Sydney Girls High School

2006 Assessment Task 2

MATHEMATICS

Extension Two

Year 12

Time allowed - 90 minutes (plus 5 minutes reading time)

Topic: Complex Numbers

Instructions

- Attempt all twelve questions.
- Questions are not of equal value.
- All necessary working should be shown in every question.
- Marks may be deducted for careless or badly arranged work.
- Write on one side of the paper only.
- Diagrams are not to scale.

Question One (5 marks)
Given $z_1 = 2-3i$ and $z_2 = 1+i$, find each of the following writing your answer in the form $a+bi$.

a) $z_1 + z_2$

b) $z_1 - z_2$

c) $z_1 z_2$

d) $\dfrac{z_1}{z_2}$

e) $\dfrac{\overline{z_1}}{i}$

Question Two (6 marks)
Sketch the locus of the following on separate diagrams

a) $\{\text{Re}(z)\}\times\{\text{Im}(z)\} \le 4$

b) $|z| < 2 \cap \text{Re}(z) \ge 0$

c) $\dfrac{3\pi}{4} \le \arg(z+1) \le \pi \cap 0 \le \arg(z+2) \le \dfrac{\pi}{4}$

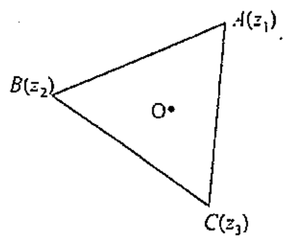
Question Three (6 marks)

a) Simplify $i^{16} - (1+i^5)^2(-i)^3$

b) Prove $\overline{z_1}\ \overline{z_2} = \overline{z_1 z_2}$ (let $z_1 = a+ib,\ z_2 = c+id$)

c) A quadratic equation with real coefficients has a root equal to $7-2i$. Find the equation.

Question Four (4 marks)
An equilateral triangle has its centre at the origin(O). The vertex at A is represented by



the complex number $z_1 = 2+2i$. Find the complex numbers that represent the points B and C

Question Five (10 marks)

a) Find the square roots of (-15-8i)

b) Hence or otherwise solve $z^2+(2i-3)z+5-i=0$

c) i) Express $(1+i)(-1+\sqrt{3}i)$ in the form $a+ib$ and mod-arg form

ii) Hence or otherwise find the exact value of $\cos\frac{11\pi}{12}$

Question Six (10 marks)

a) Find real numbers x and y such that $2x+2ixy+iy=8+3i$

b) Simplify $(1+\sqrt{3}i)^7(2-2i)^4$ (answer in the form $a+ib$)

c) Simplify $\left(\frac{1+i}{2}\right)^{24}$

d) Given $z=\sqrt{3}+i$:

i) Find the smallest positive integer n for which z^n is real

ii) Evaluate z^n for this value of n

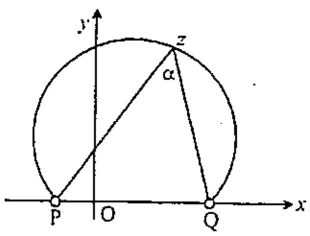
Question Seven (6 marks)

a) Find the Cartesian equation of the locus of z if $|z-4-5i|=|z+2+3i|$

b) Find the Cartesian equation of the locus of z if $|z+2-4i|=2|z-4-i|$

c) Describe the locus of b) above

Question Eight (7 marks)



P has co ordinates (-2, 0)
Q has co ordinates (4, 0)

The diagram shows the locus of the points z on the complex plane such that $\arg(z-4)-\arg(z+2)=\frac{\pi}{4}$. The locus is the major arc of a circle. The angle PzQ is α. Copy the diagram.

a) Show why $\alpha=\frac{\pi}{4}$

b) Find the equation of the locus of z (include any restrictions in your answer)

Question Nine (12 marks)

Given the equation $z^5 = -1$

a) Solve over the complex field

b) Factorise $z^5 + 1$ over the complex field

c) Factorise $z^5 + 1$ over the real field

d) Factorise $z^5 + 1$ over the rational field

e) Show $\cos\frac{\pi}{5} + \cos\frac{3\pi}{5} = \frac{1}{2}$

f) Sketch your solutions on an Argand diagram .Find the exact perimeter of the pentagon formed by the five roots.

Question Ten (7 marks)

a) Sketch the locus given by $|z - 2 - 4i| = 2$

b) Hence find: i) $\max|z|$

ii) max $\arg(z)$

iii) min $\arg(z)$ (answer to the nearest degree)

Question Eleven (6 marks)

a) Let $\frac{a-ib}{a+ib} = x + iy$ hence prove that $|x+iy| = 1$

b) Find the Cartesian equation of the locus of z if $w = \frac{z+1}{z+2i}$ and w is purely real

Question Twelve (11 marks)

De Moivre's Theorem states that $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ where n is an integer

a) Use mathematical induction to prove De Moivre's Theorem for n a positive integer.

b) Hence or otherwise show that $z^n + \frac{1}{z^n} = 2\cos n\theta$ where $z = \cos\theta + i\sin\theta$

c) Hence or otherwise express $\cos^6\theta$ in the form $p\cos 6\theta + q\cos 4\theta + r\cos 2\theta + s$

$\alpha\beta\chi\delta\varepsilon\phi\varphi\gamma\eta\iota\kappa\lambda\mu\nu o\pi\varpi\theta\vartheta\rho\sigma\varsigma\tau\upsilon\omega\xi\psi$