Question 1 *(20 marks)* **Marks**

(a) Sketch the region represented in the complex plane by **3**
$|z-1|<3$ and $0 \le \operatorname{Arg}(z-1) \le \frac{\pi}{4}$.

(b) Use De Moivre's theorem to solve the equation $z^5 = 1$. Show that the points representing the five roots of this equation on an Argand diagram form the vertices of a regular pentagon of area $\frac{5}{2}\sin\frac{2\pi}{5}$ and perimeter $10\sin\frac{\pi}{5}$. **5**

(c) If $z_1 = 24 + 7i$ and $|z_2| = 6$, find the greatest and least values of $|z_1 + z_2|$. **4**

(d) (i) Express $-1 + i\sqrt{3}$ in mod – arg form. **2**

(ii) Hence evaluate $\left(-1 + i\sqrt{3}\right)^6$. **2**

(e) Find the equation of the locus of z in the complex number plane such that $|z-5| = 2|z+1|$. Describe the locus geometrically. **4**

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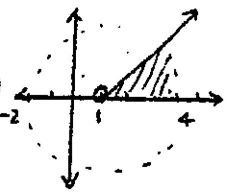
Ext 2 Assess 1 '05

Question 1

a) 

1 – dotted circle
1 – correct sector
1 – (1,0) excluded

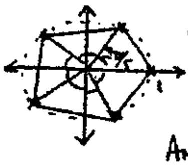
b) $z^5 = 1$

$|z| = 1$

$\arg z = \frac{2\pi k}{5}$ where $k = 0, 1, 2, 3, 4$.

$= 0, \pm\frac{2\pi}{5}, \pm\frac{4\pi}{5}$

$z = \text{cis}\left(\pm\frac{2\pi}{5}\right), \text{cis}\left(\pm\frac{4\pi}{5}\right), 1.$

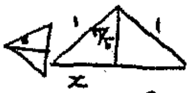


Pentagon vertices subtend an angle of $\frac{2\pi}{5}$ at centre.

Area of one sector $= \frac{1}{2}r^2\sin\theta$

$\therefore$ Area $= 5 \times \frac{1}{2} \times 1^2 \sin\frac{2\pi}{5}$

$= \frac{5}{2}\sin\frac{2\pi}{5}$



$x = \sin\pi/5$

$\therefore$ Perimeter $= 10\sin\pi/5$.

1 – use of De Moivre's
1 – correct roots.
1 – diagram of pentagon.
1 – correct derivation of area.
1 – correct derivation of perimeter.

c) $|z_1| = 25$

$|z_1 + z_2| \leq |z_1| + |z_2| = 31$

(Greatest if collinear)

Least if $z_1 + z_2$ are colliner in opp. directions.

$|z_1 + z_2| \geq \big||z_1| - |z_2|\big| = 19$

$\therefore$ Greatest $= 31$, Least $= 19$.

1 – calculates $|z_1|$
1 – adds modulus to find greatest
1 – subtracts to find least (± 19)
1 – takes abs value to obtain 19.

d) i, $|z| = 2$ $\quad \arg z = \tan^{-1}(-\sqrt{3}) = \frac{2\pi}{3}$ 

$\therefore 2\,\text{cis}\frac{2\pi}{3}$

1 – correct modulus
1 – correct angle in correct quadrant.

ii/ $(-1 + i\sqrt{3})^6 = 2^6\,\text{cis}\left(\frac{2\pi}{3} \times 6\right)$

$= 64\,\text{cis}\,4\pi$

$= 64$

1 – use of De Moivre's
1 – complete simplification to 64.

e) Let $z = x + iy$

$|x + iy - 5| = 2|x + iy + 1|$

$\sqrt{(x-5)^2 + y^2} = 2\sqrt{(x+1)^2 + y^2}$

$x^2 - 10x + 25 + y^2 = 4(x^2 + 2x + 1 + y^2)$

$x^2 - 10x + 25 + y^2 = 4x^2 + 8x + 4 + 4y^2$

$3x^2 + 18x + 3y^2 - 21 = 0$

$x^2 + 6x + y^2 - 7 = 0$

$(x+3)^2 + y^2 = 16$

Circle centre $(-3, 0)$

radius 4.

1 - substitutes $z = x + iy$ or similar

1 - simplified equation.

1 - recognition of circle form

1 - centre + radius.