Name: ____________________

Teacher: ____________________

SCEGGS Darlinghurst

HSC Assessment 1
14 February, 2003

Mathematics – Extension 1

General Instructions

- Time allowed: 60 minutes
- Weighting 25%
- This paper has **three** questions
- Attempt **all** questions and show all necessary working
- Marks will be deducted for careless or badly arranged work
- Write using blue or black pen, diagrams in pencil
- Start each question on a new page
- Write your name on each page
- Approved calculators, mathematical templates and geometrical instruments may be used

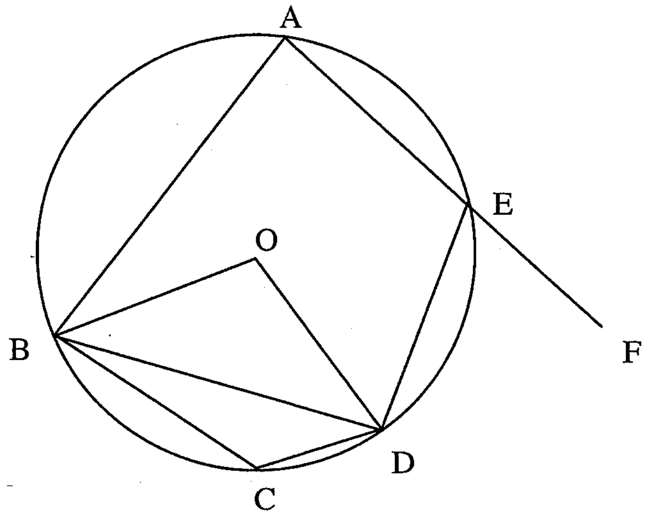
Question		Marks	Communication	Reasoning
1		/13	/5	/4
2		/11	/2	/4
3		/10	/2	/5
	TOTAL	/34	/9	/13

Question One (13 marks)

- Answer on the pad paper provided
- Write your name and at the top of the page
- Start each question on a new page
- Clearly label each question

Marks

(a)



Given: O is the centre of the circle
$\angle \text{BCD} = 150°$
$\angle \text{ABO} = 40°$

(i) mark all relevant information on the diagram (no reasons necessary) **3** C

(ii) state the value of $\angle \text{FED}$ **1**

(b) The polynomial:

$$\text{P}(x) = x^3 + 2x^2 - x + K$$

is divisible by $x - 1$

(i) Find the value of K **1**

(ii) Factorise $\text{P}(x)$ fully. **2**

(iii) Solve $\text{P}(x) \leq 0$ **2** C

(c) Prove, using Mathematical Induction, that $13 \times 6^n + 2$ is divisible by 5. **4** R
(n Positive integer.)

Question Two (11 marks)

- **START A NEW PAGE**

Marks

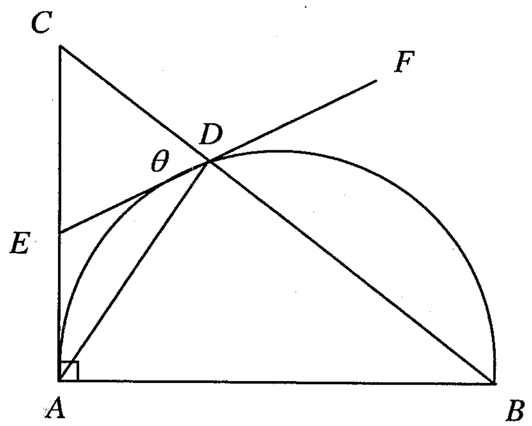
(a) The roots of the equation: **4**

$$x^3 - 12x^2 + 12x + 80 = 0$$

are 3 consecutive terms in arithmetic progression.

Find these roots.

(b)



AB is the diameter of a semi-circle. ΔABC is right-angled at A, and BC cuts the semi-circle at D. EF is a tangent to the semi-circle at D. AD is joined. $\angle CDE = \theta$.

- COPY OR TRACE THE DIAGRAM ONTO YOUR WRITING PAGE.

(i) Why is $\angle ADB = 90°$? 1 C

(ii) Why is $\angle ADE = \angle ABD$? 1 C

(iii) Name two angles equal to $\angle CDE$. 1

(iv) Prove ΔADE is isosceles. 2 R

(v) Prove that E is the midpoint of AC. 2 R

Question Three (10 marks)

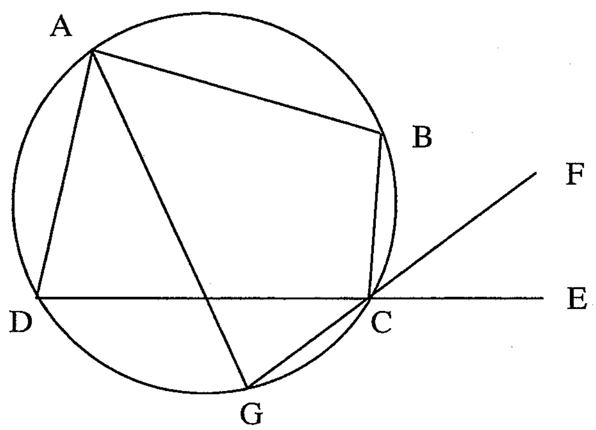
- **START A NEW PAGE**

Marks

(a) Use Mathematical Induction to prove that for all positive integers n — 5 R

$$\sum_{r=1}^{n} r\,(r\,!) = (n+1)!-1$$

(b)



Given AG bisects $\angle DAB$,

Prove that:

(i) $\angle DAG = \angle FCE$. — 2 C

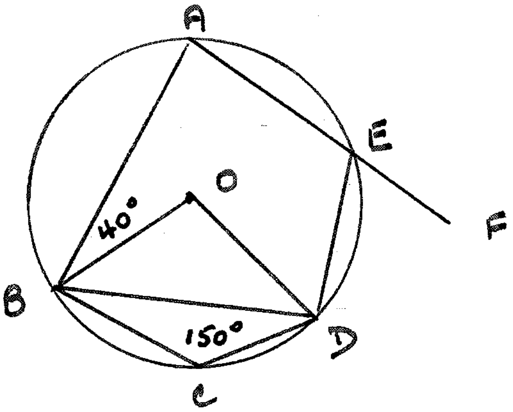
(ii) $\angle DAB = \angle BCE$. — 1

(iii) hence CF bisects $\angle BCE$. — 2

End of Paper

Extension 1 Assessment Test 1 14/2/03

1 a) (i)



✓ radii equal ($BO = OD$)

✓ reflex $\angle BOD = 300°$

($\text{or } \angle BOD = 60°$)

✓ $\angle ABD = \angle FED$

✓ (ii) $\angle FED = 100°$

b) (i) $P(1) = 1 + 2 - 1 + k = 0$

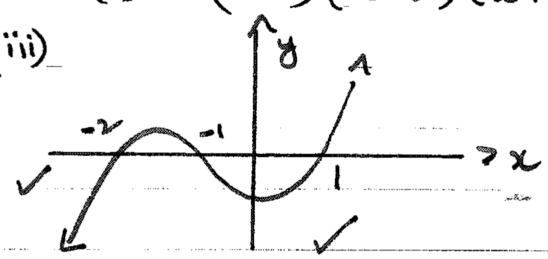
✓ $k = -2$

(ii) ✓

$$
\begin{array}{r}
x^2 + 3x + 2 \\
x - 1 \,\big)\, \overline{x^3 + 2x^2 - x - 2} \\
\underline{x^3 - x^2} \\
3x^2 - x - 2 \\
\underline{3x^2 - 3x} \\
2x - 2
\end{array}
$$

✓ $P(x) = (x-1)(x+2)(x+1)$

(iii)



✓ ✓

$x \le -2, \quad -1 \le x \le 1$

c) If $n = 1$, $13 \times 6 + 2 = 80$

which is divisible by 5

Assume true for $n = k$

i.e. $13 \times 6^k + 2 = 5M$

(M an integer)

consider $n = k + 1$

$$
\begin{aligned}
13 \times 6^{k+1} + 2 &= 13 \times 6^k \times 6 + 2 \\
&= (5M - 2) \times 6 + 2 \\
&= 30M - 12 + 2 \\
&= 30M - 10
\end{aligned}
$$

which is divisible by 5.

if true for $n = k$, the statement is true for $n = k + 1$. It is true for $n = 1$ and thus is true for $n = 2$, 3 etc.

i.e. true for all n positive integer.

2 a) If in A.P. let the roots be $a - d$, a, $a + d$. ✓

$\therefore a - d + a + a + d = 12$

$3a = 12$

$a = 4$ (i) ✓

$(a - d)\,a\,(a + d) = -80$

$4(16 - d^2) = -80$

$16 - d^2 = -20$

$d^2 = 36$

$d = 6$ or -6 ✓

if $d = 6$, terms are $-2, 4, 10$ ✓

if $d = -6$, terms are $10, 4, -2$

$\therefore$ terms are $-2, 4, 10$

b) (i) Angle in a semi circle is 90° ✓

(ii) angle between tangent and chord equals angle in alternate segment ✓

(iii) $\angle FDB$, $\angle DAB$ ✓

(iv) $\angle CDA = 90^\circ$ (adjacent angles supplementary)

$\therefore \angle EDA = 90^\circ - \theta$

$\angle EAB = 90^\circ$ (given)

$\therefore \angle EAD = 90^\circ - \theta$

$\therefore \Delta$ is isosceles (2 angles equal)

(v) EA = ED (opp. equal angles in Δ EAD)

$\angle ECD = 90^\circ - \angle ABD$ (angle sum of Δ CAB is 180°)

$= 90^\circ - \angle EDA$

But $\theta = 90^\circ - \angle EDA$ (above)

$\therefore \angle ECD = \theta$

$\therefore$ EC = ED (opp equal angles in Δ ECD)

$\therefore$ EA = EC

i.e. E is midpoint AC

(3) a) To prove that

$1.1! + 2.2! + \cdots + n.n! = (n+1)! - 1$

If $n = 1$, $1.1! = 1$

$2! - 1 = 1$

$\therefore$ true for $n = 1$

Assume true for $n = k$

i.e. $1.1! + 2.2! + \cdots k.k! = (k+1)! - 1$

Consider $n = k+1$

LHS $= 1.1! + 2.2! + \cdots k.k! + (k+1)(k+1)!$

$= (k+1)! - 1 + (k+1)(k+1)!$

$= (k+1)!\,[1 + k + 1] - 1$

$= (k+2)(k+1)! - 1$

$= (k+2)! - 1$

= RHS if $n = k+1$

$\therefore$ if true for $n = k$, the statement is true for $n = k+1$. It is true for $n = 1$ and is thus true for $n = 2, 3 \ldots$ i.e. all n +ve integers

b)

(i) $\angle DAG = \angle DCG$ (standing on same arc, equal)

$\angle FCE = \angle DCG$ (vertically opp. =)

$\therefore \angle DAG = \angle FCE$.

(ii) $\angle DAB = \angle BCE$ (ext $\angle$ of cyclic quadrilateral = int. opp angle)

(iii) $\angle DAB = 2\angle DAG$ (AG bisects $\angle DAB$, given)

$\therefore \angle BCE = 2\angle DAG$

but $\angle FCE = \angle DAG$ (proved)

$\therefore \angle BCF = \angle DAG$

i.e. CF bisects $\angle BCE$.