Sydney Girls High School

2012
MATHEMATICS EXTENSION 2
[November 2011]

YEAR 12
ASSESSMENT TASK 1

Time Allowed: 60 minutes + 5 minutes reading time.

Name: _____ **Teacher:** _____

Topics: Uniform Circular Motion, Curve Sketching.

General Instructions:

- There are EIGHT (8) Questions which are not of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work or incomplete working.
- Start each question on a new page.
- Write on one side of the paper only.
- Diagrams are NOT to scale.
- Board-approved calculators may be used.
- Write your student number clearly at the top of each question and clearly number each question.
- Use $g = 10ms^{-2}$ in all questions.

Total: 50 marks

Marks

Question 1 (6 marks)

Given $f(x) = x^2 - 1$ draw sketch graphs of the following:

(a) $y = f(x)$ 1

(b) $y = -f(x)$ 1

(c) $y = \dfrac{1}{f(x)}$ 2

(d) $y^2 = f(x)$ 2

Question 2 (8 marks)

Sketch the following showing any intercepts or asymptotes:

(a) $9y^2 + 25x^2 = 1$ 2

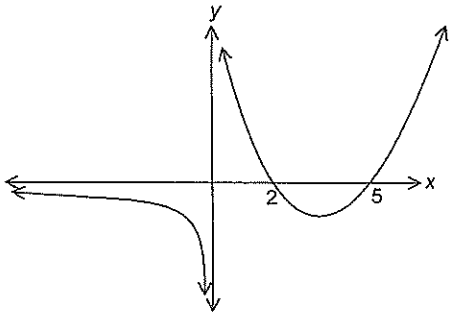
(b) $(x+3)(y+2) = 4$ 2

(c) $x^2 - 9y^2 = 1$ 2

(d) $y = \dfrac{1}{(x-2)(x+1)^2}$ 2

Marks

Question 3 (10 marks)



The diagram is a sketch of the function $y = f(x)$ which cuts the x axis at $x = 2$ and $x = 5$.

The graph has asymptotes $x = 0$ and $y = 0$. On separate diagrams sketch:

(a) $y = |f(x)|$ 1

(b) $y = f(|x|)$ 1

(c) $y = [f(x)]^2$ 2

(d) $2^{f(x)}$ 2

(e) $y = f(x^2)$ 2

(f) $y = \sqrt{f(-x)}$ 2

Question 4 (3 marks)

A girl ties a stone of mass 0.3 kg to the end of a piece of string, then swings it in a circle of radius 1.5m. If the tension in the string is 3 N, calculate:

(a) the speed of the stone 1

(b) the frequency of its motion 1

(c) its angular velocity 1

[2]

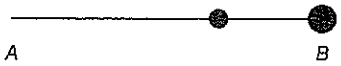
Marks

Question 5 (4 marks)

A 5 metre piece of string AB has a mass of 4kg attached at point B. The string is then rotated about A in a horizontal circle. If the string breaks when the speed of rotation reaches 15 rad/sec,

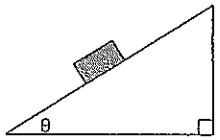
(a) Find the breaking strain of the string 1

(b) If the 4kg mass at B is replaced by a 3kg mass at the same position, and a 1kg mass 2 metres from A, find the new maximum speed of rotation (in rad/sec) 3



Question 6 (7 marks)

A railway track goes around a curve of radius 500m. The tracked is inclined at $\theta°$ to the horizontal. The forces acting on a train taking the curve are the weight force mg, a sideways frictional force F and the normal reaction force.



(a) Draw a diagram of the track showing all forces acting on a train taking the corner. 1

(b) By resolving forces vertically and horizontally derive an expression for F that is independent of N. 3

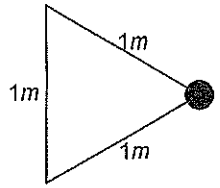
(c) At what angle should the track be banked for a designed train speed of $20 ms^{-1}$? 2

[3]

Marks

Question 7 (7 marks)

A 1kg ball is attached to a rigid vertical rod by means of two strings each one metre long. The strings are attached to the rod one metre apart. The system is rotating about the rod at ω radians per second. Both strings are taught and form an equilateral triangle with the rod.

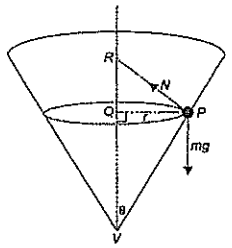


(a) What is the tension in the lower string (in terms of ω)? 4

(b) What is the speed(v) of the ball, if the ratio of the tension in the upper string to the tension in the lower string is 3:1? 3

Question 8 (6 marks)

A smooth hollow cone of semi vertex angle θ is placed with axis vertical and vertex V downwards. A particle P moves in a circle on its inner surface making x revolutions per second.



(a) Show that the distance (r) of the particle from the axis of the cone at any time is: $\frac{g\cot\theta}{4\pi^2x^2}$. 2

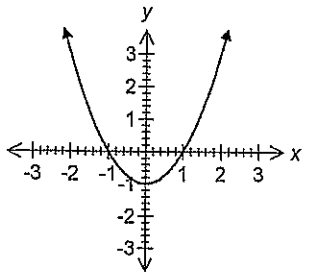
(b) If $\theta = 30°$ and $x = 1$ find VQ. 2

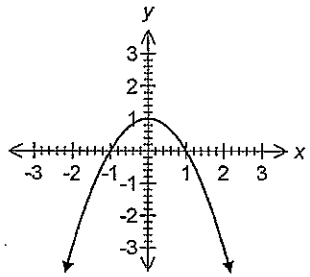
(c) If the linear velocity is to be doubled, find the minimum height of the cone 2

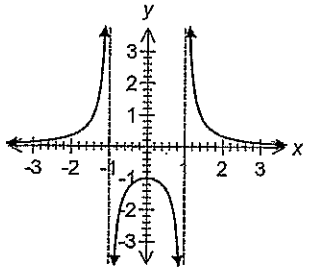
End of paper.

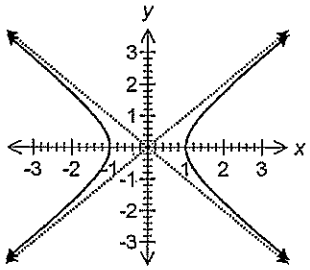
Extension 2 Mathematics Task 1 2012 Solutions

Question 1

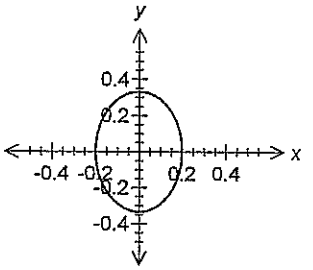


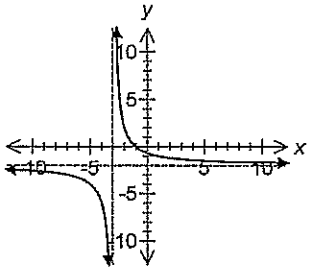


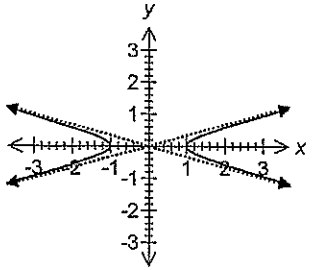


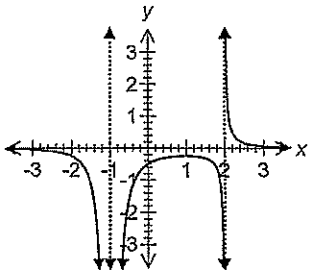


Question 2



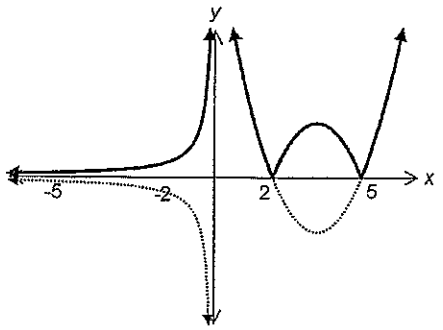




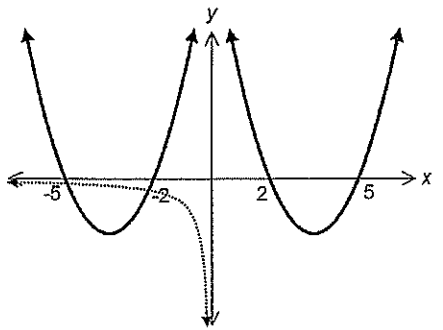


Question 3

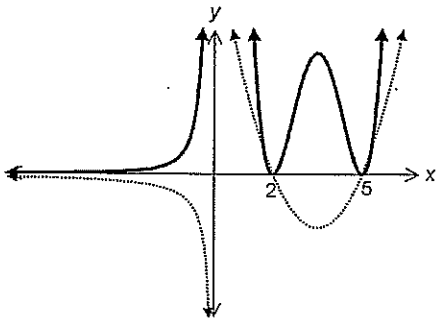
a.



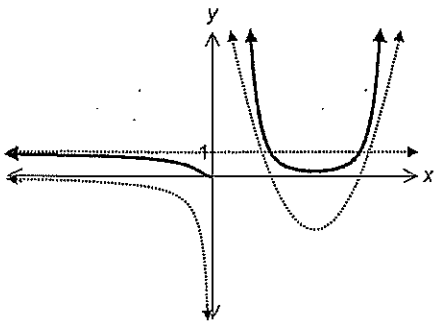
b.



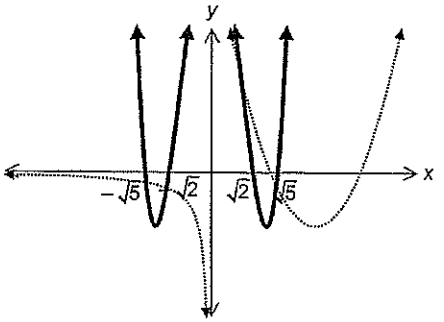
c.



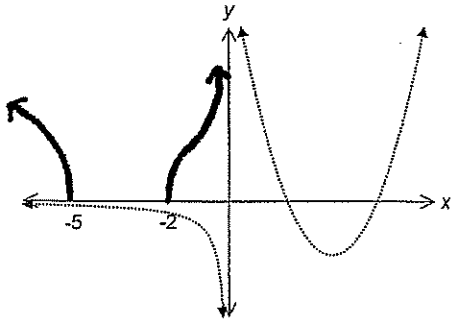
d.



e.



f.



Question 4

a. $T = \frac{mv^2}{r}$

$3 = \frac{0.3 \times v^2}{1.5}$

$v^2 = 15$

$v = \sqrt{15}\text{ms}^{-1}$

b. $v = r\omega$

$\sqrt{15} = 1.5\omega$

$\omega = \frac{\sqrt{15}}{1.5}$

$= 2.58\text{ rad/sec}$

c. $f = \frac{\omega}{2\pi}$

$= \frac{2.58}{2\pi}$

$= 0.41$

Question 5

a. $T = mr\omega^2$

$= 5 \times 3 \times 15^2$

$= 3375N$

b. $T_{BX} = mr\omega^2$

$= 3 \times 3 \times \omega^2$

$= 9\omega^2$

$T_{AX} = 9\omega^2 + 1 \times 2 \times \omega^2$

$= 11\omega^2$

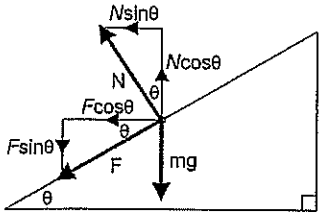
$3375 = 11\omega^2$

$\omega^2 = \frac{3375}{11}$

$\omega = 5.28$

Question 6

a.



b.

Horizontally:

$F\cos\theta + N\sin\theta = \frac{mv^2}{r} \rightarrow (1)$

Vertically:

$N\cos\theta = F\sin\theta + mg$

$N\cos\theta - F\sin\theta = mg \rightarrow (2)$

$(1) \times \cos\theta$

$F\cos^2\theta + N\cos\theta\sin\theta = \frac{mv^2}{r}\cos\theta \rightarrow (3)$

$(2) \times \sin\theta$

$N\sin\theta\cos\theta - F\sin^2\theta = mg\sin\theta \rightarrow (4)$

$(3) - (4)$

$F = \frac{mv^2}{r}\cos\theta - mg\sin\theta \rightarrow (5)$

c.

Substitute:

$F = 0$

$v = 20ms^{-1}$

$r = 500$

$0 = \frac{m \times 20^2}{500}\cos\theta - 10m\sin\theta$

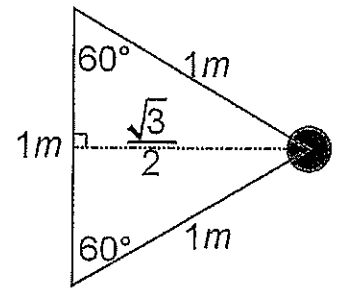
$10m\sin\theta = \frac{m \times 20^2}{500}\cos\theta$

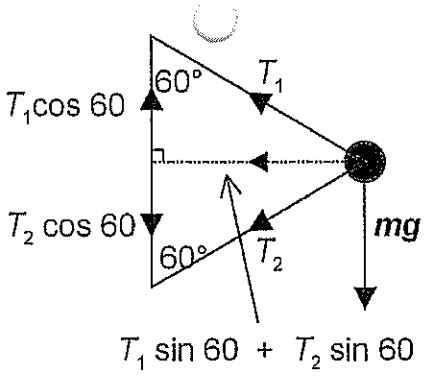
$\tan\theta = \frac{20^2}{500 \times 10}$

$\tan\theta = \frac{2}{25}$

$\theta = 4°34'$

Question 7





a.

Horizontally:

$\sin 60(T_1+T_2)=mr\omega^2 \rightarrow (1)$

Vertically:

$T_1\cos 60=T_2\cos 60+mg$

$\cos 60(T_1-T_2)=mg \rightarrow (2)$

From (1)

$\frac{\sqrt{3}}{2}(T_1+T_2)=1\times\frac{\sqrt{3}}{2}\omega^2$

$T_1+T_2=\omega^2 \rightarrow (3)$

From (2)

$\frac{1}{2}(T_1-T_2)=10$

$T_1-T_2=20 \rightarrow (4)$

(3)−(4)

$2T_2=\omega^2-20$

$T_2=\frac{\omega^2-20}{2}$

b.

$T_1-\frac{\omega^2-20}{2}=20$

$T_1=\frac{20+\omega^2}{2}$

$T_1:T_2=3:1$

$T_1=3T_2$

$\frac{20+\omega^2}{2}=3\left(\frac{\omega^2-20}{2}\right)$

$20+\omega^2=3\omega^2-60$

$2\omega^2=80$

$\omega^2=40$

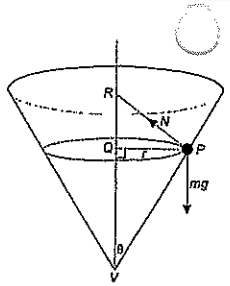
$\omega=2\sqrt{10}$ rad/sec

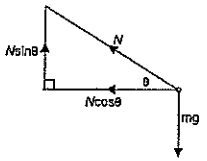
$v=r\omega$

$=\frac{\sqrt{3}}{2}\times 2\sqrt{10}$

$=\sqrt{30}ms^{-1}$

Question 8





a.

Horizontally:

$N\cos\theta=mr\omega^2 \rightarrow (1)$

Vertically:

$N\sin\theta=mg \rightarrow (2)$

$(2)\div(1)$

$\tan\theta=\frac{g}{r\omega^2}$

Substituting:

$\omega=2\pi x$

$\tan\theta=\frac{g}{r4\pi^2x^2}$

$r=\frac{g\cot\theta}{4\pi^2x^2}$

b.

If $\theta=30°$ and $x=1$

$r=\frac{10\cot 30}{4\pi^2}$

$=0.44m$

$\tan 30=\frac{0.44}{h}$

$VQ=\frac{0.44}{\tan 30}$

$=0.76m$

d.

$v=2\times 0.44\times 2\pi$

$=1.76\pi$

$\frac{r}{h}=\frac{rg}{v^2}$

$h=\frac{v^2}{g}$

$=3.06m$