FINAL MARK

GIRRAWEEN HIGH SCHOOL
MATHEMATICS
YEAR 12 HSC TASK 2 2017
ANSWERS COVER SHEET

Name: ______________________________

QUESTION	MARK	H2	H3	H4	H5	H6	H7	H8	H9
Q1 to Q5	/5				✓	✓	✓		✓
Q6	/13				✓				✓
Q7	/18				✓		✓		✓
Q8	/16				✓				✓
Q9	/10				✓				✓
Q10	/15				✓			✓	✓
Q11	/10				✓			✓	✓
Q12	/13				✓			✓	✓
TOTAL									
	/100				/100	/5	/23	/38	/100

GIRRAWEEN HIGH SCHOOL

YEAR 12 – Task 2

2017

MATHEMATICS

Time allowed – 90 minutes

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used.
- Start each question on a new page.

SECTION 1(Multiple Choice – 5 marks)

For questions 1 to 5, fill in the circle corresponding to the correct answer on your answer sheet.

1. If the graph of $g(x)$ has the following properties:

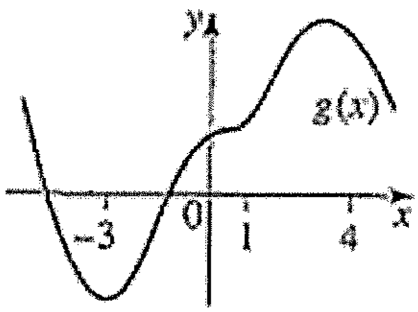
(i) $g'(x) = 0$ if $x = -3, 1$ and 4

(ii) $g'(x) < 0$ if $x < -3$ and $1 < x < 4$

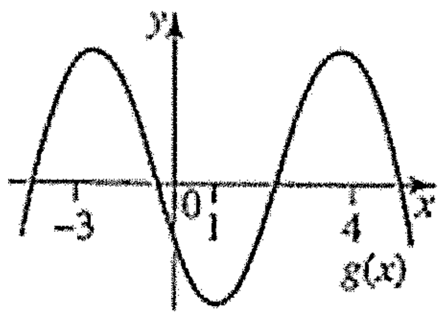
(iii) $g'(x) > 0$ for all other x

Then the graph of $g(x)$ could be

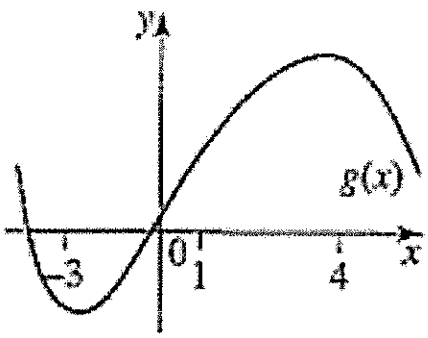
(A)



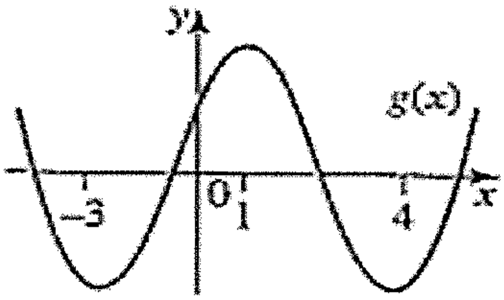
(C)



(B)



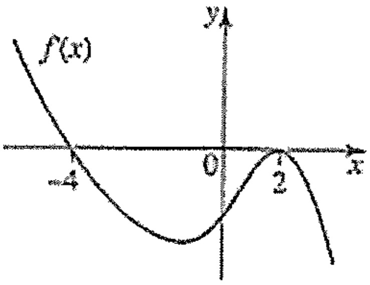
(D)



2. Evaluate $\int_1^5 (f(x)+1)\,dx$, given that $\int_1^5 f(x)\,dx = 6$.

(A) 16 **(B)**10 **(C)** 11 **(D)** 19

3. The graph of $f'(x)$ shown below indicates that the graph of $f(x)$ has



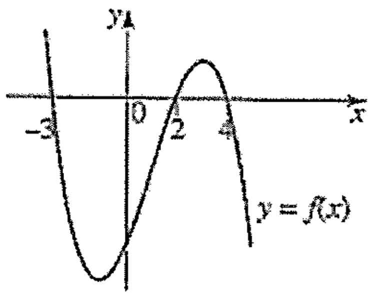
(A) A turning point at $x = 2$ and $x = -4$.

(B) A turning point at $x = 2$ and point of inflexion at $x = -4$.

(C) A turning point at $x = -4$ and point of inflexion at $x = 2$.

(D) Two points of inflexion at $x = -4$ and $x = 2$.

4. The area between the curve, the $x-$axis and the lines $x = -3$ and $x = 4$ is equal to



(A) $\int_{-3}^{4} f(x)dx$

(B) $\int_{-3}^{2} f(x)dx + \int_{2}^{4} f(x)dx$

(C) $\int_{2}^{4} f(x)dx + \int_{2}^{-3} f(x)dx$

(D) $\int_{-3}^{2} f(x)dx - \int_{2}^{4} f(x)dx$

5. An antiderivative or a primitive of $2(3x+4)^{-4}$ is

(A) $-\frac{2}{3}(3x+4)^{-3}$

(B) $-\frac{2}{3}(3x+4)^{-3}+5$

(C) $-\frac{2}{9}(3x+4)^{-3}$

(D) $-\frac{2}{9}(3x+4)^{-5}$

Question 6 (13 marks) **Marks**

(a) Determine the values of x for which $y = x^3 - 5x^2 + 3x + 2$ is decreasing. 3

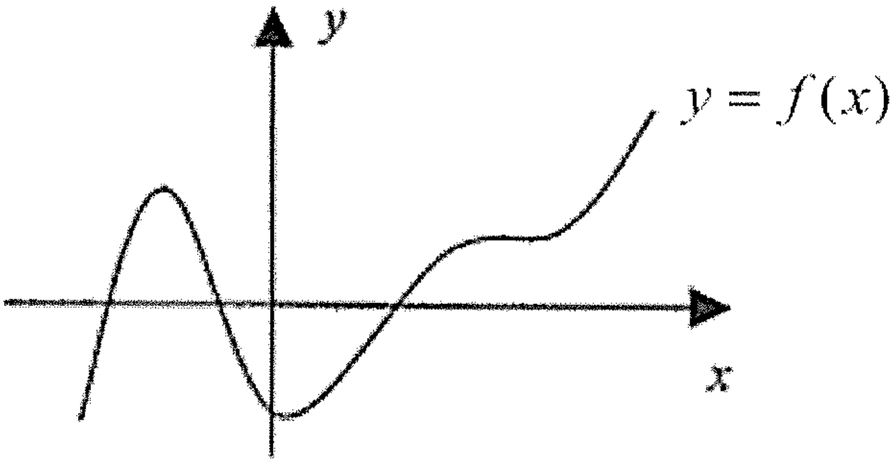
(b) Show that the curve $y = \dfrac{x}{x-1}$ has no stationary points. 2

(c) Find $f'(x)$ and $f''(x)$ if $f(x) = \sqrt{1-3x}$ 4

(d) For what values of x is the curve $f(x) = 2x^3 - 7x^2 - 5x + 4$ concave up? 4

Question 7 (18 marks)

(a) Sketch the graph of $f'(x)$ (on separate sheet provided in the answer booklet) 4



(b) For the curve $y = 6x^2 - 2x^3$, find:

(i) the coordinates of the turning points and determine their nature. 4

(ii) the coordinates of the point of inflexion. 3

(iii) Sketch the curve showing the turning points, the point of inflexion and the points where the curve meets the x- axis. 5

(iv) Determine the absolute maximum and minimum values of $f(x)$ for $-1 \le x \le 4$. 2

Question 8 (16 marks)

(a) The point (2,-1) is a point of inflexion on the curve $y = x^3 + ax^2 + bx + 3$. Find the values of a and b. 4

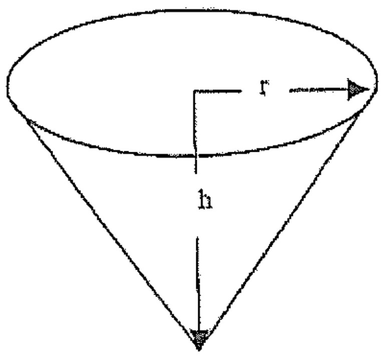
(b) Evaluate the following: 12

(i) $\int (x-2)(x-5)\ dx$ **(ii)** $\int \left(\frac{2}{x^4} - \frac{3}{x^3}\right) dx$

(iii) $\int_{-1}^{4} (2x+3)^4\ dx$ **(iv)** $\int_{-2}^{1} \frac{4x^4 - x}{x}\ dx$

Question 9 (10 marks)

(a) The diagram represents a conical water Tower. The radius of the cone is r and the height h. The volume of the cone is given by $V = \frac{1}{3}\pi r^2 h$. It is given that $2r + h = 60$.



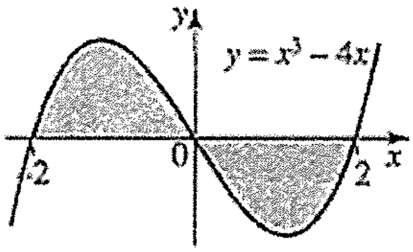
(i) Show that the volume (V) of the cone is $V = 20\pi r^2 - \frac{2}{3}\pi r^3$. 2

(ii) Find the maximum volume of the cone. 4

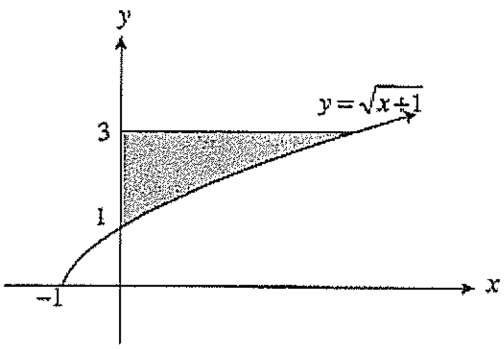
(b) At any point (x, y) on a curve, $\frac{d^2y}{dx^2} = 12x + 6$. Find the equation of the curve if it passes through the point $(-1,-2)$ and the gradient of the tangent at this point is 1. 4

Question 10 (15 marks)

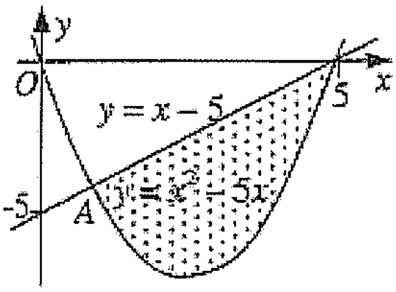
(a) Find the area bounded by the curve $y = x^3 - 4x$ and the x- axis. 5



(b) Find the area bounded by the curve $y = \sqrt{x+1}$, the $y-$ axis and the lines $y = 1$ and $y = 3$. 4



(c) The graphs of $y = x - 5$ and $y = x^2 - 5x$ intersect at the points $(5,0)$ and A, as shown in the diagram.

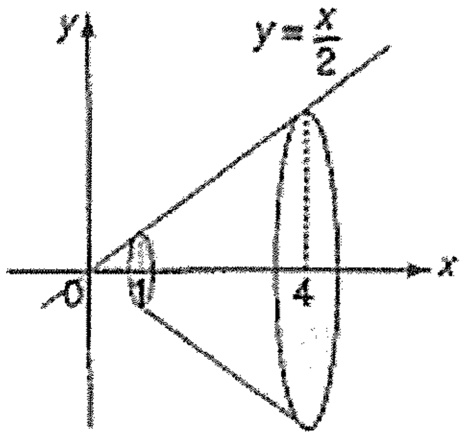


(i) Find the coordinates of A. 2

(ii) Find the area of the shaded region bounded by $y = x - 5$ and $y = x^2 - 5x$. 4

Question 11 (10 marks)

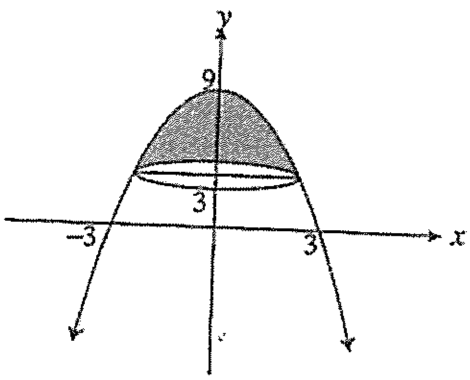
(a) Find the volume of the solid generated when the section of the line $y = \frac{x}{2}$ between $x = 1$ and $x = 4$ is rotated about the $x-$axis. 3



(b) Find the volume generated when the area bounded by the curve

$y = 9 - x^2$ for $x \geq 0$, the $y-$axis and the line $y = 3$ is rotated about the

$y-$axis. 3



(c) **(i)** Differentiate $(x^4 - 1)^9$. 2

(ii) Hence find $\int 2x^3(x^4 - 1)^8 \, dx.$ 2

Question 12 (13 marks)

(a) The table shows points on a continuous curve $y = f(x)$. Use the Trapezoidal Rule

to find the approximate value of $\int_3^4 f(x)dx$ correct to three decimal places. 3

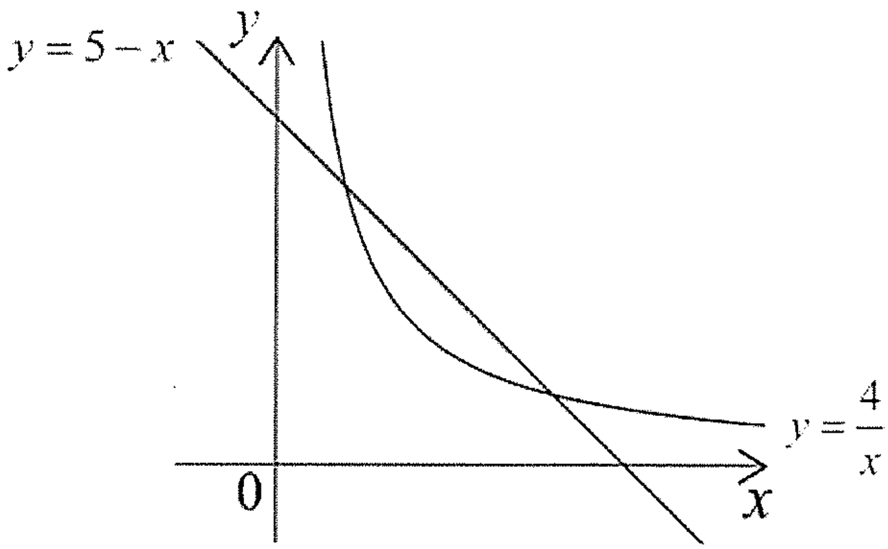
x	3	3.2	3.4	3.6	3.8	4
y	7.19	7.62	8.41	8.74	9.26	9.78

(b) Evaluate $\int_0^2 \frac{1}{x+2}\,dx$ by Simpson's rule and by taking five function values.

Write the answer correct to three decimal places. 5

(c) Find the volume of the solid formed when the area between $y = \frac{4}{x}$ and

$y = 5 - x$ is rotated about the x-axis. 5



END OF EXAMINATION

Year 12 Mathematics Test 2 2017 Solutions

Multiple choice (5 marks)

1 D 2 B 3 C 4 C 5 C

Question 6 (13 marks)

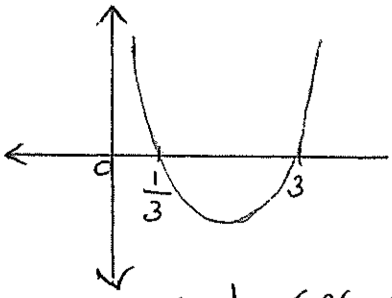
(a) $y = x^3 - 5x^2 + 3x + 2$

$y' = 3x^2 - 10x + 3$

For decreasing, $y' < 0$

$3x^2 - 10x + 3 < 0$

$(3x-1)(x-3) < 0$



(3)

$\therefore \frac{1}{3} < x < 3$

(b) $y = \frac{x}{x-1}$

$y' = \frac{(x-1) \times 1 - x \times 1}{(x-1)^2}$

$= \frac{x - 1 - x}{(x-1)^2}$

$= \frac{-1}{(x-1)^2}$ (2)

$y' = 0 \Rightarrow \frac{-1}{(x-1)^2} = 0$ which has no solution.

$\therefore y = \frac{x}{x-1}$ has no stationary points.

(c) $f(x) = \sqrt{1-3x}$

$f'(x) = \frac{1}{2\sqrt{1-3x}} \times -3 = \frac{-3}{2\sqrt{1-3x}}$

$f''(x) = \frac{2\sqrt{1-3x} \times 0 - (-3) \times 2 \times \frac{1}{2\sqrt{1-3x}} \times -3}{4(1-3x)}$

$= \frac{-9}{4(1-3x)\sqrt{1-3x}} = \frac{-9}{4(1-3x)^{\frac{3}{2}}}$

$= \frac{-9}{4\sqrt{(1-3x)^3}}$ (4)

Alternative Method

$f'(x) = \frac{-3}{2\sqrt{1-3x}}$

$= \frac{-3}{2}(1-3x)^{-\frac{1}{2}}$

$f''(x) = \frac{-3}{2} \times -\frac{1}{2}(1-3x)^{-\frac{3}{2}} \times -3$

$= \frac{-9}{4} \times \frac{1}{(1-3x)^{\frac{3}{2}}}$

$= \frac{-9}{4\sqrt{(1-3x)^3}}$

(d) $f(x) = 2x^3 - 7x^2 - 5x + 4$

$f'(x) = 6x^2 - 14x - 5$

$f''(x) = 12x - 14$

For concave up, $f''(x) > 0$

$12x - 14 > 0$

$12x > 14$

$x > \frac{14}{12}$ (4)

$x > \frac{7}{6}$

Question 7 (18 marks)

(a) See last page

(b) $y = 6x^2 - 2x^3$

(i) $y' = 12x - 6x^2$

$y' = 0 \implies 12x - 6x^2 = 0$

$6x(2 - x) = 0$

$x = 0$ or $x = 2$

when $x = 0$, $y = 0$

when $x = 2$, $y = 8$

stationary points are $(0,0)$ and $(2,8)$

$y'' = 12 - 12x$

when $x = 0$, $y'' = 12 > 0$

$\therefore (0,0)$ is a minimum turning point

when $x = 2$, $y'' = -12 < 0$ (4)

$\therefore (2,8)$ is a maximum turning point.

(ii) Possible points of inflexion are given by $y'' = 0$

$12 - 12x = 0$

$x = 1$

x	0	1	2
y''	12	0	−12

Concavity changes. (3)

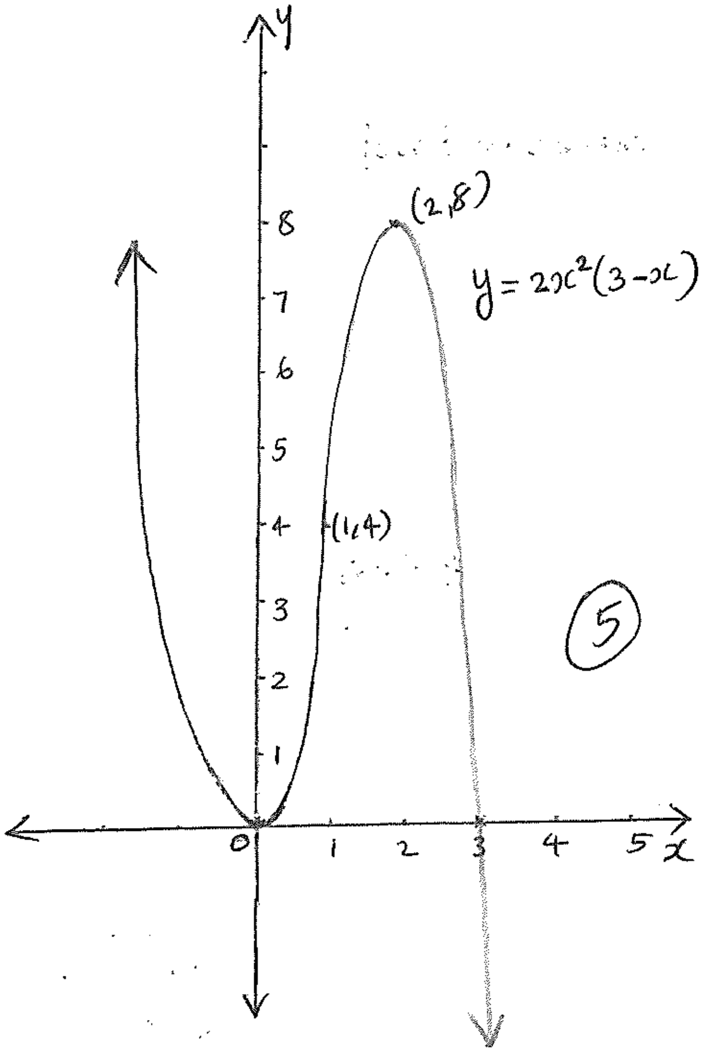
When $x = 1$, $y = 6 - 2 = 4$

$\therefore (1,4)$ is a point of inflexion.

(iii) x intercepts

$6x^2 - 2x^3 = 0$

$2x^2(3 - x) = 0$ $x = 0, 3$



(5)

(iv) when $x = -1$

$y = 6 \times 1 - 2(-1)$

$= 8$

when $x = 4$

$y = 32 \times -1 = -32$ (2)

Absolute maximum is 8 and absolute minimum is -32.

Question 8 (16 marks)

$y = x^3 + ax^2 + bx + 3$

$y' = 3x^2 + 2ax + b$

$y'' = 6x + 2a$

when $x = 2$, $y'' = 0$

$0 = 12 + 2a$

$2a = -12$ $\therefore a = -6$

$y = x^3 - 6x^2 + bx + 3$

when $x = 2$, $y = -1$

$-1 = 8 - 24 + 2b + 3$ (4)

$2b = 12$; $b = 6$

$a = -6,\ b = 6$

(b)(i) $\int (x-2)(x-5)\,dx$

$= \int (x^2 - 7x + 10)\,dx$

$= \frac{x^3}{3} - \frac{7x^2}{2} + 10x + C$ (3)

(ii) $\int \left(\frac{2}{x^4} - \frac{3}{x^3}\right) dx$

$= \int (2x^{-4} - 3x^{-3})\,dx$

$= 2\frac{x^{-3}}{-3} - 3 \times \frac{x^{-2}}{-2}$ (3)

$= \frac{-2}{3x^3} + \frac{3}{2x^2} + C$

(iii) $\int_{-1}^{4} (2x+3)^4 \, dx$

$= \left[\frac{(2x+3)^5}{10} \right]_{-1}^{4}$

$= \frac{1}{10} \left[(2x+3)^5 \right]_{-1}^{4}$

$= \frac{1}{10} \left(11^5 - 1 \right)$ (3)

$= 16105$

(iv) $\int_{-2}^{1} \frac{4x^4 - x}{x} \, dx$

$= \int_{0}^{3} (4x^3 - 1) \, dx$

$= \left[4\frac{x^4}{4} - x \right]_{-2}^{1}$

(3)

$= \left[x^4 - x \right]_{-2}^{1}$

$= (1-1) - (16+2)$

$= -18$

Question 9 (10 marks)

(i) $2r + h = 60$

$h = 60 - 2r$

$V = \frac{1}{3} \pi r^2 h$

$= \frac{1}{3} \pi r^2 (60 - 2r)$

$= \frac{60\pi r^2}{3} - \frac{2}{3} \pi r^3$

$= 20\pi r^2 - \frac{2}{3} \pi r^3$ (2)

(ii) $\frac{dV}{dr} = 40\pi r - 2\pi r^2$

$= 2\pi r (20 - r)$

$= 0$ when $r = 0$ or $r = 20$

$\frac{d^2V}{dr^2} = 40\pi - 4\pi r$

when $r = 20$, $\frac{d^2V}{dr^2} = 40\pi - 4\pi \times 20$

$= 40\pi - 80\pi$

$= -40\pi < 0$

$\therefore$ Volume is maximum when $r = 20$

Maximum volume

$= \frac{1}{3} \pi (20)^2 \times 20$ (4)

$= \frac{8000\pi}{3}$ cubic units.

(b) $\frac{d^2y}{dx^2} = 12x + 6$

$\frac{dy}{dx} = \frac{12x^2}{2} + 6x + C$

$= 6x^2 + 6x + C$

when $x = -1$, $y' = 1$

$1 = 6 - 6 + C$ $\therefore C = 1$

$\frac{dy}{dx} = 6x^2 + 6x + 1$

$$y = \frac{6x^3}{3} + \frac{6x^2}{2} + x + D$$

$$= 2x^3 + 3x^2 + x + D$$

when $x=-1$, $y=-2$

$$-2 = -2+3-1+D$$

$$D = -2$$ (4)

$$\therefore y = 2x^3 + 3x^2 + x - 2$$

Question 10 (15 marks)

Shaded area

$$= \underbrace{\int_{-2}^{0} (x^3 - 4x)\,dx}_{I_1} - \underbrace{\int_{0}^{2} (x^3 - 4x)\,dx}_{I_2}$$

$$I_1 = \left[\frac{x^4}{4} - \frac{4x^2}{2}\right]_{-2}^{0}$$

$$= \left[\frac{x^4}{4} - 2x^2\right]_{-2}^{0}$$

$$0 - \left(\frac{16}{4} - 2\times 4\right)$$

$$= 4$$

$$I_2 = \left[\frac{x^4}{4} - 2x^2\right]_{0}^{2}$$

$$= \left(\frac{16}{4} - 8\right) - 0$$

$$= -4$$ (5)

$$I_1 - I_2 = 4 - (-4)$$

$$= 8 \text{ square units}$$

(b) $y = \sqrt{x+1}$

$y^2 = x+1$; $x = y^2 - 1$

Required area $= \int_{1}^{3} x\,dy$

$$= \int_{1}^{3} (y^2 - 1)\,dy$$

$$= \left[\frac{y^3}{3} - y\right]_{1}^{3}$$ (4)

$$= \left(\frac{3^3}{3} - 3\right) - \left(\frac{1}{3} - 1\right)$$

$$= (9-3) - \frac{1}{3} + 1 = 6\frac{2}{3} \text{ square units}$$

(c)(i) $x^2 - 5x = x - 5$

$$x^2 - 6x + 5 = 0$$

$$(x-1)(x-5) = 0$$

$$x = 1 \text{ or } x = 5$$

when $x = 1$, $y = 1 - 5 = -4$

$$\therefore A(1, -4)$$ (2)

(ii) Shaded area

$$= \int_{1}^{5} (x - 5 - x^2 + 5x)\,dx$$

$$= \int_{1}^{5} (-x^2 + 6x - 5)\,dx$$

$$= \left[\frac{-x^3}{3} + 3x^2 - 5x\right]_{1}^{5}$$ (4)

$$= \left(\frac{-125}{3} + 75 - 25\right) - \left(\frac{-1}{3} + 3 - 5\right)$$

$$= \frac{32}{3} \text{ square units}$$

Question 11 (10 marks)

(a) $V = \pi \int_1^4 \frac{x^2}{4}\, dx$

$= \frac{\pi}{4}\left[\frac{x^3}{3}\right]_1^4$

$= \frac{\pi}{12}\left[x^3\right]_1^4$

$= \frac{\pi}{12}(64-1)$ (3)

$= \frac{21\pi}{4}$ cubic units

(b) $y = 9 - x^2$

$x^2 = 9 - y$

$V = \pi \int_3^9 (9-y)\, dy$ (3)

$= \pi\left[9y - \frac{y^2}{2}\right]_3^9$

$= \pi\left[\left(81 - \frac{81}{2}\right) - \left(27 - \frac{9}{2}\right)\right]$

$= 18\pi$ cubic units

Question 12 (13 marks)

(a) $\int_3^4 f(x)\, dx$

$= \frac{0.2}{2}\left\{7.19 + 9.78 + 2(7.62 + 8.41 + 8.74 + 9.26)\right\}$ (3)

$= 8.503$

(b) $n = 4$ $h = \frac{2-0}{4} = 0.5$

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{3}$	$\frac{2}{7}$	$\frac{1}{4}$

(5)

$\int_0^2 \frac{1}{x+2}\, dx$

$= \frac{0.5}{3}\left\{\left(\frac{1}{2}+\frac{1}{4}\right) + 2\left(\frac{1}{3}\right) + 4\left(\frac{2}{5}+\frac{2}{7}\right)\right\}$

$= 0.693$

(c)(i) $\frac{d}{dx}(x^4-1)^9 = 9(x^4-1)^8 \times 4x^3 = 36x^3(x^4-1)^8$ (2)

(ii) $\int 36x^3(x^4-1)^8\, dx = (x^4-1)^9 + C$

$18\int 2x^3(x^4-1)^8\, dx = (x^4-1)^9 + C$

$\int 2x^3(x^4-1)^8\, dx = \frac{(x^4-1)^9}{18} + C$ (2)

(c) Points of intersection

$$\frac{4}{x} = 5-x$$

$$4 = x(5-x)$$

$$4 = 5x - x^2$$

$$x^2 - 5x + 4 = 0$$

$$(x-4)(x-1) = 0$$

$$x = 4 \text{ or } x = 1$$

$$V = \pi \int_1^4 (5-x)^2 dx - \pi \int_1^4 \left(\frac{4}{x}\right)^2 dx$$

$$= \pi \int_1^4 \left(25 - 10x + x^2 - \frac{16}{x^2}\right) dx$$

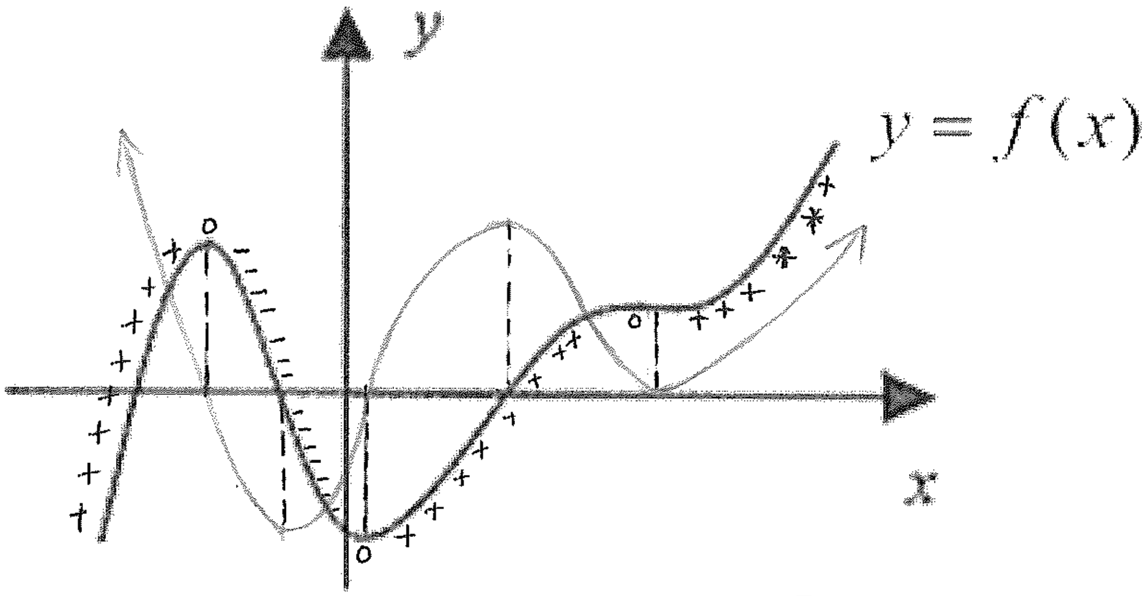
(5)

$$= \pi \left[25x - 5x^2 + \frac{x^3}{3} + \frac{16}{x}\right]_1^4$$

$$= \pi \left\{\left(100 - 80 + \frac{64}{3} + 4\right) - \left(25 - 5 + \frac{1}{3} + 16\right)\right\}$$

$= \underline{\underline{9\pi \text{ cubic}}}$ units

Question 7(a)



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