HURLSTONE AGRICULTURAL HIGH SCHOOL

MATHEMATICS

2009

YEAR 12

ASSESSMENT TASK 3

EXAMINERS ~ S. FAULDS, P. BICZO, S. HACKETT

GENERAL INSTRUCTIONS

- Reading Time – 3 minutes.
- Working Time – 40 minutes.
- Attempt **all** questions.
- Questions are of equal value.
- **All** necessary working should be shown in every question.
- This paper contains three (3) questions.
- Marks may not be awarded for careless or badly arranged work.
- Board approved calculators and MathAids may be used.
- **Each question is to be started in a new answer booklet.** Additional booklets are available if required.
- This examination must **NOT** be removed from the examination room

STUDENT NAME: _____

CLASS TEACHER: _____

QUESTION 1 **10 marks** Start a NEW answer booklet

Marks

(a) Use the trapezoidal rule with 4 function values to find an approximation to **2**

$$\int_1^4 x \log_{10} x \, dx .$$

Give your answer to 2 decimal places.

(b) The probability that Jessmindar will score a goal in any of her soccer matches is 15%.
Jessmindar plays three games of soccer.

(i) Draw a **probability tree** to demonstrate the possible outcomes. **2**

(ii) Find the probability that Jessmindar scores in exactly two of the matches. **2**

(c) The probability that a polar bear kept in captivity will produce an offspring that will survive to infancy is 0.3. If the zoo has n pregnant polar bears:

(i) Explain why the probability that **at least one** polar bear will produce an offspring that survives to infancy is $1 - 0.7^n$. **2**

(ii) Hence, find the least value of n that gives a greater than 75 % chance of producing at least one polar bear offspring that survives to infancy. **2**

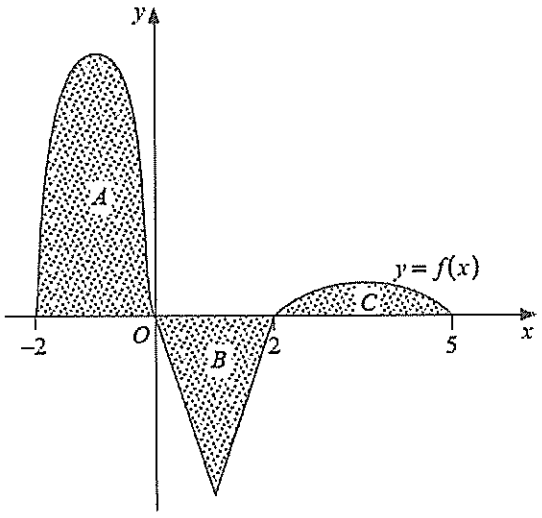
QUESTION 2 **10 marks** Start a NEW answer booklet

Marks

(a) Find a primitive function of x^4. **1**

(b) Evaluate $\int_1^4 \frac{1}{\sqrt{x}}\,dx$. **2**

(c) The graph of the function $y = f(x)$ is shown. **1**

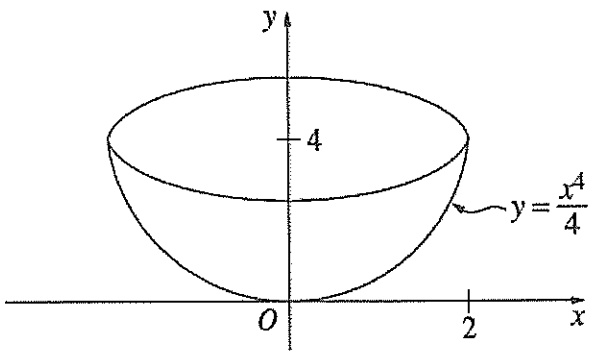


The shaded areas are bounded by $y = f(x)$ and the x axis. The shaded area A is 10 square units, the shaded area B is 6 square units and the shaded area C is 2 square units.

Evaluate $\int_{-2}^{5} f(x)dx$.

(d) Find the area bounded by the line $y = x$ and the curve $y = x^2$. **2**

(e)



NOT TO SCALE

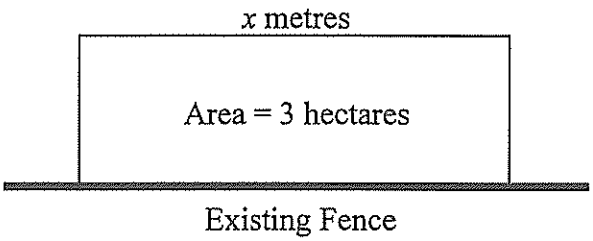
A bowl is formed by rotating the part of the curve $y = \frac{x^4}{4}$ between $x = 0$ and $x = 2$ about the y axis. **4**

Find the volume of the bowl using integration.

QUESTION 3 **10 marks** Start a NEW answer booklet

Marks

(a)



A farmer wishes to fence off a new paddock of area 3 hectares as shown in the diagram. New fencing material will be required at a cost of \$5/metre on three sides of the paddock but he can use an existing fence for the remaining side, which will cost \$2.50/metre to repair.

(i) If the length of the paddock is x metres, find an expression for the width of the paddock in terms of x and hence show that the cost of this new paddock can be expressed in dollars as: **1**

$$C = \frac{15x}{2} + \frac{300000}{x}$$

(ii) Use calculus to find the value of x which will minimise the cost of fencing. **2**

(iii) Calculate the minimum cost of fencing the paddock. **1**

(b) For the curve $y = x^3 - 6x^2 + 9x - 4$, find:

(i) the co-ordinates of any stationary points and determine their nature. **3**

(ii) the co-ordinates of the point of inflexion, and **1**

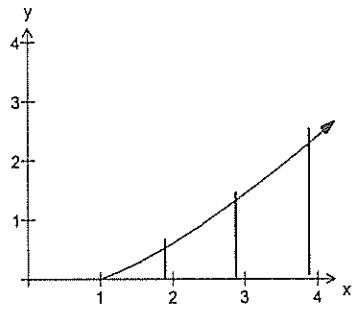
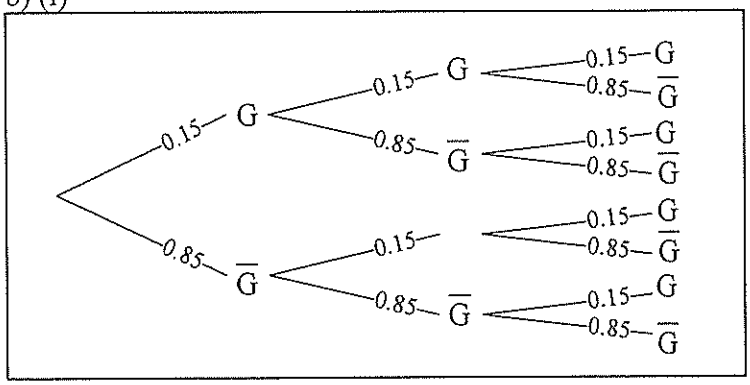
(iii) draw a neat sketch of the curve showing these features. **2**

Year 12 Task 3	Mathematics	Examination 2009
Question No.1	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

H5 applies appropriate techniques from the study of probability to solve problems

H8 uses techniques of integration to calculate areas and volumes

Outcome	Solutions	Marking Guidelines
H8	a)  $\int_1^4 x\log_{10}x\,dx = \frac{1}{2}\times 1\left(1\log_{10}1+2\log_{10}2\right)+$ $\frac{1}{2}\times 1\left(1\log_{10}1+2\log_{10}2\right)+\frac{1}{2}\times 1\left(1\log_{10}1+2\log_{10}2\right)$ $=\frac{1}{2}\left(0+2\left(2\log_{10}2+3\log_{10}3\right)+4\log_{10}4\right)$ = 3.24 to 2 d.p.	2 marks : correct calculation and answer 1 mark: substantial progress towards correct answer
H5	b) (i) 	2 marks: correct diagram 1 mark : substantial progress towards correct answer
	(ii) P(G,G, $\overline{G}$ or G, $\overline{G}$,G or $\overline{G}$,G,G) = $0.15\times 0.15\times 0.85+0.15\times 0.15\times 0.85+0.15\times 0.15\times 0.85)$ $=\frac{459}{8000}$	2 marks : correct answer 1 mark : substantial progress towards correct answer
H5	c)(i) If surviving cub = S, with P(S) = 0.3, then $P(\overline{S})=0.7$. For *n* cubs, P(at least one surviving) = $1-P(\text{none surviving})$ $=1-P(\overline{S},\overline{S},\overline{S},\ldots)$ $=1-0.7\times 0.7\times 0.7\times\ldots$ (n times) $=1-0.7^n$	2 marks : correct justification 1 mark : substantial progress towards correct justification

	(ii) For $1-0.7^n > 0.75$, $0.25 > 0.7^n$, $\log_{10} 0.25 > \log_{10} 0.7^n$ $\log_{10} 0.25 > n \log_{10} 0.7$ $\frac{\log_{10} 0.25}{\log_{10} 0.7} < n$ (as $\log_{10} 0.7$ is negative) $\therefore n > 3.88....$ $\therefore$ least value of n is 4	2 marks : correct answer with justification 1 mark : substantial progress towards correct answer or correct justification

Year 12	Mathematics	Task 2 ~ 2008/9
Question No. 2	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

Outcome	Sample Solution	Marking Guidelines
	a) $\frac{x^5}{5}$	**1 mark** ~ Correct answer with or without constant term
	b) $\int_1^4 \frac{1}{\sqrt{x}}dx = \int_1^4 x^{-\frac{1}{2}}dx$ $= \left[\frac{x^{\frac{1}{2}}}{1/2}\right]_1^4$ $= \left[2\sqrt{x}\right]_1^4$ $= 4-2$ $= 2$	**2 mark** ~ Correct solution **1 mark** ~ Correct integration of $\frac{1}{\sqrt{x}}$
	c) 6	
	d) Points of intersection (0,0) and (1,1) $Area = \int_0^1 (x-x^2)dx$ $= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$ $= (\frac{1}{2} - \frac{1}{3}) - (0-0)$ $= \frac{1}{6} units^2$	**1 mark** ~ Correct answer. **2 marks** ~ Correct solution **1 mark** ~ Correctly states points of intersection
	e) $y = \frac{x^4}{4} \rightarrow x^2 = 2\sqrt{y}$ $Volume = \pi \int_0^4 2\sqrt{y}dy$ $= 2\pi \left[\frac{y^{3/2}}{3/2}\right]_0^4$ $= \frac{4\pi}{3}(8-0)$ $= \frac{32\pi}{3} units^3$	**4 marks** ~ Correct solution **3 marks** ~ Correctly integrates $2\sqrt{y}$ showing correct bounds **2 marks** ~ Shows correct bounds on correct integral **1 mark** ~ Correctly rearranges $y = \frac{x^4}{4}$

Year 12 Mathematics Task 3 Examination 2009

Question No. 3 Solutions and Marking Guidelines

Outcomes Addressed in this Question

H5 applies appropriate techniques from the study of calculus to solve problems
H6 uses the derivative to determine the features of the graph of a function

Outcome	Solutions	Marking Guidelines
H5	**(a) (i)** Length of paddock $= x$ m Area of paddock $= 30000\text{m}^2$ $\therefore$ Breadth $= \frac{30000}{x}$ m Cost of fencing $C = 5\left(2\times\frac{30000}{x}+x\right)+\frac{5}{2}x$ $= \frac{300000}{x}+5x+\frac{5x}{2}$ $= \frac{300000}{x}+\frac{10x}{2}+\frac{5x}{2}$ $= \frac{15x}{2}+\frac{300000}{x}$ As required	**1 mark** Correctly shows result in a clear and logical way.
H5	**(ii)** $\frac{dC}{dx} = \frac{15}{2}-\frac{300000}{x^2}$ $\frac{d^2C}{dx^2} = \frac{600000}{x^3}$ Since $x>0$, $\frac{d^2C}{dx^2}>0$, any stationary points will be minimums Stationary pts. occur when $\frac{dC}{dx}=0$ ie. $\frac{15}{2}-\frac{300000}{x^2}=0$ $15x^2 = 600000$ $x^2 = 40000$ $x = 200$m $\therefore$ Minimum cost occurs when $x = 200$m.	**2 marks** Correct solution **1 mark** Demonstrates some understanding of finding the minimum value of a function using calculus.
H5	**(iii)** Cost of fencing: $C = \frac{15\times200}{2}+\frac{300000}{200}$ $= 1500+1500$ $= \$3000$	**1 mark** Correct answer OR answer consistent with minimum value found in (ii)
H6	**(b) (i)** $y = x^3-6x^2+9x-4$ $\frac{dy}{dx} = 3x^2-12x+9$ $= 3(x^2-4x+3)$ $\frac{d^2y}{dx^2} = 6x-12$ Stationary points occur when $\frac{dy}{dx}=0$ ie. $3(x^2-4x+3)=0$ $3(x-1)(x-3)=0$ $\therefore x = 1,3$ When $x=1$, $y=0$, $\frac{d^2y}{dx^2}=-6<0$ $\therefore$ Maximum turning point at (1, 0) When $x=3$, $y=-4$, $\frac{d^2y}{dx^2}=6>0$ $\therefore$ Minimum turning point at (3, -4)	**3 marks** Correct solution **2 marks** Solution substantially correct with a single error in finding co-ordinates of stationary points or in their classification **1 mark** Finds the first two derivatives of the function.

H6 (ii)

Possible points of inflexion occur when $\frac{d^2y}{dx^2} = 0$

ie. $6x - 12 = 0$

$x = 2$

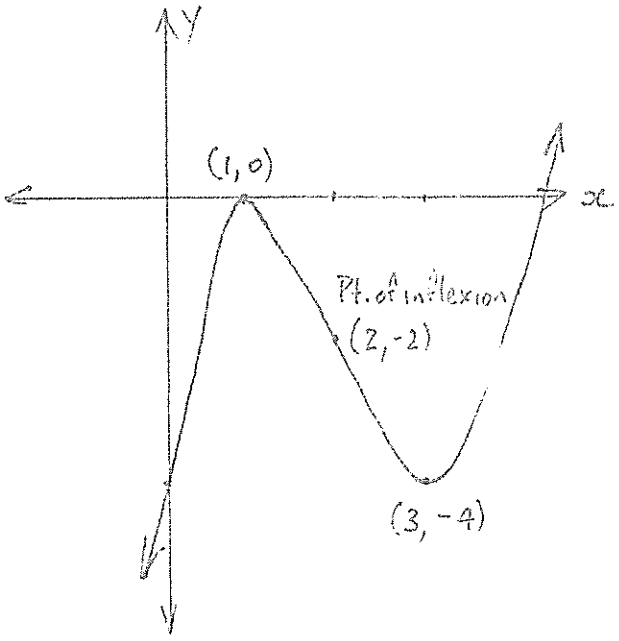
x	1	2	3
$\frac{d^2y}{dx^2}$	$-ve$	0	$+ve$

Sign change in $\frac{d^2y}{dx^2} \Rightarrow$ Change in concavity

$\therefore$ Point of inflexion at (2, -2)

1 mark
Correct solution

H6 (iii)



2 marks
Sketch of function drawn correctly, showing stationary points and point of inflexion labelled.

1 mark
Sketch substantially correct and neatly drawn with a single minor error in showing the critical points.