2 Unit Term 2 - Assessment 1997

Time allowed: 85 minutes.

Question 1: (START A NEW PAGE)

A particle moves in a straight line so that it's position x metres from a fixed point O at time t seconds is given by

$$x = 4t^3 - 6t^2 - 9t + 5$$

Find (i) the velocity function in terms of t
(ii) the acceleration function in terms of t
(iii) the initial position, velocity, and acceleration
(iv) the position of the particle when it is stationary
(v) the velocity of the particle when the acceleration is zero
(vi) the average speed in the first 5 seconds

Question 2: (START A NEW PAGE)

(a) If the roots of $2x^2 + 4x + 3 = 0$ are α and β find the value of

(i) $\alpha + \beta$

(ii) $\alpha\beta$

(iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$

(iv) $\alpha^2 + \beta^2$

(b) Given the quadratic equation $2x^2 - kx + 5 = 0$, find:
(i) the discriminant,
(ii) the values of k for the quadratic equation to have real roots.
(iii) the range of the function $y = 2x + \dfrac{5}{x}$.

Question 3: (START A NEW PAGE)

(a) The population P of a town increases at a rate proportional to the population.

(i) Write the differential equation to describe the growth of population P with the annual growth constant k.
(ii) Show that the equation $P = P_0e^{kt}$ is a solution to this differential equation, where P_0 is the initial population, and t is the number of years.
(iii) If the initial population is 21000 and the annual growth rate is 8% per annum find the population in 11 years.
(iv) Find the time when the population reaches 100000.

(b) The acceleration a (ms^{-2}) of a body moving in a straight line at time t is given by :

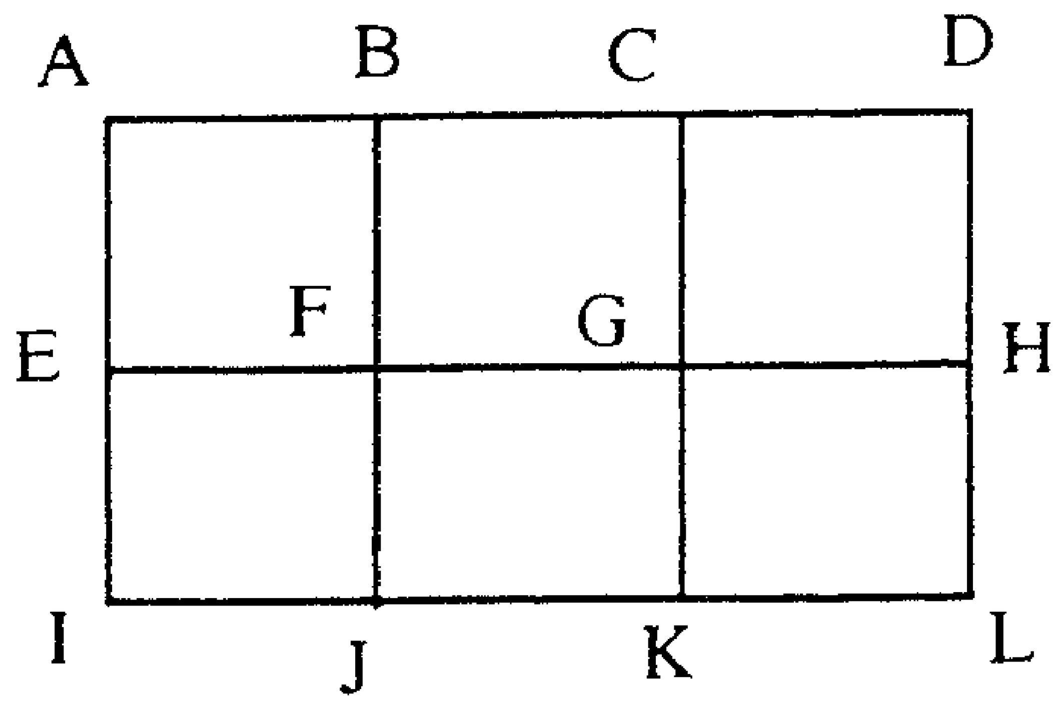
$$a = 20e^{5t} + 6\sin 3t$$

If initially the body is at distance x = 3 metres and velocity $v = 2\ ms^{-1}$ find
(i) the velocity in terms of t.
(ii) the distance travelled in terms of t.

Question 4: (START A NEW PAGE)

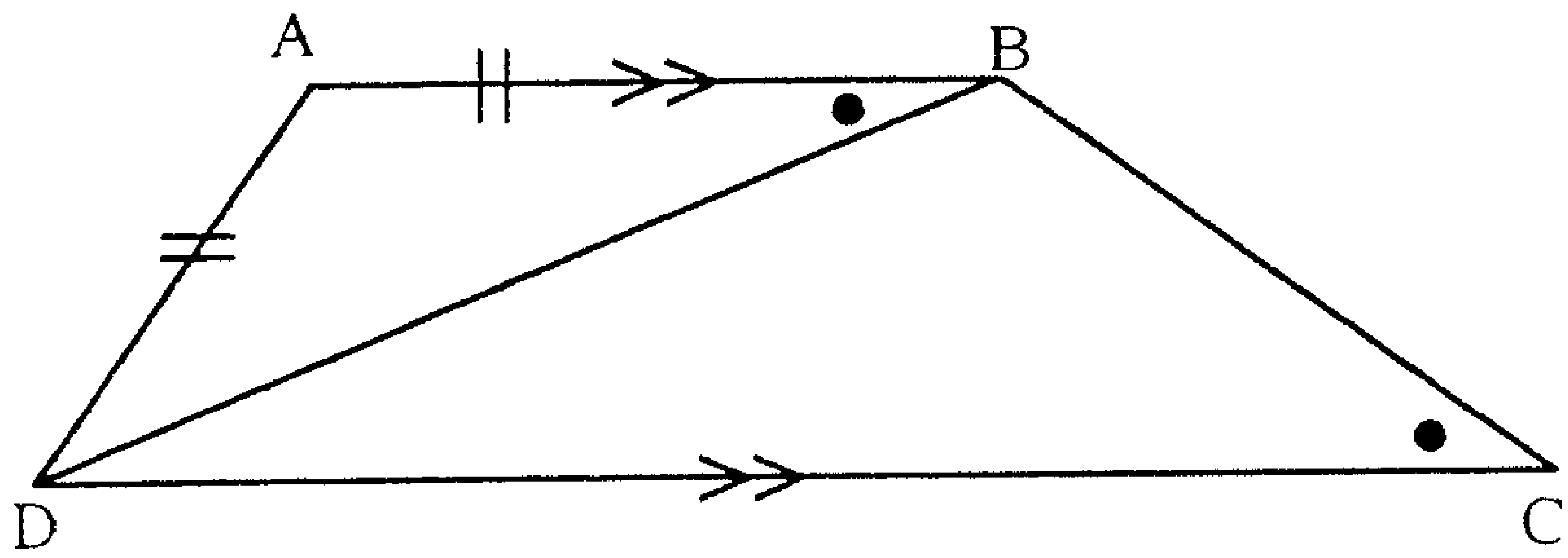
(a) A radioactive particle decays at a rate proportional to the amount remaining.
If the particle's half life is 15 years find the percentage remaining after 100 years.

(b)



A person goes from A to L, always getting closer to L.
Find the path, with reasons, which has the highest probability, and state the probability.

(c)



(i) Show that $\Delta ABD \;|||\; \Delta BDC$ if $AB = AD$, $AB \parallel DC$ and $\angle ABD = \angle BCD$.

(ii) If the perimeter of ΔABD is 12 cm and $DC = 25$ cm, find the lengh of AB.

Question 5: (START A NEW PAGE)

(a) The letters from the word ' LITTLE ' are placed in Bag 1 and the letters from the word ' LION ' are placed in Bag 2. A letter is drawn from each bag.
Find the probability that:
(i) both letters are consonants,
(ii) both letters are the same,
(iii) both letters are consonants if it is known that at least one of the letters is an L.

(b) A particle which moves in a straight line has a velocity function of :

$$v = 9(3t + 5)^2$$

Find:
(i) the acceleration in terms of t,
(ii) the distance travelled in terms of t if the initial position is 125,
(iii) the acceleration in terms of x.

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2U ASSESS. (97) T2

Question 1

(i) $x = 4t^3 - 6t^2 - 9t + 5$

$v = 12t^2 - 12t - 9$ [1]

(ii) $a = 24t - 12$ [1]

(iii) $t = 0$, $x = 5\,m$, $v = -9\,ms^{-1}$ $a = -12\,ms^{-2}$ [3]

(iv) $v = 0 \Rightarrow 3(4t^2 - 4t - 3) = 0$

$3(2t+1)(2t-3) = 0$

$t = -\frac{1}{2}, \frac{3}{2}$

but $t > 0$

$\therefore t = 1\frac{1}{2}$ sec.

$\therefore x = 4(1\frac{1}{2})^3 - 6(1\frac{1}{2})^2 - 9(1\frac{1}{2}) + 5$

$= -8.5$ [3]

(v) $a = 0 \Rightarrow t = \frac{1}{2}$ $\underline{V = -12}$

$V = \cancel{12 \times (\frac{1}{2})^2 - 12(\frac{1}{2}) - 9}$

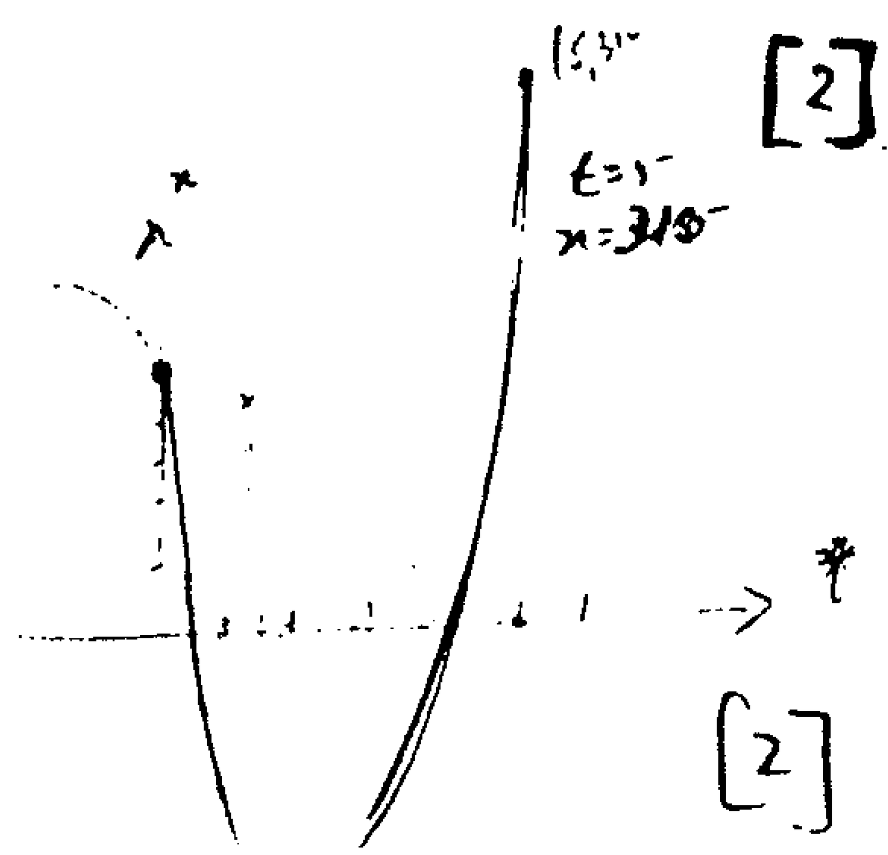
$= \cancel{3 - 6 + 9 = 6}$ m/s. [2]

(vi) Total dist trav $= (5 + 8\frac{1}{2}) + (8\frac{1}{2} + 310)$

$= 332$

Av. speed $= \frac{332}{5}$

$= 66.4$ m/s.



[2]

(a) (i) $\alpha + \beta = -\frac{4}{2} = -2$ [1]

(ii) $\alpha\beta = \frac{3}{2}$ [1].

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$

$= \frac{-2}{3/2}$

$= -\frac{4}{3}$ [2]

(iv) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (-2)^2 - 2(3/2)$

$= 4 - 3$

$= 1$. [2].

(b) (i) $\Delta = K^2 - 40$ [2].

(ii) Real roots $\Delta \geq 0 \Rightarrow K^2 - 40 \geq 0$

$K^2 \geq 40$

$K \leq -\sqrt{40}$ or $K \geq \sqrt{40}$ [2]

(iii) values for y exist only if $2x^2 - yx + 5 = 0$ has solution

$\Delta = y^2 - 40$

$\therefore$ roots if $y \leq -\sqrt{40}$ or $y \geq \sqrt{40}$

$\therefore$ range is $y \leq -\sqrt{40}$ or $y \geq \sqrt{40}$ [2]

(a) (i) $\frac{dP}{dt} = kP$ [1]

(ii) $P = P_0 e^{kt}$ (1)

$\frac{dP}{dt} = kP_0 e^{kt}$

$= kP$ since $P = P_0 e^{kt}$

(iii) $P = 21000 e^{0.08t}$

$P = 21000 e^{0.08 \times 11}$

$\doteqdot 50630$ (to nearest 10) [50628.89] [2]

(iv) $100000 = 21000 e^{0.08t}$

$e^{0.08t} = \frac{100000}{21000}$

$0.08t = \ln\left(\frac{100}{21}\right)$

$t = \frac{\ln\left(\frac{100}{21}\right)}{0.08}$ [2]

$t \doteqdot 19.5$ yrs.

(b) $\ddot{x} = 20e^{+5t} + 6\sin 3t$

(i) $v = 4e^{5t} - 2\cos 3t + c_1$ (3)

$t=0, v=2 \Rightarrow 2 = 4 - 2 + c_1$ $\quad * \ \underline{c=0}$

$c_1 = 0$

$\therefore v = 4e^{5t} - 2\cos 3t$

(ii) $\frac{dx}{dt} = 4e^{5t} - 2\cos 3t$

$x = \frac{4}{5}e^{5t} - \frac{2}{3}\sin 3t + c_2$ $\quad x = \frac{4}{5}e^{5t} - \frac{2}{3}\sin 3t + \frac{4}{5}$ $\quad 2\frac{1}{5}$ [3]

(a) $A = A_0 e^{-kt}$

$\frac{1}{2}A_0 = A_0 e^{-15k}$

$\frac{1}{2} = e^{-15k}$

$-15k = \ln\left(\frac{1}{2}\right)$

$k = \frac{\ln\left(\frac{1}{2}\right)}{-15}$

$= \frac{\ln 2}{15}$

when $t = 100$ [4]

$A = A_0 e^{-\frac{\ln 2}{15} \times 100}$

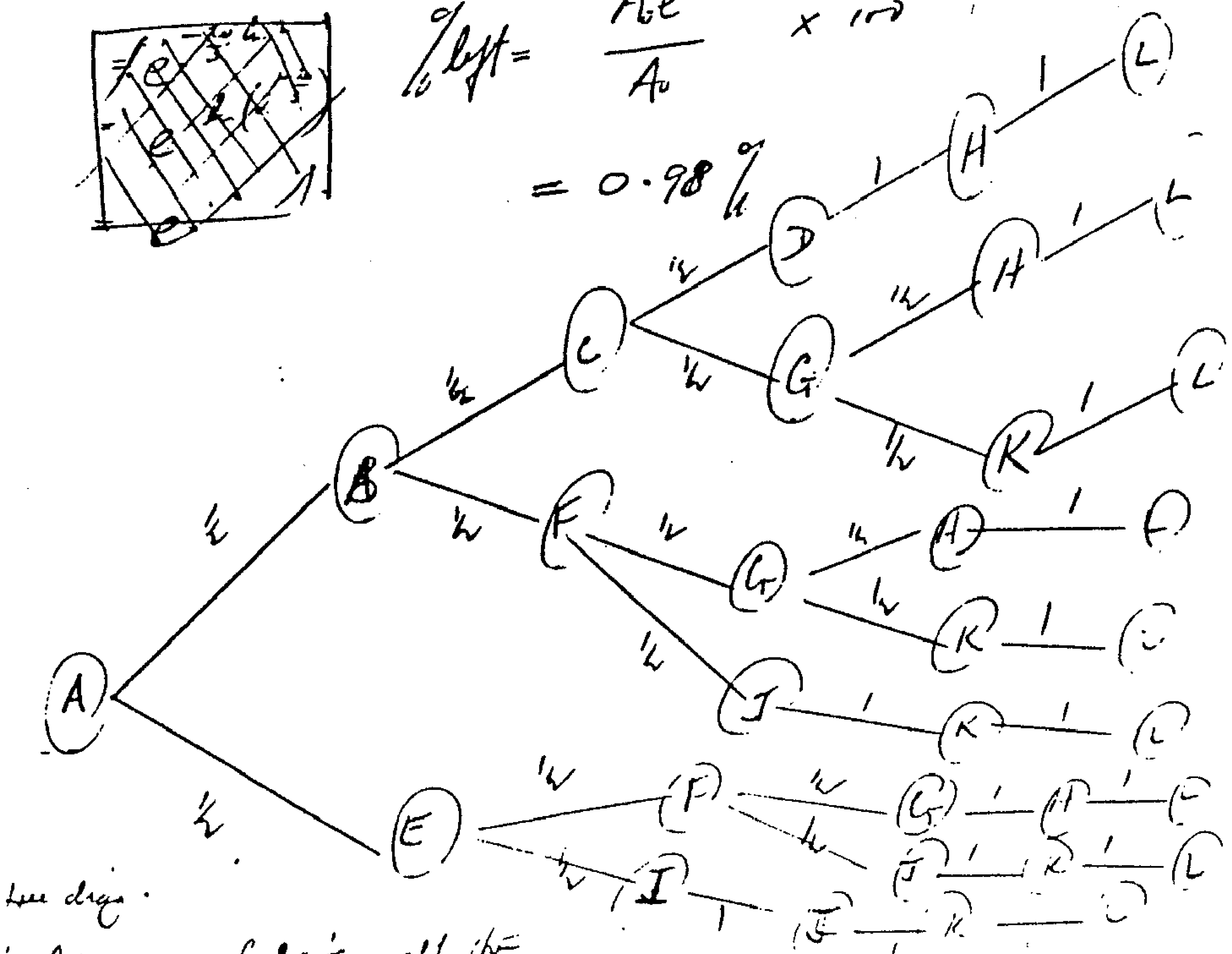
$= A_0 e^{-\frac{20}{3}\ln 2}$

$\therefore$ ratio $= \frac{A_0 e^{-\frac{20}{3}\ln 2}}{A_0}$

% left $= \frac{A_0 e^{-\frac{20}{3}\ln 2}}{A_0} \times 100$

$= 0.98\%$

(b) [3]



use tree diagram.

Path AEIJKL Prob $= \frac{1}{4}$ all other

1(c)(i) Let ABD

$A\hat{D}B = x°$ (equal angles opp. equal sides)

$B\hat{D}C = x°$ (AB∥CD, alt. angles are equal)

$\therefore A\hat{D}B = B\hat{D}C$ (both $x°$) [2]

$\therefore \triangle ABD \,|||\, \triangle BCD$ (equiangular)

(ii) let $AB = x$

$\therefore AD = x$ & $DB = 12 - 2x$

$$\frac{AB}{BC} = \frac{DB}{DC}$$ (ratio of corresp. sides in similar $\triangle$'s)

$$\frac{x}{12-2x} = \frac{12-2x}{25}$$

$25x = 144 - 48x + 4x^2$

$4x^2 - 73x + 144 = 0$

$$x = \frac{73 \pm \sqrt{73^2 - 4(4)(144)}}{8}$$

$$= \frac{73 \pm \sqrt{3025}}{8}$$

$$= \frac{73 \pm 55}{8}$$

$x = 16$ or 2.25 [3]

but $x < 12$

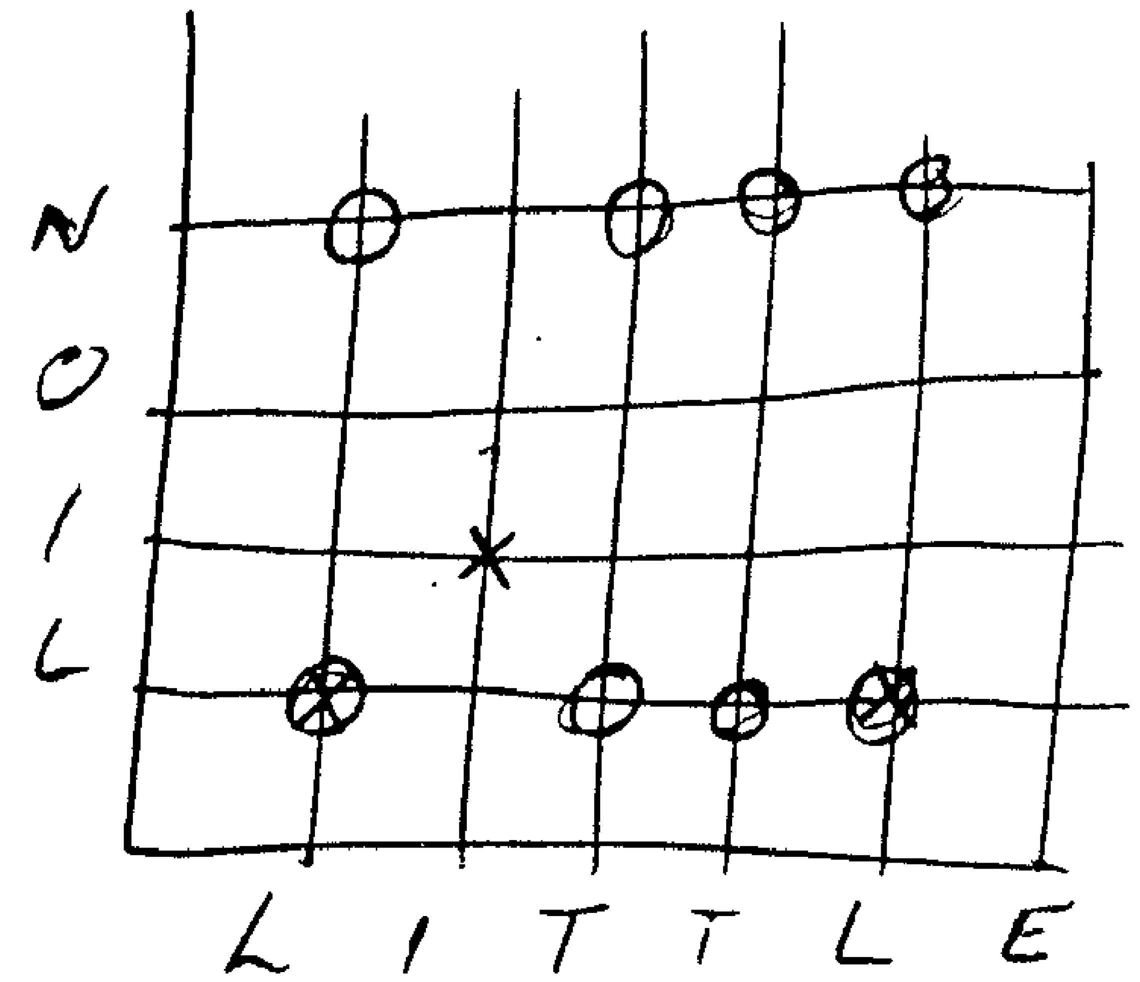
$\therefore x = 2.25$ cm

Question 3

(a) (i) $n(S) = 24$

$P(\text{2 consonant}) = \frac{8}{24}$

$= \frac{1}{3}$ [2]



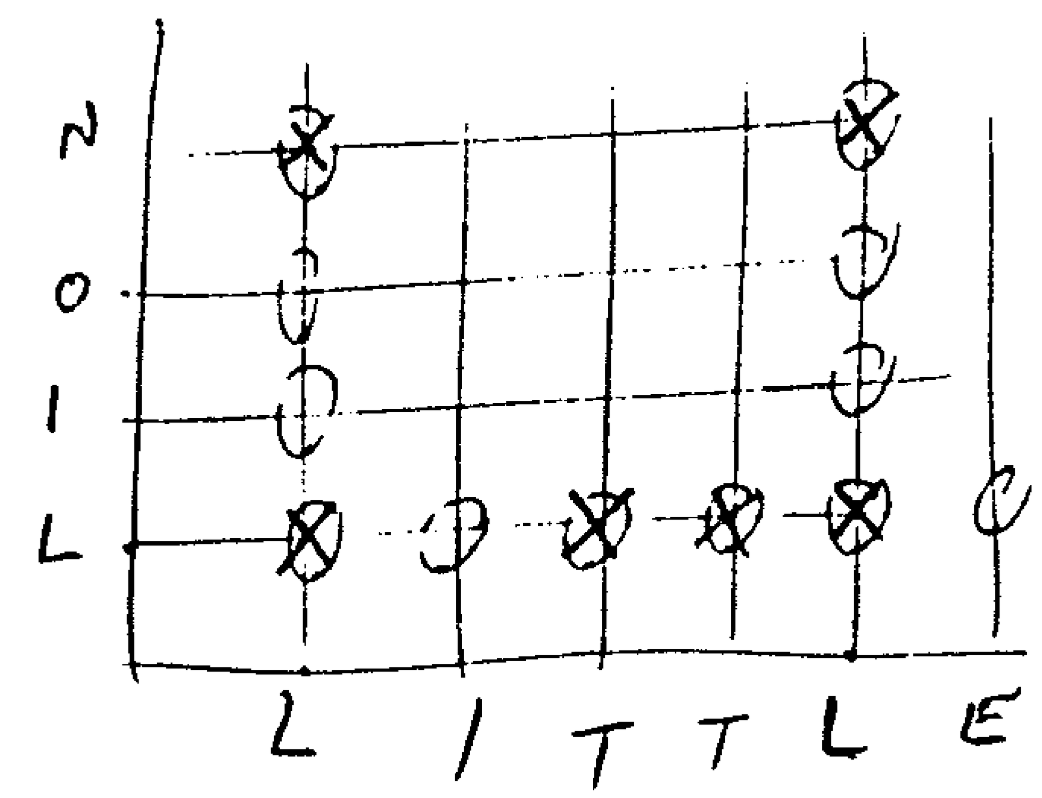
(ii) $P(\text{both same} \; \cancel{\text{2 vowels}}) = \frac{3}{24}$

$= \frac{1}{8}$ [~~2~~] 1

(iii) $n(S) = 12$

$P(\text{2 consonants}) = \frac{6}{12}$

$= \frac{1}{2}$ [2].



(b) (i) ~~[illegible]~~

$V = 9(3t+5)^2$

$a = 9 . 2(3t+5)^1 . 3$

$= 54(3t+5)$ [2].

(ii) $\frac{dx}{dt} = 9(3t+5)^2$

$x = \frac{9(3t+5)^3}{3.3} + c$ [3]

$x = (3t+5)^3 + c$

$t = 0, \; x = 125$

$125 = 125 + c$

$x = (3t+5)^3$

(iii) $3t + 5 = x^{\frac{1}{3}}$

$\therefore a = 54x^{1/3}$ [2].