YEAR 12 TERM 2 ASSESSMENT 1996
MATHEMATICS 2UNIT.

TIME ALLOWED 85 MINS

INSTRUCTIONS:
All necessary working should be shown.
This is an OPEN BOOK test
Each section should be started on a new page.

Section A (Start a new page)

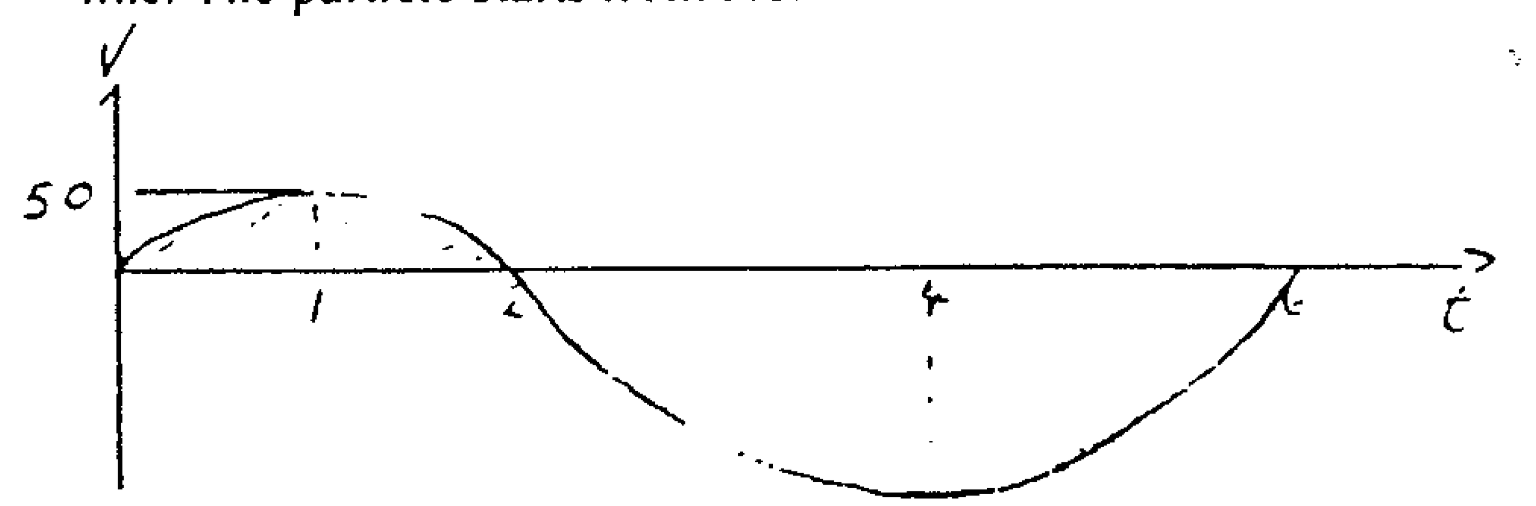
(a) A particle moves in a straight line in such a way that its distance in metres from the origin after t secs is given by $x = 2t^3 + 3t^2 - 36t + 10$.
(i) Find the velocity and acceleration at any time t
(ii) Find its initial position.
(iii) In which direction is the particle initially moving? Give a reason.
(iv) When and where does it come to rest?

(b) A bag contains green, black and red jellybeans. Therefore if I choose one jellybean at random from the bag, the probability that it is black is $\frac{1}{3}$. Is this statement true or false? Explain why, in no more than one sentence.

(c) A box contains 5 black, 7 red and 4 white marbles. One marble is drawn. Find the probability that :
(i) it is red or white
(ii) it is not white

Section B (Start a new page)
(a) A particle moves in a straight line in such a way that its velocity in ms^{-1} at time t secs is given by $v(t) = at^2 - bt + c$ where a,b and c are constants.
Knowing $v(1) = 1$, $v(2) = 9$ and $v(0) = 3$:
(i) find a,b and c
(ii) by considering the discriminant show that the particle never comes to rest.

(b) The graph represents the velocity v m/s of a particle after t seconds travelling in a straight line. The particle starts from rest.



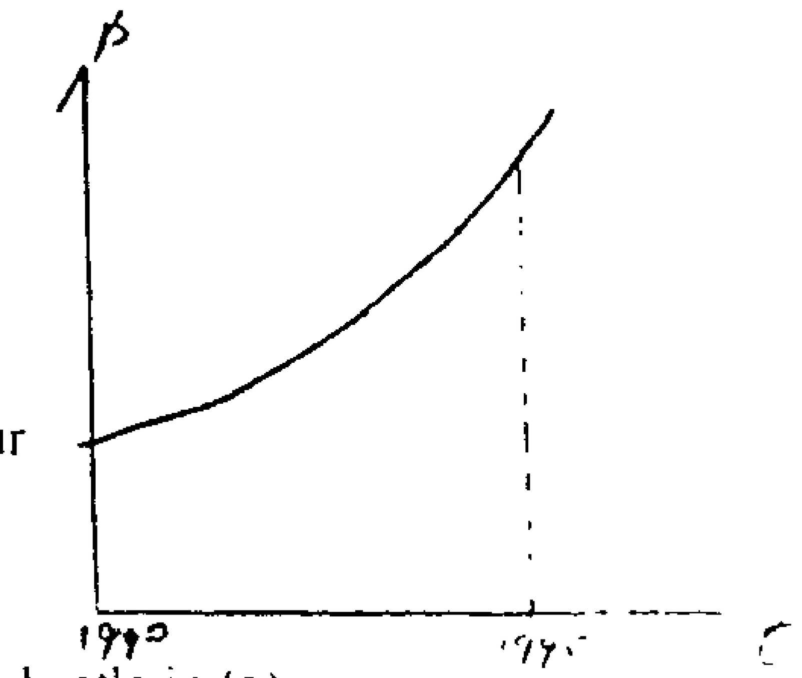
(i) What is the velocity of the particle after 1 second?
(ii) When does the particle change direction ?
(iii) When is the acceleration of the particle zero ?
(iv) What happens to the particle after 6 seconds ?
(v) Explain what is represented by the shaded region in the diagram

Section C (Start a new page)

(a) The population of kangaroos on an island is decreasing according to the formula $N = N_0e^{-kt}$ where N is the kangaroo population and k is a constant,t is time in years.
(i) If the population at the end of 1980 was 7000 and at the end of 1990 was 4000, show that the value of k is 0.0560 correct to 4 decimal places.
(ii) Find in how many years (from 1980) will it take for the population to be 40% of the original population
(iii) What will the population be after 5 years?
(iv) What is the rate of change at $t = 5$?

(b) A box contains 5 black, 3white and 2 pink plastic discs. By drawing a tree diagram or otherwise find the probability that, if two are drawn in succession without replacement:
(i) both are the same colour
(ii) one is black and the other white

Section D (Start a new page)
(a) The graph shows the population of a particular beetle between 1990 and 1995.



(i) Briefly describe how the beetle population has changed over the period (include the rate of change of the population in your discussion)

(ii) Comment on the sign of $\frac{d^2B}{dt^2}$

(b) A second type of insect (I) has the same population in 1990 as the beetle in (a).
Between 1990 and 1995 (inclusive) $\frac{dI}{dt} < 0$
(i) What does this say about the number of this second type of insect during the period 1990-95?
(ii) It is also observed that in the period 1990-95, $\frac{d^2I}{dt^2} > 0$
Sketch I against t for the time period 1990-95.

(c)The probability that a particular type of shrub will flower in the first year after planting is $\frac{1}{4}$
(i) Find the probability that:
(α) a shrub will not flower
(β) if three shrubs are planted all three will not flower
(γ) if three shrubs are planted at least one will flower
(ii) Show that if 9 shrubs are planted then the probability of having at least one flowering plant is greater than 90%.

Section E(Start a new page)
(a)The acceleration a ms^{-2} of a particle is given by $a = 4\pi^2 \cos 2\pi t$ The particle starts from rest at the origin. Find:
(i) the velocity and displacement at any time t
(ii) Draw a graph of x against t for $0 \le t \le 3$
(iii) A second particles position is give by $x = 12 - 4t + t^2$
Show that the two particles never meet.

(b) (i)Two dice are thrown together. Find the probability that.
(α) the sum of the dice is 4
(β)the sum of the dice is 6
(γ) the sum of the dice is neither a 4 nor a 6.
(ii) If the pair of dice are thrown twice find the probability that neither a sum of 4 nor a sum of six is thrown on the first throw and a sum of four is thrown on the second throw.
(iii) Find the probability that a sum of 4 is thrown before a sum of 6 (considering an unlimited number of throws). Give answer in simplified form.

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Section A (13 marks)

(a) (i) $x = 2t^3+3t^2-36t+10$

$v = 6t^2+6t-36$ 1

$a = 12t+6$ 1

(ii) $x=10$ when $t=0$ 1

(iii) $v=-36$ when $t=0$ ∴ moving left 2

(iv) when $v=0$

$0 = 6(t^2+t-6)$

$= 6(t+3)(t-2)$

$t=2$ only 1

$x = 2\times 8+3\times 4-36\times 2+10$

$= -34$ 1

(b) FALSE: There is no guarantee that the numbers of each colour are equal 2

(i) $\frac{7}{16}+\frac{4}{16} = \frac{11}{16}$ 2

(ii) $1-\frac{4}{16} = \frac{12}{16}$ 2

Section B (13 marks)

$v(t) = at^2-bt+c$

$1 = a-b+c$, $9 = 4a-2b+c$ 1

∴ $c=3$ 1

$-2 = a-b$

$6 = 4a-2b$

$-4 = 2a-2b$

$2a=10$

$a=5$ 1

$b=7$ 1

(b) $5t^2-7t+3$

$\Delta = -11$ 1

If $v=0$ No real soln for t

∴ Never comes to rest. 1

(b) (i) $v=50$ 1

(ii) $t=2$ (NOT $t=6$) 1

(iii) $t=1, 4$ 2

(iv) particle stops 1

(v) Distance travelled (pos. direction)

Section C (14 marks)

(a) $N=N_0e^{-kt}$

(i) $4000 = 7000e^{-10k}$

$\ln\frac{4}{7} = -10k$

$k = 0.0560$ (4D) 3

(ii) $\frac{4}{10} = e^{-0.0560t}$

$\ln\frac{4}{10} = -0.0560t$ 3

$t \doteqdot 16.4$ yrs

(iii) $N = 7000e^{-0.0560\times 5}$

$N = 5291.5$ 2

(iv) $\frac{dN}{dt} = -kN_0e^{-kt}$

$= -0.0560\times 7000e^{-0.0560\times 5}$ 2.

$= -296.12$

decreasing at 296 Kangaroos/yr.

(b) (i) $\frac{5}{10}\cdot\frac{4}{9}+\frac{3}{10}\cdot\frac{2}{9}+\frac{2}{10}\cdot\frac{1}{9}$

$= \frac{14}{45}$ 2

(ii) $2\times\frac{5}{10}\cdot\frac{3}{9}$

$= \frac{1}{3}$ 2

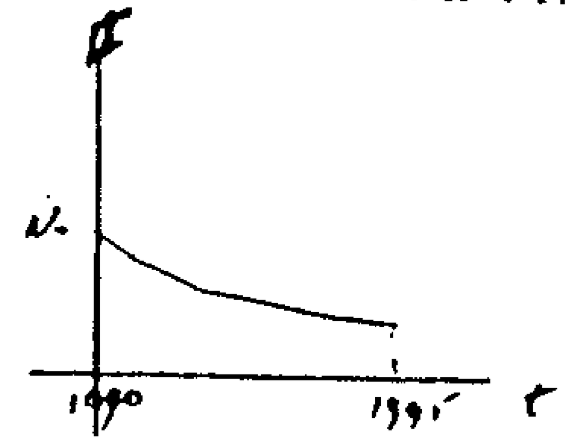
Section D (13 marks)

(a) (i) Population has increased (at an ever increasing rate) 2

(ii) $\frac{dP}{dt} > 0$ 1

(b) (i) Number is decreasing 1

(ii)



2

(c) (i) (α) $3/4$ 1

(β) $\left(\frac{3}{4}\right)^3$ 1

(γ) $1-\left(\frac{3}{4}\right)^3$ 2

(ii) $1-\left(\frac{3}{4}\right)^3$ 2

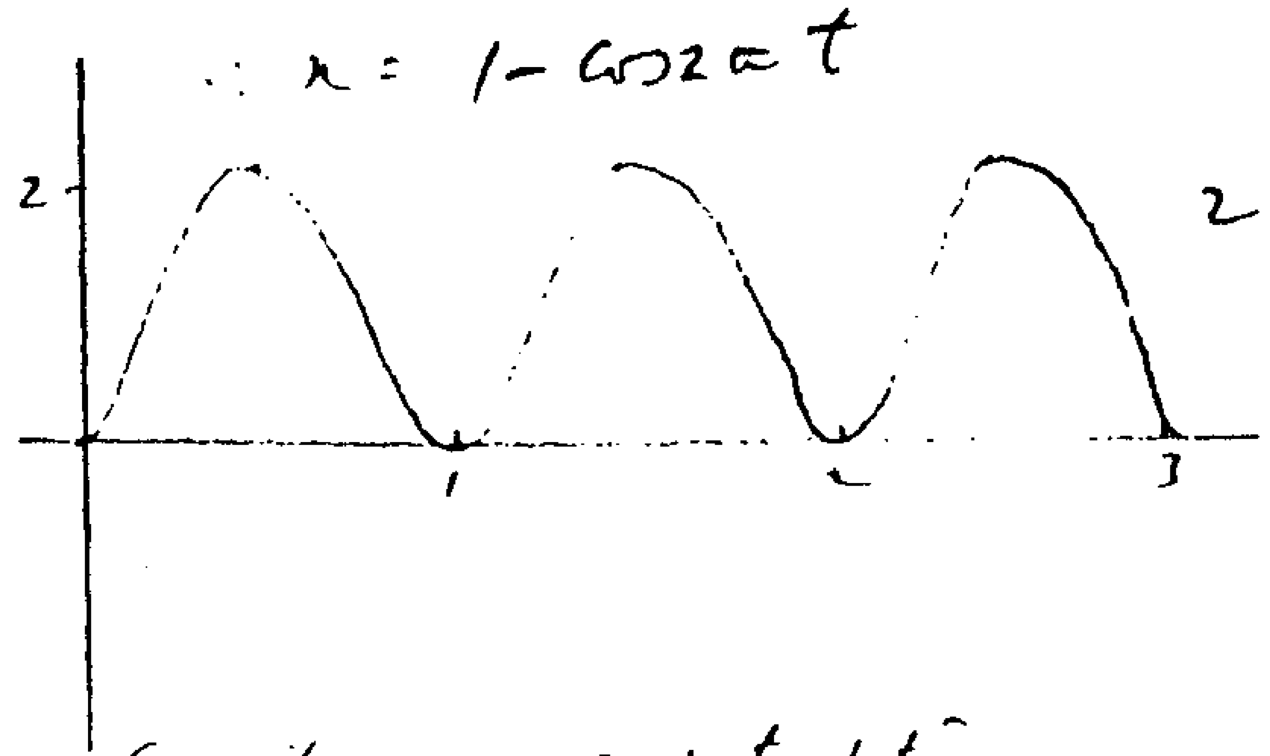
Section E (17 marks)

(a) $a = 4\pi^2\cos 2\pi t$

$v = 2\pi\sin 2\pi t + c_1$, $t=0, v=0$ ∴ $c_1=0$ 2

$x = -\cos 2\pi t + c_2$, $t=0$ $x=0$ ∴ $c_2=1$ 2

∴ $x = 1-\cos 2\pi t$



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Consider $x = 12-4t+t^2$ 2

Min value of 8 when $t=2$

Does not meet $y = 1-\cos\pi t$ which has max value of 2

(b) (i) (α) $\frac{1}{12}$ 1

(β) $\frac{5}{36}$ 1

(γ) $\frac{28}{36}$ 2

(ii) $\frac{28}{36}\times\frac{1}{12}$ 2

(iii) $\frac{3}{8}$ $\left[\frac{1}{12}+\frac{28}{36}\cdot\frac{1}{12}+\left(\frac{28}{36}\right)^2\frac{1}{12}+\ldots\right.$