Question 1 (11 Marks) **Marks**

a) Solve for x: $\dfrac{x-2}{x+3}+2\geq 0$ 3

b) Simplify $\dfrac{y}{x^2-xy}+\dfrac{1}{x}$ 2

c) Find the acute angle between the lines $5y=3x+1$ and $4x-y=3$ 2

d) Suppose that P is the point (-4,7) and Q is the point (1,-3).

(i) Find the point R which divides the interval PQ internally in the ratio $c:1$. 1

(ii) Hence, or otherwise, find the ratio in which the line $3x+4y=6$ divides the interval PQ. 3

Question 2 (11 Marks) Start a new page

a) What are the co-ordinates of the focus of the parabola $x^2+6x+8y-7=0$ 2

b) If α and β are the roots of the equation $4x^2-2x-1=0$ find (without solving for x)

(i) $\dfrac{1}{2\alpha}+\dfrac{1}{2\beta}$ 3

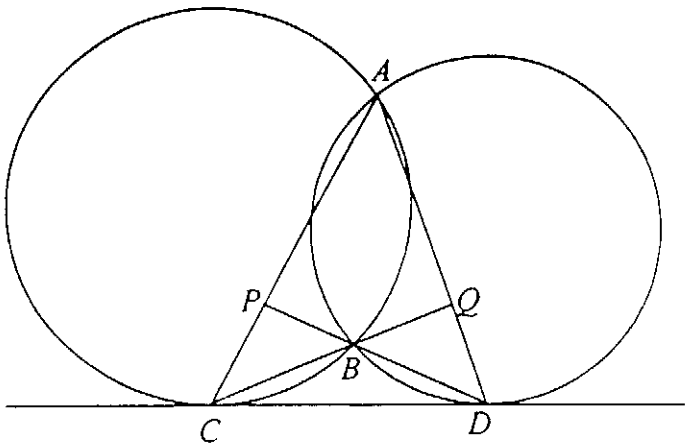
(ii) $\alpha^2+\beta^2$ 1

(iii) $\alpha-\beta$ 2

c) What value(s) of k will make the expression $(k+1)x^2-2(k-1)x+(2k-5)$ a perfect square? 3

Question 3 (10 Marks) Start a new page **Marks**

a)



Two circles intersect at A and B. A common tangent touches both circles at C and D, as shown in the diagram above. The line DB meets the chord AC at P, and the line CB meets the chord AD at Q.

(i) Make a large, neat copy of the diagram on your answer sheet, and draw the common chord AB. 1

(ii) Let $\angle BCD = \alpha$ and $\angle BDC = \beta$. 1
Give a reason why $\angle CAB = \alpha$ and $\angle DAB = \beta$.

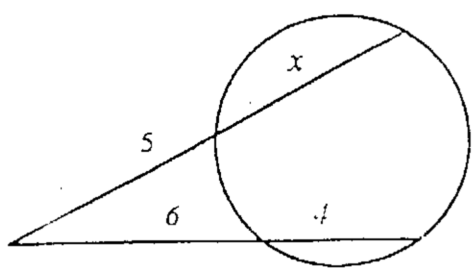
(iii) Show that $\angle PBQ = 180^0 - (\alpha + \beta)$. 2

(iv) Give a reason why $APBQ$ is a cyclic quadrilateral. 1

(v) Show that $\angle PQB = \alpha$. 2

(vi) Hence show that PQ is parallel to CD. 1

b) Find the value of x in the diagram below. 2

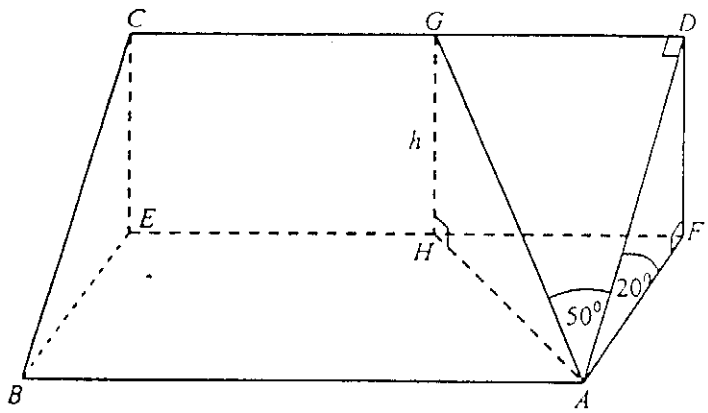


Question 4 (10 Marks) Start a new page **Marks**

a) Show that $\dfrac{\cos\theta}{1-\sin\theta} - \sec\theta = \tan\theta$. 3

b) Solve for $0^0 \le x \le 360^0$: $\sin^2 2x = \dfrac{1}{4}$ 3

c) A plane hillside $ABCD$ makes an angle of 20^0 with the horizontal
A path AG makes an angle of 50^0 with a line of greatest slope.
If $DF = GH = h$:



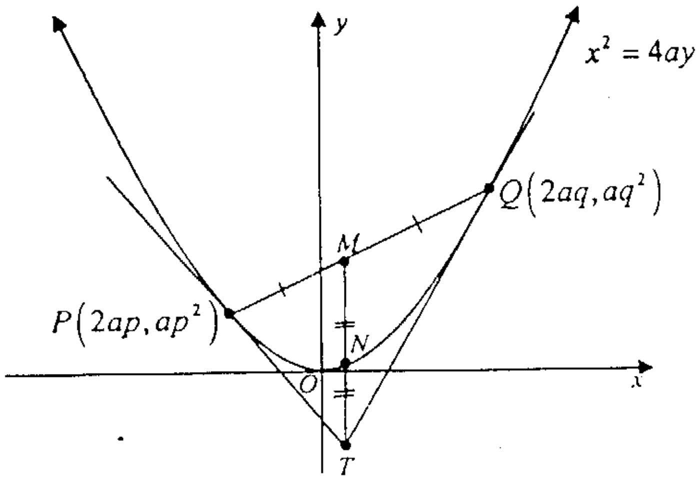
(i) Show that $AD = \dfrac{h}{\sin 20^0}$ 1

(ii) Show that $AG = \dfrac{h}{\sin 20^0 \cos 50^0}$ 1

(iii) Hence, find the inclination of the path AG to the horizontal.
(Leave your answer to the nearest degree) 2

Question 5 (12 Marks) Start a new page **Marks**

a)



In the diagram, $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are two points on the parabola $x^2 = 4ay$.

(i) Show that the tangent at P has equation $y = px - ap^2$. 2

(ii) The tangents at P and Q meet at T. Assuming that the tangent at Q is $y = qx - aq^2$, show that T is the point $(a(p+q), apq)$. 2

(iii) M is the midpoint of the chord PQ. Show that MT is parallel to the axis of symmetry of the parabola. 2

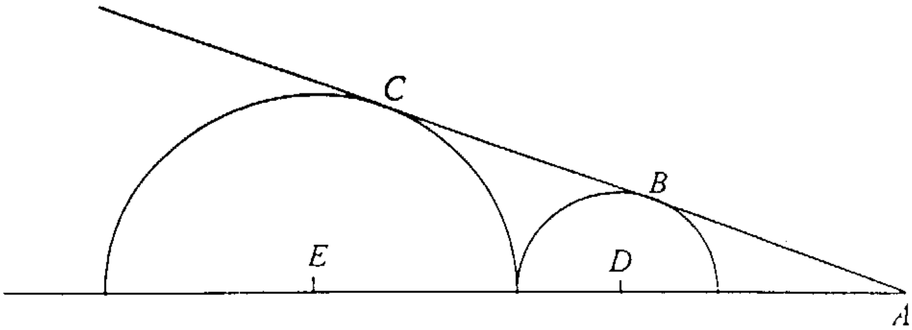
b) Find $\lim_{x \to -5} \dfrac{\sqrt{20-x} - 5}{5+x}$ 3

c) If the roots of the equation $4(p^2+q^2)x^2 + 4prx + (r^2 - 4q^2) = 0$ are real and $q \neq 0$, then show that 3

$$p^2 + q^2 \geq \frac{r^2}{4}$$

Question 6 (10 Marks) Start a new page **Marks**

a)



The diagram above shows two semicircles centred at D and E with radii 1cm and 3cm respectively. ABC is a common tangent to both semicircles.

(i) Find the length of AD 4

(ii) Find the size of $\angle DAB$ 1

(iii) Find the length of AB 1

b) (i) Show that $\dfrac{12}{4-x} - 2 = \dfrac{2x+4}{4-x}$ 1

(ii) Hence, or otherwise sketch $y = \dfrac{2x+4}{4-x}$ 2

(iii) Hence, or otherwise solve for x: $\dfrac{2x+4}{4-x} \geq 0$ 1

End of Paper

2002 - AP2 Extension

Solutions

1. a) $\frac{x-2}{x+3} \geq -2$

$(x-2)(x+3) \geq -2(x+3)^2$ (1)

$(x-2)(x+3) + 2(x+3)^2 \geq 0$

$(x+3)(x-2+2x+6) \geq 0$

$(x+3)(3x+4) \geq 0$ (1)

$\therefore \; x < -3 \text{ or } x \geq -\frac{4}{3}$ (1)

b) $\frac{xy + x^2 - xy}{x(x^2-xy)}$ (1)

$= \frac{x^2}{x(x^2-xy)}$

$= \frac{x^2}{x^2(x-y)}$

$= \frac{1}{x-y}$ (1)

c) $3x - 5y + 1 = 0$ ①

$4x - y - 3 = 0$ ②

$m_1 = \frac{3}{5} \quad m_2 = 4$ (1)

$\therefore \tan\theta = \left| \frac{\frac{3}{5} - 4}{1 + \frac{3}{5}\times 4} \right|$

$= \frac{17}{5} \times \frac{5}{17} = 1$

$\theta = 45°.$ (1)

d) (i) $R: \left(\frac{c-4}{1+c}, \frac{-3c+7}{1+c} \right)$

$= \left(\frac{c-4}{c+1}, \frac{7-3c}{c+1} \right)$ (1)

(ii) $3\left(\frac{c-4}{c+1}\right) + 4\left(\frac{7-3c}{c+1}\right) = 6$ (1)

$3c - 12 + 28 - 12c = 6c + 6$ (1)

$10 = 15c$

$c = \frac{10}{15} = \frac{2}{3}$ (1)

$\therefore$ Ratio is ~~[illegible]~~ 2 : 3

2. a) $x^2 + 6x + 9 = 7 - 8y + 9$

$(x+3)^2 = 16 - 8y$ (1)

$= -8(y-2)$

$\therefore$ Focus $= (-3, 0)$ (1)

b) (i) $\alpha + \beta = \frac{1}{2}, \; \alpha\beta = -\frac{1}{4}$ (1)

$\frac{1}{2\alpha} + \frac{1}{2\beta} = \frac{2(\alpha+\beta)}{4\alpha\beta}$

$= \frac{\alpha+\beta}{2\alpha\beta}$ (1)

$= \frac{1}{2} \times \frac{-2}{1}$

$= -1$ (1)

(ii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \frac{1}{4} + \frac{1}{2}$

$= \frac{3}{4}$ (1)

(ii) $\alpha - \beta = \pm\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$ (1)

$= \pm\sqrt{\frac{1}{4} + 1}$

$= \pm\frac{\sqrt{5}}{2}$ (1)

c) $\Delta = 0$

$4(k-1)^2 - 4(k+1)(2k-5) = 0$ (1)

$(k-1)^2 - (k+1)(2k-5) = 0$

$k^2 - 2k + 1 - 2k^2 + 3k + 5 = 0$

$-k^2 + k + 6 = 0$

$k^2 - k - 6 = 0$

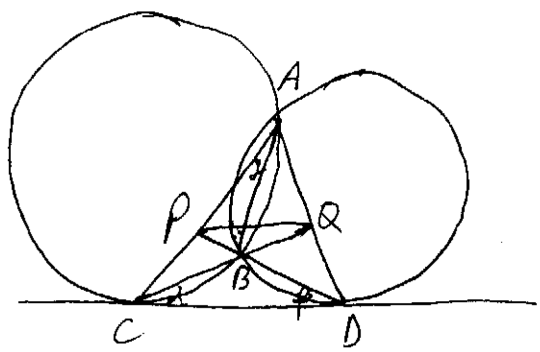
$(k-3)(k+2) = 0$ (1)

$\therefore k = +3$ or -2

But $k \neq -2$ as $a < 0$

$\therefore k = 3$ (1)

3 a) (i)



(1)

(ii) Alternate segment theorem (1)

(iii) $C\hat{B}D = 180 - (\alpha+\beta)$ ($\angle$ sum of Δ) (1)

$\therefore P\hat{B}Q = 180 - (\alpha+\beta)$ (vertically opposite $\angle$s) (1)

(iv) Opposite $\angle$s are supplementary (1)

(v) Since APBQ is a cyclic quad (1)

$P\hat{Q}B = \alpha$ ($\angle$s in same segment) (1)

(vi) $\therefore PQ \parallel CD$ (alternate $\angle$'s are equal) (1)

b) $5(x+5) = 6 \times 10$ (product of intercepts are equal) (1)

$5x + 25 = 60$

$x = $ [redacted] 7 (1)

4. a) $LHS = \frac{\cos\theta}{1-\sin\theta} - \frac{1}{\cos\theta}$

$= \frac{\cos^2\theta - 1 + \sin\theta}{\cos\theta(1-\sin\theta)}$ (1)

$= \frac{1 - \sin^2\theta - 1 + \sin\theta}{\cos\theta(1-\sin\theta)}$ (1)

$= \frac{\sin\theta - \sin^2\theta}{\cos\theta(1-\sin\theta)}$

$= \frac{\sin\theta(1-\sin\theta)}{\cos\theta(1-\sin\theta)}$ (1)

$= \tan\theta$

$= RHS$

b) $\sin 2x = \pm\frac{1}{2}$ (1)

$\therefore 2x = 30°, 150°, 210°, 330°, 390°, 510°, 570°, 690°$ (1)

$x = 15°, 75°, 105°, 165°, 195°, 255°, 285°, 345°$ (1)

c) (i) $DF = h$

$\therefore \frac{h}{AD} = \sin 20$

$h = AD \sin 20$ (1)

$AD = \frac{h}{\sin 20}$

(ii) $\frac{AD}{AG} = \cos 50°$

$AG = \frac{AD}{\cos 50°}$ (1)

$= \frac{h}{\sin 20 \cos 50}$

(iii) $\frac{h}{AG} = \sin\theta$

$\therefore \frac{AG \sin 20 \cos 50}{AG} = \sin\theta$ (1)

$\therefore \sin\theta = \sin 20 \cos 50$

$\theta = 13°$ (1)

5. (i) $y = \frac{x^2}{4a}$

$\frac{dy}{dx} = \frac{2x}{4a}$

$= \frac{x}{2a}$ (1)

At P $\frac{dy}{dx} = p$

$\therefore y - ap^2 = p(x - 2ap)$ (1)

$= px - 2ap^2$

$y = px - ap^2$

(ii) $y = px - ap^2$

$y = qx - aq^2$

$(p-q)x = a(p^2 - q^2)$ (1)

$x = a(p+q)$

$y = ap(p+q) - ap^2$

$= ap^2 + apq - ap^2$

$= apq$ (1)

$\therefore T$ is $(a(p+q), apq)$

$M = \left(\frac{2ap+2aq}{2}, \frac{ap^2 - aq^2}{2}\right)$

$= \left(a(p+q), \frac{a(p^2-q^2)}{2}\right)$ (1)

Since M has same x co-ordinate as T, MT is vertical. (1)

$\therefore$ MT is parallel to axis of parabola

b) $= \lim_{x \to -5} \frac{\sqrt{20-x} - 5}{5+x} \times \frac{\sqrt{20-x}+5}{\sqrt{20-x}+5}$ (1)

$= \lim_{x \to -5} \frac{20 - x - 25}{(5+x)(\sqrt{20-x}+5)}$

$= \lim_{x \to -5} \frac{-5-x}{(5+x)(\sqrt{20-x}+5)}$

$= \lim_{x \to -5} \frac{-1}{\sqrt{20-x}+5}$ (1)

$= -\frac{1}{10}$ (1)

c) $\Delta \geq 0$

$\therefore 16p^2r^2 - 16(p^2+q^2)(r^2-4q^2) \geq 0$ (1)

$16p^2r^2 - 16(p^2r^2 - 4p^2q^2 + q^2r^2 - 4q^4) \geq 0$

$64p^2q^2 - 16q^2r^2 + 64q^4 \geq 0$ (1)

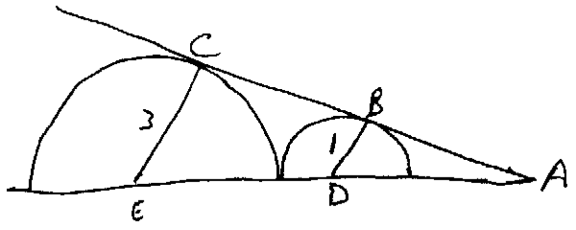
$4p^2q^2 - q^2r^2 + 4q^4 \geq 0$

$\therefore 4p^2q^2 + 4q^4 \geq q^2r^2$

$4q^2(p^2+q^2) \geq q^2r^2$ (1)

$p^2 + q^2 \geq \frac{r^2}{4}$

6. a) (i)



In Δ's ABD & ACE

$\hat{A}$ is common

$D\hat{B}A = E\hat{C}B = 90°$ (∠ between tangent & radius) (1)

∴ ΔABD ||| ΔACE (equiangular) (1)

∴ $\frac{AD}{AE} = \frac{1}{3}$ (sides in proportion)

$\frac{AD}{AD+4} = \frac{1}{3}$ (1)

$3AD = AD + 4$

$2AD = 4$

$AD = 2$ (1)

(ii) $\sin D\hat{A}B = \frac{1}{2}$

∴ $D\hat{A}B = 30°$ (1)

(iii) $AB^2 = 2^2 - 1^2$

$AB = \sqrt{3}$ cm (1)

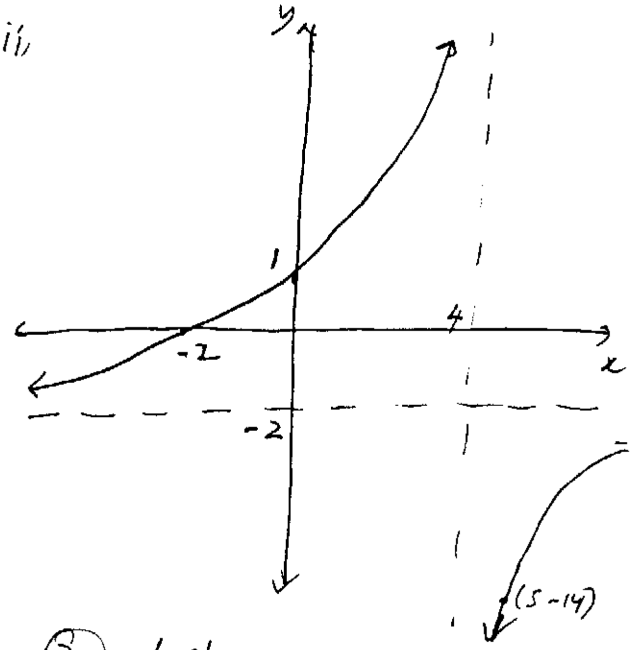
b) (i) $LHS = \frac{12 - 2(4-x)}{4-x}$

$= \frac{12 - 8 + 2x}{4-x}$

$= \frac{4 + 2x}{4-x}$ (1)

$= RHS$

(ii)



(2) 1 shape
1 asymptotes and axes

(iii) $-2 \leq x < 4$

(1)