THE SCOTS COLLEGE

Extension 1 Mathematics

HSC Assessment 2

25th February 2011

Time Allowed: 45 minutes

Instructions:

- Show all necessary workings
- Approved non-programmable calculators may be used
- Begin a new sheet of paper for each question

Outcomes to be assessed:

Methods of Integration: P2, P8, H4, H5,H8, PE2, HE6 - E11.5, E13.6
Polynomials: PE3 - 16.1 – 16.4

Calculus	Q1, Q2	/20
Functions	Q3	/12
	TOTAL	/32

QUESTION ONE (10 MARKS) BEGIN A NEW SHEET OF PAPER

a) *i)* Differentiate $y = xsin3x$ *(1)*

ii) Hence find $\int xcos(3x)dx$ *(2)*

b) Using the substitution $u = 2 + x$, find $\int x(2+x)^4 dx$ *(3)*

c) Using the substitution $x = 2cos\theta$, find $\int_0^1 \sqrt{4-x^2}dx$ *(4)*

QUESTION TWO (10 MARKS) BEGIN A NEW SHEET OF PAPER

a) *(i)* Show that the curve $f(x) = 3 - x\log_e x$ has an *x-intercept* between $x = 2$ and $x = 3$. *(2)*

(ii) Use two applications of the halving the interval method to find between which two numbers the curve cuts the x-axis, with a difference of 0.25. *(3)*

b) The curve $y = 2\sin\left(\frac{x}{2}\right)$ and the line $y = \frac{x}{3}$ are on the same number plane.

(i) Show that the *x-coordinates* of their points of intersection satisfy the equation $6sin\left(\frac{x}{2}\right) - x = 0$. *(1)*

(ii) By sketching the graphs of the two functions, show that the equation in part *(i)* has a root between 4 and 5. *(2)*

(iii) Using 4.5 as a first approximation, use Newton's method once to find another approximation to the root correct to one decimal place. *(2)*

QUESTION THREE (12 MARKS) BEGIN A NEW SHEET OF PAPER

a) *(i)* Using long division, prove that $(2x - 3)$ is a factor of $P(x) = 6x^3 + 5x^2 - 33x + 18$. *(2)*

(ii) Hence completely factorise $P(x)$. *(2)*

b) $(x - k)$ is a factor of $x^2 - 5x + (2k + 2)$. Using the factor theorem, find the values of k. *(2)*

c) When the polynomial $P(x) = ax^3 + ax^2 + 3x + 3$ is divided by $(x + 2)$, the remainder is 5. Find the value of a, using the remainder theorem. *(2)*

d) If α, β, γ are the roots of $x^3 - 2x^2 + 3x + 7 = 0$, find the values of:

(i) $\frac{2}{\alpha} + \frac{2}{\beta} + \frac{2}{\gamma}$ *(2)*

(ii) $\alpha^2 + \beta^2 + \gamma^2$ *(2)*