
Student Number

YEAR 12

EXTENSION 2 MATHEMATICS

ASSESSABLE TASK 1
TERM IV 2006

GRAPHS AND COMPLEX NUMBERS

Time Allowed: 60 minutes

ARB.LJF
20.11.06

General Instructions

- Write using black or blue pen.
- Board approved calculators may be used.
- All *necessary* working should be shown in every question if full marks are to be awarded.
- Marks may **NOT** be awarded for messy or badly arranged work.

Marks

1. Sketch the graphs of the following functions:

(a) $y = 1 - x + \dfrac{1}{1 + x}$ **2**

(b) $y = |x - 1| + |x - 2|$ **2**

(c) $y = \log_3 (2 - x)$ **2**

(d) $y = \dfrac{x^2 + 6x}{x - 2}$ **2**

(e) $y = \dfrac{x^3}{x^2 - 1}$ **2**

2. (a) Solve the equation

$2z^2 + 3iz - 1 = 0$ **2**

(b) Simplify the following by expressing in the form $x + iy$, where x and y are real:

(i) $\dfrac{5 - i}{2 + 3i}$ **2**

(ii) $\dfrac{1}{1 + \cos\theta - i\sin\theta}$ **2**

(c) If $3 + 4i$ is a root of $x^3 - 5x^2 + 19x + 25 = 0$, find the other roots. **2**

Marks

(d) If $z = x + iy$, sketch and describe the locus of z if $2|z| = z + \bar{z} + 4$ **2**

3. (a) Given $z = (1 + i\sqrt{3})(1 + i)$ **4**

(i) Express z in the form $x + iy$ where x and y are real

(ii) By expressing $1 + i\sqrt{3}$ and $1 + i$ in mod-arg form first, show that

$z = \sqrt{8}cis\left(\frac{7\pi}{12}\right)$

(iii) Hence find the exact values of $\cos\frac{7\pi}{12}$ and $\sin\frac{7\pi}{12}$

(b) $z_1 = 4 - i$, $z_2 = 2i$ and z_3 form the vertices of an isosceles right-angled triangle whose right angle is at z_3. Find z_3. **2**

(c) Illustrate on the Argand diagram the region

$0 \le Arg(z + 4) \le \frac{2\pi}{3}$ and $|z + 4| \le 4$ **2**

(d) Find the locus of z if

$Arg\left(\frac{z - i}{z + 1}\right) = \frac{\pi}{2}$ **2**

4. (a) Simplify the following: **4**

(i) $(1 - i\sqrt{3})^7(2 - 2i)^4$

(ii) $\frac{(\sqrt{3} + i)^{10}}{(2 - 2i)^8}$

Marks

(b) (i) Find the three cubic roots of 1. **6**

(ii) If w is the non-real cubic root of 1 of the smallest positive argument, show that $1 + w + w^2 = 0$.

(iii) Show that

$$(1 - w)(1 - w^2)(1 - w^4)(1 - w^5) = 9$$

END OF PAPER