pi/2$	①	$\pi/2$.
(β) $\arg\left[\dfrac{z_4}{z_3}\right] = -\pi/2$ $\arg z_4 - \arg z_3 = -\pi/2$ $\arg z_3 = \arg z_4 + \pi/2$ $\therefore \frac{\pi}{2}$ is angle between z_3 and z_4	①	difference in args is $\pi/2$
$z_3 = OC$ and $z_4 = AB$ are diagonals of OACB $\therefore$ diagonals are perpendicular	①	z_3, z_4 are diagonals.

OTHER METHODS POSSIBLE BUT MUST REFER to part (α)