# Sydney Girls High School

## 2014

# MATHEMATICS EXTENSION 2

# HSC Assessment Task 1

# November 2014

**Time Allowed: 60 minutes + 5 minutes reading time**

**Topics: Circular Motion, Curve Sketching**

There are FOUR (4) Questions which are of equal value
Attempt all questions
Show all necessary working. Marks may be deducted for badly arranged work or incomplete working
Start each question on a new page.
Write on one side of the paper only.
Diagrams are NOT to scale.
Board-approved calculators may be used
Use $g = 10\ ms^{-2}$

**Total: 48 marks**

**Student Name:_____ Teacher Name:_____**

**Marks**

**Question 1 (start a new page) (12 marks)**

(a) Sketch the following without using calculus and showing all important features on separate number planes

(i) $\frac{x^2}{9} - \frac{y^2}{4} = 1$ **2**

(ii) $y = (x+4)(x-1)^3$ **2**

(iii) $16x^2 + 9y^2 = 144$ **2**

(b) A particle moves 20m on a circular arc in 5 seconds. If the particles angular velocity is a constant *4* rad $s^{-1}$ find its

(i) Radius of motion 1

(ii) Linear velocity 1

(iii) Acceleration towards the centre of motion 1

(iv) Period 1

(v) Angular distance after 7 seconds 1

(vi) Frequency in revolutions per minute 1

**Question 2 (start a new page) (12 marks)** **Marks**

(a) Sketch $y = f^{-1}(x)$ showing all important features if $f(x) = 2^x + 1$ **2**

(b) $f(x) = \dfrac{x-3}{x^2-4}$

(i) Find the equations of the asymptotes of $y = f(x)$ **2**

(ii) Sketch $y = f(x)$ clearly showing the asymptotes and the intercepts on the $x$ and $y$ axis **2**

(c) A piece of string 0.5m can just sustain a mass of 8kg when it is hung vertically. What is the maximum speed in radians per second that a 0.4 kg mass attached to one end of the string can be rotated in a circle on a smooth horizontal table? **3**

(d) The gravitational force F between two objects of masses M and m is given by

$$F = \frac{GMm}{r^2}$$

where $G = 6.67 \times 10^{-11}$ and $r$ is the distance between the objects. If $M$ represents the mass of the sun $(2 \times 10^{30} kg)$ and $m$ the mass of the earth.

(i) Show that the time for the earth to orbit the sun can be given by **1**

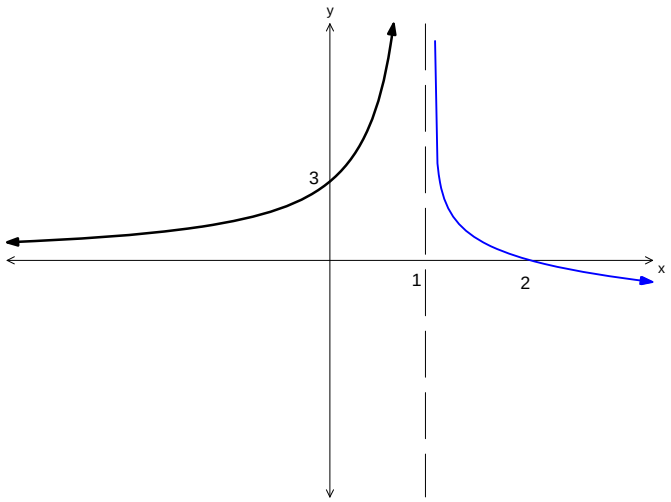
$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

(ii) Find the radius of the orbit of the earth around the sun. **1**

(iii) Find the velocity of the earth in metres per second. **1**

**Question 3 (start a new page) (12 marks)** **Marks**

(a) The graph of $y = f(x)$ is shown below




On separate number planes graph

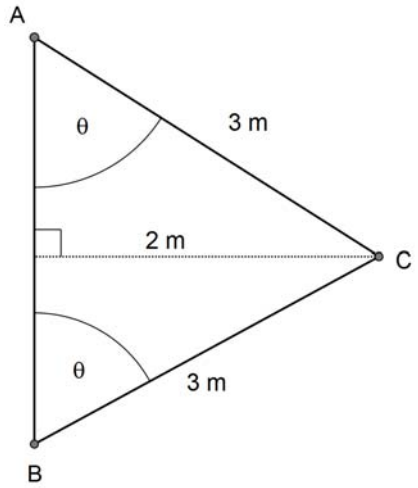
(i) $y = |f(x)|$ 1

(ii) $y = f(x+2)$ **2**

(iii) $y = \{f(x)\}^2$ **2**

(iv) $y = \dfrac{1}{f(x)}$ **2**

(b) The diagram below shows a 6 metre string *ACB* fixed at *A* and *B*.
A mass of 4 kg is attached at the midpoint *C*.
The mass at *C* rotates in a circle of radius 2m with angular velocity *w* rad $s^{-1}$.




The forces acting on the mass at *C* are $T_1$ (the tension in the string *AC*), $T_2$ (the tension in the string *BC*) and *mg* (gravity).

(i) Draw a diagram showing the forces acting on the mass at *C*. 1

(ii) By resolving the forces in (i) horizontally and vertically show that **2**

$$T_1 - T_2 = 24\sqrt{5}$$
$$T_1 + T_2 = 12\omega^2$$

(iii) Find the maximum speed in rad $s^{-1}$ that the mass at *C* can travel before the string at *BC* becomes taut. **2**

**Question 4 (start a new page) (12 marks)** **Marks**

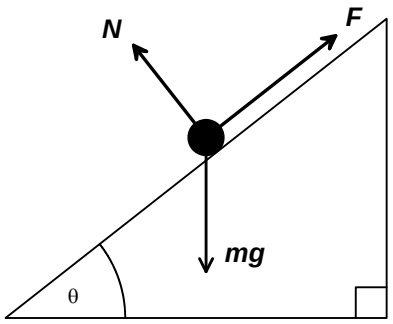
(a) Sketch the following on separate number planes showing all important features

(i) $y = 3^{x^2}$ **1**

(ii) $(x-2)^2 y = 1$ **2**

(iii) $x^3 - x^2 - y^2 = 0$ **2**

(b) A road contains a bend that is part of a circle of radius *r*. At the bend, the road is banked at an angle $\theta$ to the horizontal. A truck travels around the bend at constant speed *v*. Assume that the truck is represented by a point of mass *m*, and that the forces acting on the truck are the gravitational force *mg*, a sideways friction force *F* (acting up the road as drawn) and a normal reaction *N* to the road.




(i) By resolving the horizontal and vertical components of force, find expressions for $F\cos\theta$ and $F\sin\theta$. **2**

(ii) Show that $F = \dfrac{m(gr\tan\theta - v^2)}{r}\cos\theta$ **3**

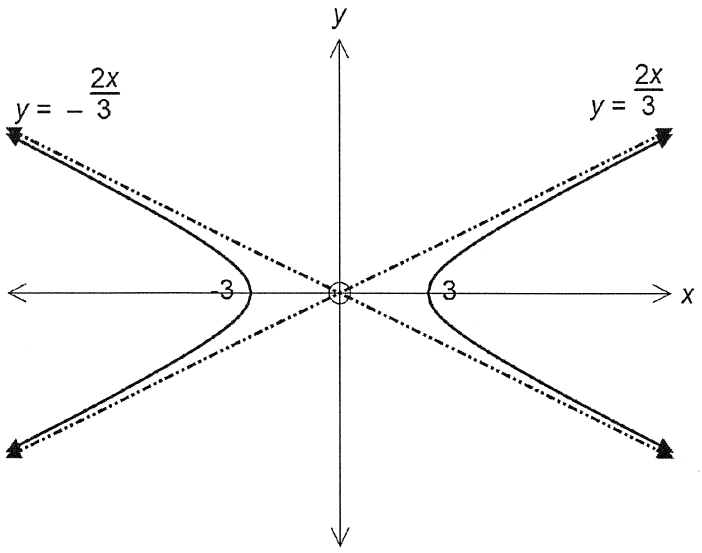
(iii) Suppose that the radius of the bend is 100m and that the road is banked at an angle of $5°30'$. **2**
Find the speed that the track can travel at in km/h so that there is no sideways friction force (correct to 2 decimal places).

**-- END OF EXAM --**

## Question 1

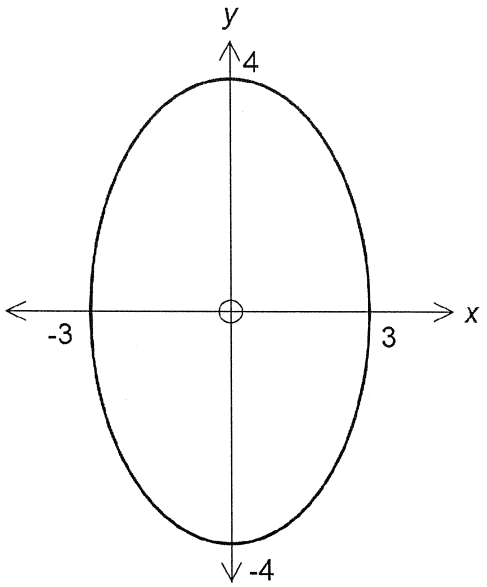
Students must show the $x$-intercepts, the asymptotes and the equation of the asymptotes. Take care that your curve approaches the asymptotes. A common error was students drawing an ellipse instead of a hyperbola.

(a) (i) $\dfrac{x^2}{9}-\dfrac{y^2}{4}=1$



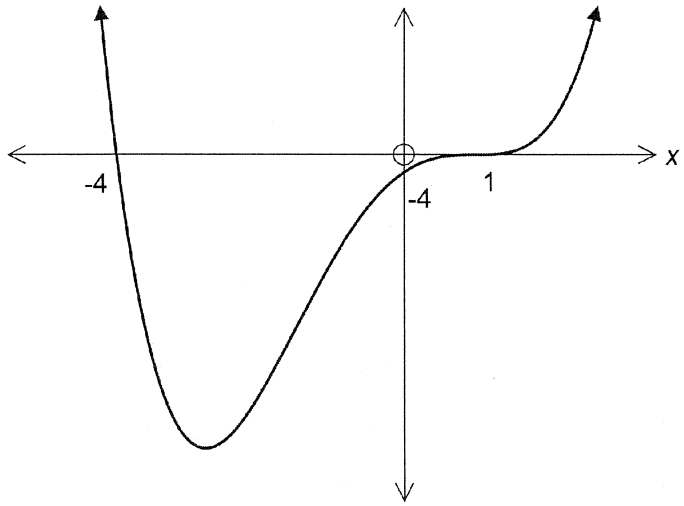

Generally done well but some students calculated the intercepts incorrectly. Also some students need to take more care with drawing the elliptical shape.

(iii) $16x^2+9y^2=144$




(ii) $y=(x+4)(x-1)^3$

Most graphs were reasonable but you should recognise there is a **horizontal** inflexion point at $x=1$. Some solutions did not reflect this feature properly. A common misconception was that the $y$ intercept was a minimum – some graphs ended up looking almost sharp at this point as a result.




(b) This question was successfully completed though there were many responses that included the wrong units. In (v) this was a particular problem. The units must be radians - 28 m was an answer relating to distance not angular distance and it is only because the radius is 1 that both answers were the same. Another common error occurred in (vi) where many students suggested the following **incorrect** relationship :

$\dfrac{2}{\pi}\times 60=120\pi$ .

(b) $\ell=20\,\text{m}$ , $t=5$ s , $4\,\text{rad/s}$

(i) $\theta=5\times 4=20$ rads

$\ell=r\theta$

$\therefore r=\dfrac{\ell}{\theta}=\dfrac{20}{20}=1\,\text{m}$

(ii) $v=r\omega=1\times 4=4$ m/s

(iii) $a=r\omega^2=1\times 4^2=16\,\text{m/s}^2$

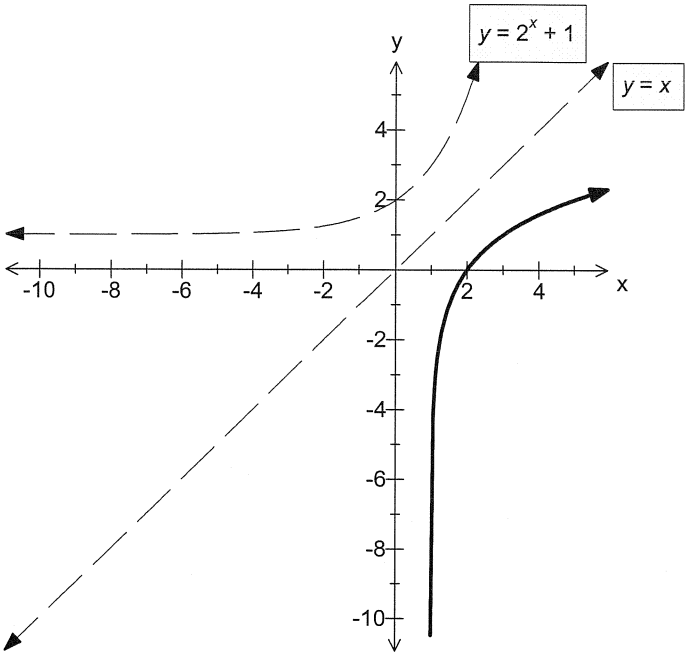
(iv) $T=\dfrac{2\pi}{\omega}=\dfrac{2\pi}{4}=\dfrac{\pi}{2}\text{s}$

(v) $\theta=\omega t=4\times 7=28$ rads

(vi) $f=\dfrac{1}{T}=\dfrac{2}{\pi}\text{revs/s}=\dfrac{120}{\pi}$ rpm

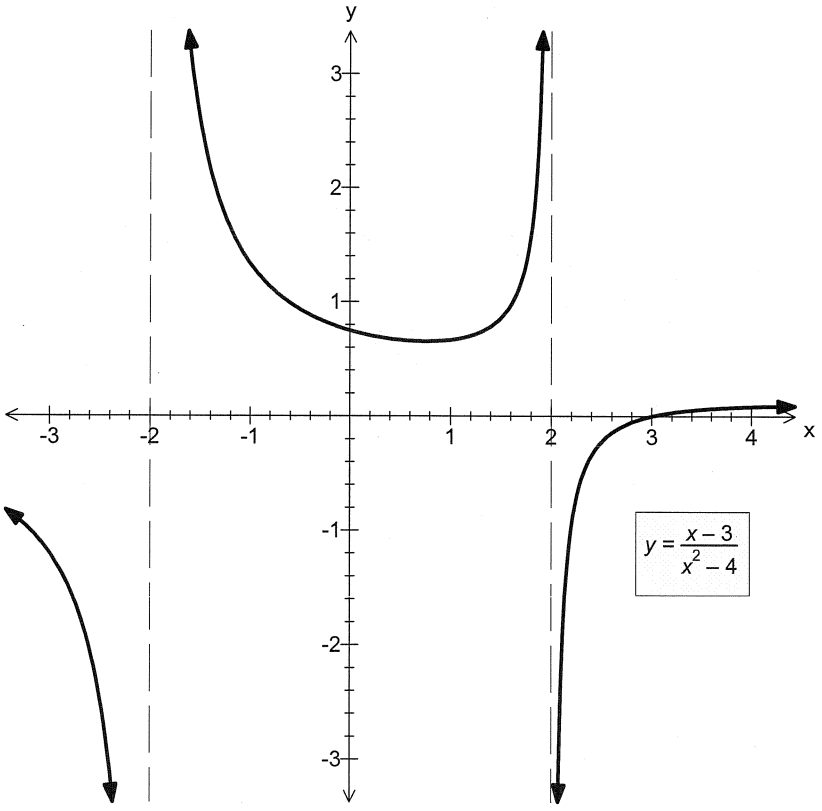
## Question 2

a) Sketch $y = f^{-1}(x)$ given $f(x) = 2^x + 1$.




b) i) $f(x) = \dfrac{x-3}{x^2-4} = \dfrac{x-3}{(x-2)(x+2)}$ $\therefore$ Asymptotes $x = \pm 2,\ y = 0$

ii) Sketch showing intercepts: $y = \dfrac{3}{4}$ and $x = 3$.



# Question 2

c)


$F = ma$

$= 8 \times 10$

$F = 80N$ ✓

$F = mr\omega^2$

$80 = 0.4 \times 0.5 \times \omega^2$ ✓

$\omega^2 = \frac{80}{0.2}$

$\omega^2 = 400$

$\therefore \omega = 20$ rad/sec ✓

d) i) $F = \frac{GMm}{r^2}$

$mr\omega^2 = \frac{GMm}{r^2}$

$\frac{r^3}{GM} = \frac{1}{\omega^2}$

$T = \frac{2\pi}{\omega}$

$\therefore \omega = \frac{2\pi}{T}$

$\frac{r^3}{GM} = \left(\frac{T}{2\pi}\right)^2$ ✓

$\frac{T}{2\pi} = \sqrt{\frac{r^3}{GM}}$

$T = 2\pi\sqrt{\frac{r^3}{GM}}$

## Question 2

d) ii)

$T = 365$ days

$= 365 \times 24 \times 3600$ secs

$= 31536000$ secs.

$G = 6.67 \times 10^{-11}$ N

$M = 2 \times 10^{30}$ kg

$T = 2\pi\sqrt{\frac{r^3}{GM}}$

$\left(\frac{31536000}{2\pi}\right)^2 = \frac{r^3}{GM}$

$\therefore r^3 = \left(\frac{31536000}{2\pi}\right)^2 \times 6.67 \times 10^{-11} \times 2 \times 10^{30}$

$r = \sqrt[3]{3.360541839 \times 10^{33}}$ ✓

$r = 1.497\,854\,984 \times 10^{11}$ m.

** Students needed to take care reading the question.

d)(ii)

$w = \frac{2\pi}{T}$

$\therefore w = \sqrt{\frac{GM}{r^3}}$

** Other methods to find v could be used.

Velocity of Earth in metres/sec

$v = rw$

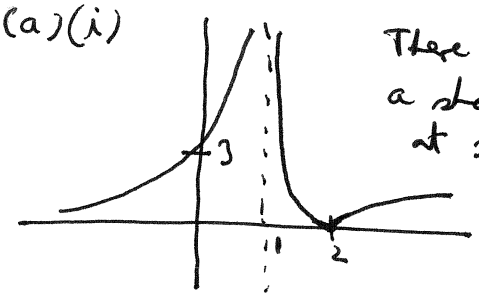
$= r \cdot \sqrt{\frac{GM}{r^3}}$

$= \sqrt{\frac{r^2 GM}{r^3}}$

$= \sqrt{\frac{6.67 \times 10^{-11} \times 2 \times 10^{30}}{1.497\,854\,984 \times 10^{11}}}$

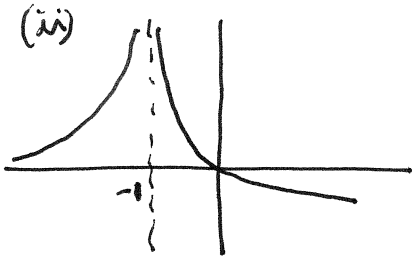
$\therefore v = 29843.0389$ m/sec. ✓

3(a)(i)

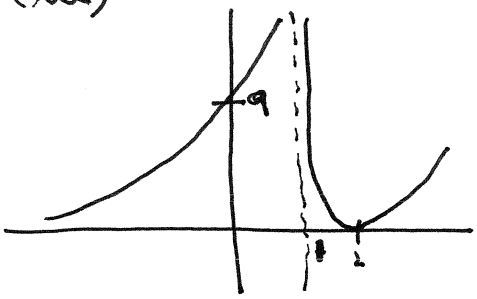



There needs to be a sharp corner at $x = 2$

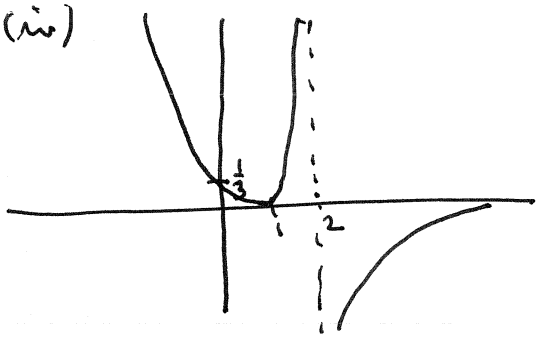
(ii)



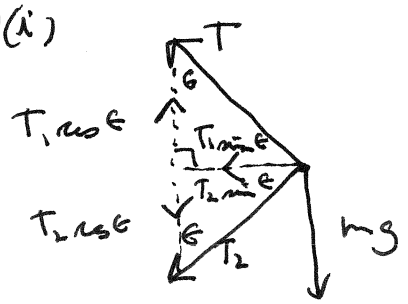

(iii)




(iv)




(b)(i)




(ii) $T_1 \cos\theta - T_2 \cos\theta - mg = 0$

$$T_1 \times \frac{\sqrt{5}}{3} - T_2 \times \frac{\sqrt{5}}{3} = 4 \times 10$$

$$T_1 - T_2 = \frac{120}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$= 24\sqrt{5}$$

$\sqrt{5}$ is found by Pythagoras Theorem

(iii) $T_2 = 0$

$$T_1 = 24\sqrt{5}$$

$$24\sqrt{5} = 12\omega^2$$

$$\omega^2 = 2\sqrt{5}$$

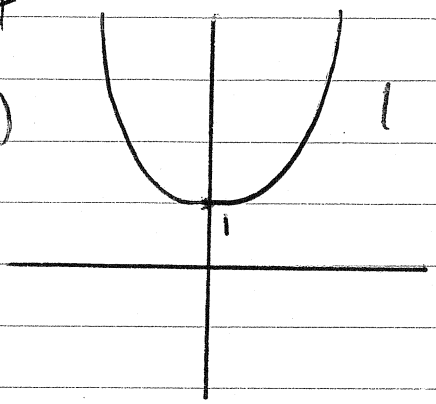
$$\omega = \sqrt{2\sqrt{5}} \text{ rads}^{-1}$$

The string is not taut when $T_2 = 0$

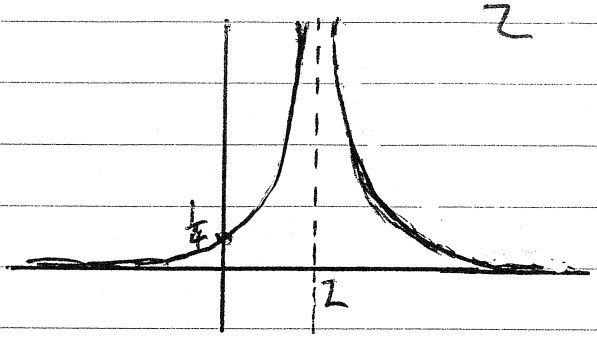
EXT 2 TASK 1

Q4

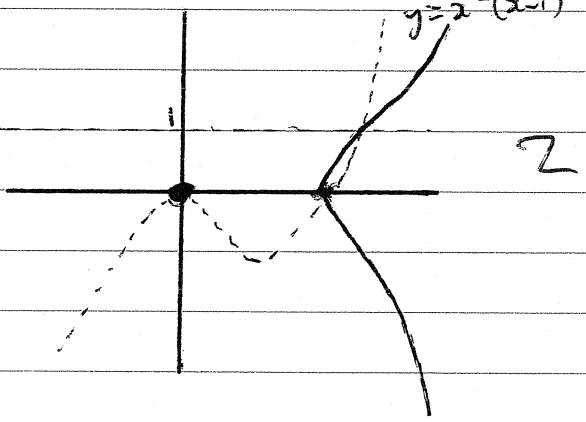
(a)




(b) $y = \frac{1}{(x-2)^2}$



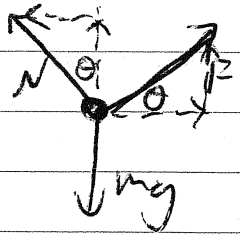

(c) $y^2 = x^2(x-1)$




COMMENTS: (a) $e^2 \neq (e^2)^2$

(c) many missed point $(0,0)$.

(b)(i)




Vertical

$F\sin\theta + N\cos\theta = mg$ 2

$F\sin\theta = mg - N\cos\theta.$

Horizontal

$N\sin\theta - F\cos\theta = \frac{mv^2}{r}$

$F\cos\theta = N\sin\theta - \frac{mv^2}{r}$

(ii) $F\sin^2\theta = mg\sin\theta - N\cos\theta\sin\theta$

$F\cos^2\theta = N\sin\theta\cos\theta - \frac{mv^2}{r}\cos\theta.$

$F = mg\sin\theta - \frac{mv^2}{r}\cos\theta$

$= \frac{m}{r}(rg\sin\theta - v^2\cos\theta)$

$F = \frac{m}{r}(rg\tan\theta - v^2)\cos\theta$ 3

(iii) $F = 0.$

$(rg\tan\theta - v^2)\cos\theta = 0.$ 2

$\theta = 5°30' \quad r = 100 \quad g = 10.$

$v^2\cos\theta = rg\sin\theta$

$v^2 = rg\tan\theta.$

$v = \sqrt{rg\tan\theta}$

$v = \sqrt{100 \times 10\tan(5°30')}$

$\approx 9.81$ m/s

$= 35.33$ km/h.