THE SCOTS COLLEGE

Assessment Task

Year 12

Ext-1 Mathematics

HSC Task 1: Tuesday 21st February,
Week 4, Term 1 2012

Weighting of Task:10%

Time allowed: 45 minutes

Directions:

- All 12 questions must be attempted on your own paper.
- Questions 1 to 5 are Multiple Choice A, B, C, or D only.
- Write your name on each page.
- Start a new page for questions 6 to 12.
- Show all necessary working in questions 6 to 12.
- Approved calculators may be used.

Topics Assessed

Integration of Trigonometric functions.(Q,9)	**/5**
Solution of trigonometric equations.(Q,3,6,12)	**/8**
Polynomials and Numerical Estimation of Roots.(Q,1,7,10)	**/10**
Inverse Functions and Inverse Trigonometric Functions.(Q,2,4,5,8,11)	**/17**

Total Marks: **/40**

Questions 1 to 5 ONLY are multiple choice (1 mark each).
For questions 1 to 5, write A, B, C or D on your answer sheet.

1. When $x^2+4x+5b$ is divided by x the remainder is -10.

   The value of b is

(A) 2 (B) -12 (C) -2 (D) 0

2. The domain of $y=-2\sin^{-1}(1-4x)$ is

(A) $\frac{-\pi}{2}\le x\le\frac{\pi}{2}$ (B) $\frac{-1}{2}\le x\le\frac{1}{2}$ (C) $0\le x\le\frac{1}{2}$ (D) $\frac{-\pi}{8}\le x\le\frac{\pi}{8}$

3. $2\cos(t-\frac{\pi}{3})$ can be rewritten as

(A) $\cos t-\sqrt{3}\sin t$ (B) $2\cos t-2\cos\frac{\pi}{3}$

(C) $\cos t+\sqrt{3}\sin t$ (D) $\sin t-\sqrt{3}\cos t$

4. $\cos^{-1}(-x)=$

(A) $\pi-\cos^{-1}x$ (B) $\pi+\cos^{-1}x$ (C) $\sin^{-1}x$ (D) $-\sin^{-1}x$

5. $\sin(2\tan^{-1}\frac{2}{3})=$

(A) $\frac{\sqrt{13}}{12}$ (B) $\frac{12}{\sqrt{13}}$ (C) $\frac{6}{\sqrt{13}}$ (D) $\frac{12}{13}$

End of Multiple Choice Section

For Questions 6 to 12 start each question on a new page and show all working.

6.

$5\text{Cos}\theta + 12\text{Sin}\theta \equiv R\text{Cos}(\theta - \alpha)$ where R>0 and α is acute.

(a) Find the value of R. (2)

(b) Find the size of α to 3 decimal places. (1)

(c) Hence solve to 3 decimal places

$5\text{Cos}\theta + 12\text{Sin}\theta = 13$ for $0 \le \theta \le 4\pi$. (2)

7.

(a) Show that $\sin x = x - 1$ has a root near $x = 2$ (2)

(b) Use Newton's Method once to find a better approximation to this root.
(Answer to 2 decimal places) (2)

8.

The function $f(x) = 3x - x^3$ has a minimum turning point at (-1,-2) and a maximum turning point at (1,2).

(a) Sketch $y = f(x)$ showing given turning points and intercepts on both axes. (2)

(b) Find the largest domain containing the origin for which $f(x)$ has an inverse function $y = f^{-1}(x)$. (1)

(c) Find the domain and range of $y = f^{-1}(x)$. (2)

(d) Sketch $y = f^{-1}(x)$ clearly showing the end points. (3)

9.

(a) Find $\int \sin^2 3x\,dx$ (2)

(b) Find in terms of π the volume of the solid formed when $y = \cos x$ is rotated about the x-axis from $x = 0$ to $x = \frac{\pi}{2}$. (3)

10.

(a) If α, β, γ are the roots of $x^3 + 4x - 9 = 0$ find $\alpha(\beta+1)+\beta(\gamma+1)+\gamma(\alpha+1)$ (2)

(b) Given that $Q(x) = 4x^3 + kx + 6$ has a root at $x = -3$.

(i) Find k. (1)

(ii) Write $Q(x)$ in the form $(x+3)(ax^2+bx+c)$ (2)

11. Given the function $f(x) = 2\sin^{-1}(\frac{x}{3})$

(a) Find $f(0)$ (1)

(b) State the domain and range of $y = f(x)$ (2)

(c) Draw the graph of $y = f(x)$ showing the end points. (3)

12. Given $\sqrt{2}\cos\theta = 1$

(a) Write the general solution for this equation in terms of π. (1)

(b) Solve for n = -1 and n = 2. Answer in terms of π. (1)

END OF EXAMINATION

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

2012 Ext-1 HSC Task 1 - Solutions

Q1 $P(0) = -10$

$\therefore 5b = -10$

$b = -2$ (C)

Q2 $-1 \leq 1 - 4x \leq 1$

$-2 \leq -4x \leq 0$

$\frac{1}{2} \geq x \geq 0$ (C)

Q3 $2\cos\left(t - \frac{\pi}{3}\right) = 2\cos t \cos\frac{\pi}{3} + 2\sin t \sin\frac{\pi}{3}$

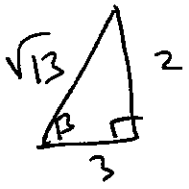
$= 2\cos t \times \frac{1}{2} + 2\sin t \times \frac{\sqrt{3}}{2}$

$= \cos t + \sqrt{3}\sin t$. (C)

Q4 $\cos^{-1}(-x) = \pi - \cos^{-1}x$ (A)

Q5 $\sin\left(2\tan^{-1}\frac{2}{3}\right) = \sin 2B$

where $\tan B = \frac{2}{3}$



$\therefore \cos B = \frac{3}{\sqrt{13}}$

$\sin B = \frac{2}{\sqrt{13}}$

$= 2\sin B \cos B$

$= 2 \times \frac{2}{\sqrt{13}} \times \frac{3}{\sqrt{13}}$

$= \frac{12}{13}$. (D)

Q6 (a) $\frac{5}{R}\cos\theta + \frac{12}{R}\sin\theta \equiv \cos(\theta - \alpha)$ (1)

$\equiv \cos\theta\cos\alpha + \sin\theta\sin\alpha$

$\therefore \cos\alpha = \frac{5}{R}$ $\sin\alpha = \frac{12}{R}$

$\therefore R = \sqrt{12^2 + 5^2}$

$R = 13$ (1)

(b) $\tan\alpha = \frac{12}{5}$

$\alpha = 1.176$ (1)

(c) (1)

$\therefore 13\cos(\theta - 1.176) = 1$

$\cos(\theta - 1.176) = 1$

$\theta - 1.176 = 0, 2\pi$

$\theta = 1.176, 2\pi + 1.1176$

$\theta = 1.176^c, 7.459^c$

7. $f(x) = \sin x - x + 1$

(a) $f(1.9) = 0.046$ above x-axis (1)

$f(2.1) = -0.0237$ below x-axis (1)

$\therefore$ root exists between 1.9 and 2.1.

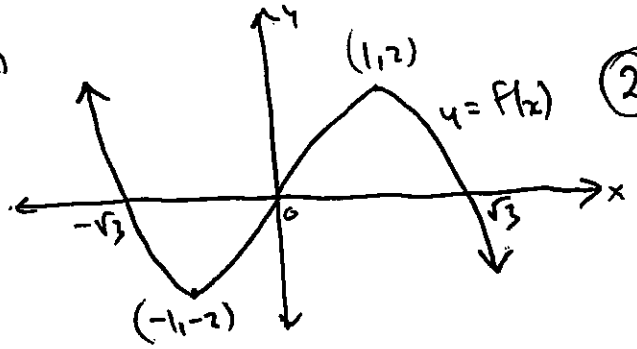
(b) $a_1 = a - \dfrac{f(a)}{f'(a_1)}$

$= \dfrac{2 - (\sin 2 - 2 + 1)}{\cos 2 - 1}$ (1)

$= 1.94$ (1)

8. $f(x) = 3x - x^3$

(a)



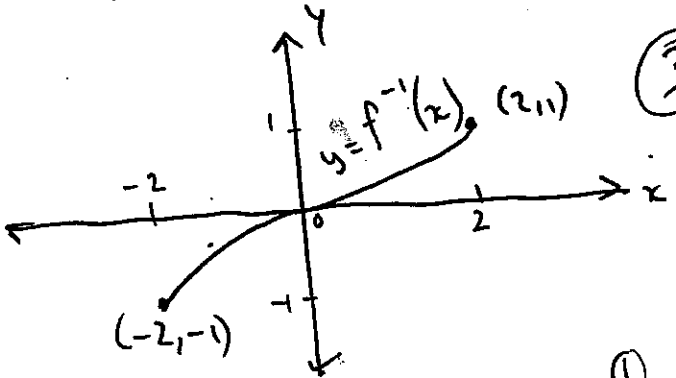
(2) – roots / y intercept
– tp

(b) D_f : $-1 \le x \le 1$ (1)

(c) $D_{f^{-1}}$: $-2 \le x \le 2$ (2)

$R_{f^{-1}}$: $-1 \le y \le 1$

(d)



(3) – end pts
– concavity
– origin

9. (a) $\int \sin^2 3x\,dx = \frac{1}{2}\int 1 - \cos 6x\,dx$ (1)

$= \frac{1}{2}\left(x - \frac{1}{6}\sin 6x\right) + c$

$= \frac{1}{2}x - \frac{1}{12}\sin 6x + c$ (1)

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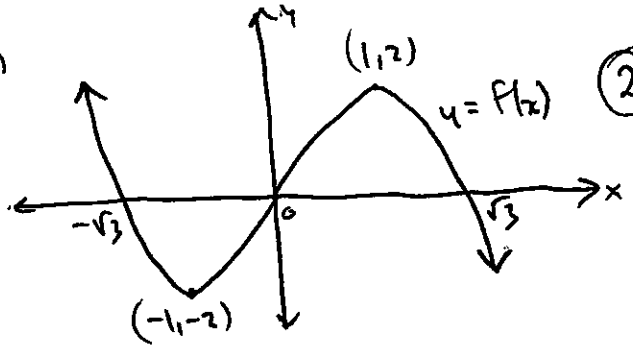

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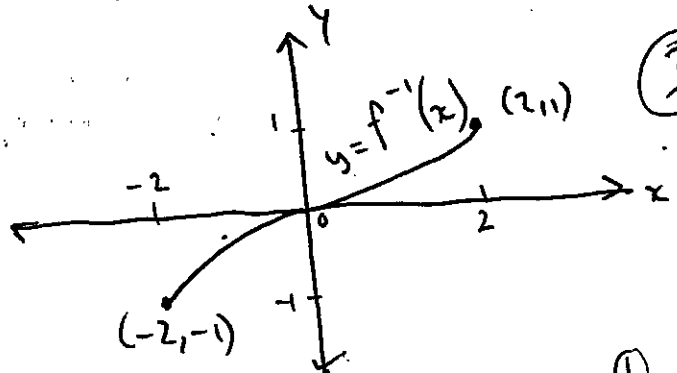

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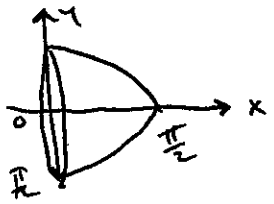
9. (b)



$V = \pi \int_0^{\frac{\pi}{2}} \cos^2 x \, dx$ ✓

$= \frac{1}{2}\pi \int_0^{\frac{\pi}{2}} (1 + \cos 2x) \, dx$

$= \frac{1}{2}\pi \left[x + \frac{1}{2}\sin 2x \right]_0^{\frac{\pi}{2}}$

$= \frac{1}{2}\pi \left[\left(\frac{\pi}{2} + \frac{1}{2}\sin \pi \right) - \left(0 + \frac{1}{2}\sin 0 \right) \right]$ ✓

$= \frac{1}{2}\pi \left[\frac{\pi}{2} + \frac{1}{2} \right]$

$V = \frac{\pi^2 + \pi}{4}$ u³. ✓

10 (a) $\alpha(\beta+1) + \beta(\gamma+1) + \gamma(\alpha+1) = \alpha\beta + \alpha + \beta\gamma + \beta + \gamma\alpha + \gamma$ ✓

$= \alpha\beta + \beta\gamma + \gamma\alpha + \alpha + \beta + \gamma$

$= 4 + 0$

$= 4.$ ✓

(b) (i) $0 = -108 - 3k + 6$

$3k = -102$

$k = -34.$ ✓

(ii) $x+3 \overline{)\, 4x^3 - 34x + 6}$ ✓ $\quad 4x^2 - 12x + 2$

0

$\therefore Q(x) = (x+3)(4x^2 - 12x + 2)$ ✓

11 (a) $f(0) = 2\sin^{-1}(0)$ ✓

$f(0) = 0$

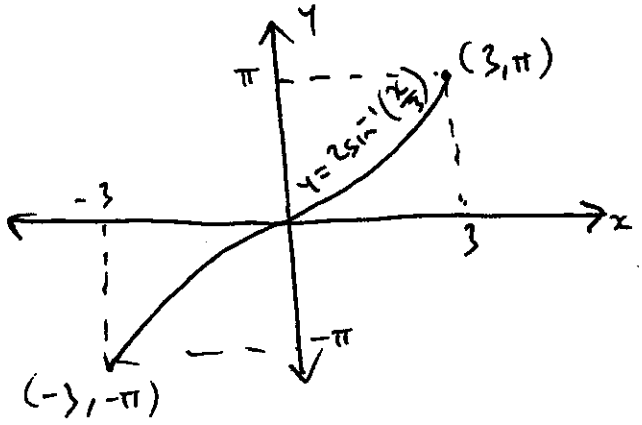
(b) $D_f: -1 \leq \frac{x}{3} \leq 1$

$-3 \leq x \leq 3$

$R_f: -\frac{\pi}{2} \leq \frac{y}{2} \leq \frac{\pi}{2}$

$-\pi \leq y \leq \pi.$

(c)



3 - endpts

- origin

- concavity.

12 $\cos\theta = \frac{1}{\sqrt{2}}$

(a) $\theta = 2n\pi \pm \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ ✓

$\theta = 2n\pi \pm \frac{\pi}{4}$

(b) $n = -1$ $\quad n = 2$ ✓

$\theta = -2\pi \pm \frac{\pi}{4}$, $\theta = 4\pi \pm \frac{\pi}{4}$.