JAMES RUSE AGRICULTURAL HIGH SCHOOL

2 Unit Term 2 Assessment 1998

Open Book Assessment

Time Allowed : 85 mins

All questions are of equal value

All necessary working to be shown.

SECTION A **(Start a new page)**

(a) A particle moves in a straight line in such a way that its distance in metres from the origin after t seconds is given by

$$x = 2t^3 + 3t^2 - 36t + 10$$

(i) In which direction is the particle moving initially ? Give reason.

(ii) When and where is the particle instantaneously at rest ?

(iii) What total distance has the particle travelled in the first three seconds of its motion ?

(b) A box contains 5 white balls, 3 black balls and 2 red balls. Two balls are drawn at random. What is the probability that:-

(i) Both balls are white

(ii) One ball is white and the other is black

(iii) Both are the same colour.

SECTION B **(Start a new page)**

(a) Find the quadratic equation whose roots are -3 and $\frac{2}{3}$ (answer to be written in the form $ax^2 + bx + c = 0$)

(b) Given α and β are the roots of the equation $x^2 - 8x + 5 = 0$, find the value of:

(i) $\alpha + \beta$

(ii) $\alpha\beta$

(iii) $\frac{2}{\alpha} + \frac{2}{\beta}$

(iv) $\alpha^2 + \beta^2$

(c) Solve $8 - 2x - x^2 > 0$

(d) Determine whether $2x^2 - 3x + 2$ is positive definite , negative definite or indefinite.Give reasons .

SECTION C **(Start a new page)**

(a) (i) The rate of growth of the population N of a city at any time t is directly proportional to the population at that time, thus $dN/dt = kN$. . Show that after time t, then $N = N_0 e^{kt}$ satisfies the equation $dN/dt = kN$

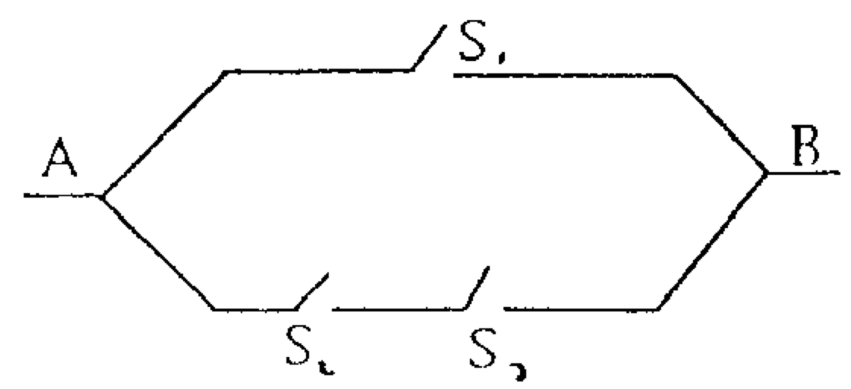
(ii) (α) If at the beginning of 1997, the population of a city is 1 000 000 and the population is increasing by 4% per annum, find the values of N_0 and k giving k to 4 dec places.

(β) Hence determine the expected population of the city at the start of 2007 to the nearest thousand.

(γ) In what year would you expect the poopulation to first exceed 2 000 000

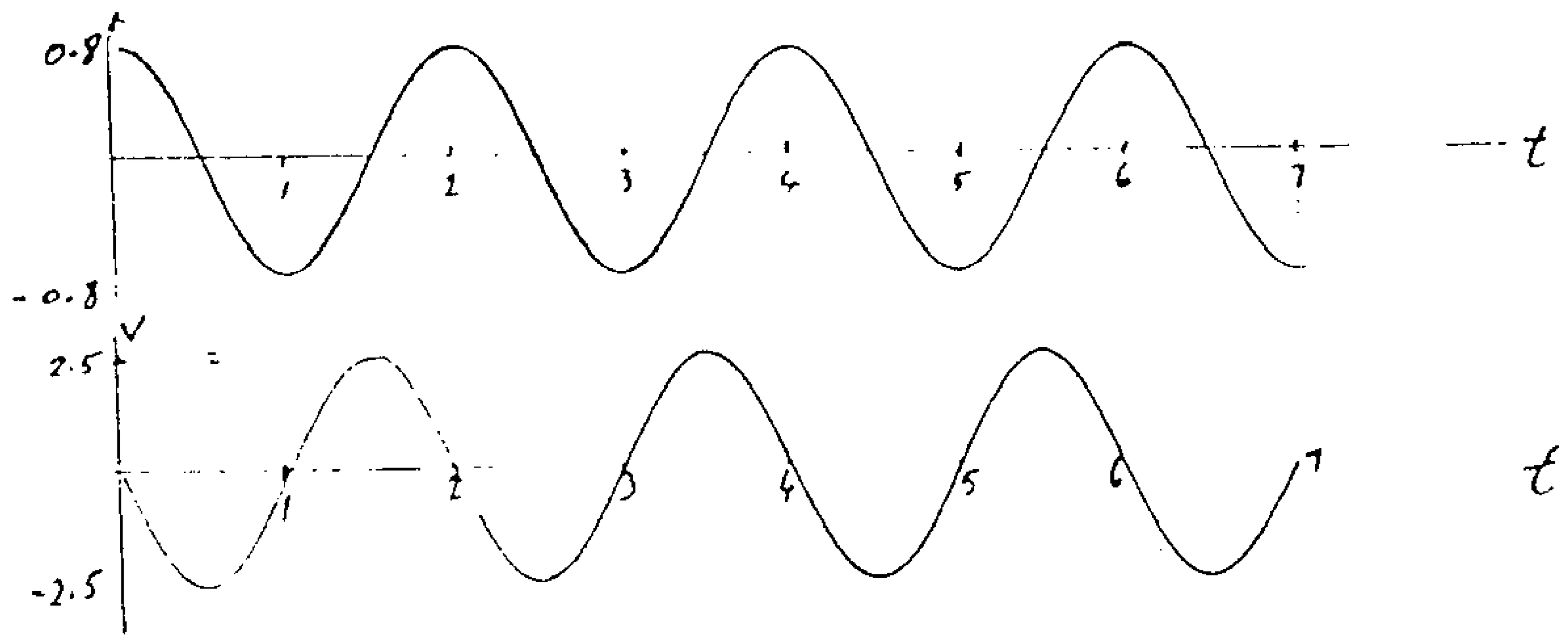
(b) The diagram shows a switching circuit. The probability that any switch is open is $\frac{1}{4}$

Determine the probability that the current cannot flow from A to B



SECTION D **(Start a new page)**

(a) A particle moves in a straight line. The diagrams below are the displacement and velocity graphs of the particle where x is in metres and t in seconds.



(i) Find the initial displacement and velocity.

(ii) State the time (s) when the particle has greatest speed.

(iii) Where does the particle first change direction ?

(iv) State the time (s) when the particle has zero acceleration ?

(b) Two cards are selected at random from five cards bearing the numbers 2, 2, 4, 5, 6.

Find the probability that the numbers on the cards:

(i) are different

(ii) have a sum of 9

(iii) have a difference of 1

SECTION E **(Start a new page)**

(a) The acceleration of a particle moving in a straight line is given by $a = \frac{-1}{t^2}$, where the displacement (x) measured in cm and the time (t) in seconds. If the particle is moving with velocity 6 cm/s, when t = 1 find the limiting velocity of the particle.

(b) The line $y = x + k$ is drawn on the same diagram as the circle $x^2 + y^2 = 16$. Find the values of k that make this line

(i) touch the circle

(ii) intersect the circle in two points.

(c) The line $x + 2y - 3 = 0$ forms the chord AB of the parabola $y = x^2 - 2$. Find the midpoint of AB

END OF PAPER.

(9) 2012

$x = 2t^3+3t^2-36t+10$

(i) $V = 6t^2+6t-36$ (1)

when $t=0$ $V=-36$

∴ Moving left (Neg Vel) (1)

(ii) $V = 6(t+3)(t-2)$

∴ comes to rest after $t=2$ (1)

when $x = 2.8+3.4-72+10$

$= -34$ m. (1)

(iii) $t=3$ $x = 2.27+27-108+10$

$= -17$ (1)

∴ Dist Travelled (1)

$10+34+34-17 = 61$m.

1 (i) $P(WW) = \frac{5}{10}\cdot\frac{4}{9} = \frac{2}{9}$ (2)

(ii) $P(WB)+P(BW) = 2\times\frac{5}{10}\cdot\frac{3}{9} = \frac{1}{3}$ (2)

(iii) $P(BB)+P(WW)+P(RR)$

$= \frac{3}{10}\cdot\frac{2}{9}+\frac{5}{10}\cdot\frac{4}{9}+\frac{2}{10}\cdot\frac{1}{9}$

$= \frac{28}{90}$

$= \frac{14}{45}$. (2)

Section B.

(i) $(x+3)(x-\frac{2}{3})=0$ (1)

$(x+3)(3x-2)=0$

$3x^2+7x-6=0$ (1)

(ii)(i) $\alpha+\beta = 8$ (1)

(iii) $\frac{2(\alpha+\beta)}{\alpha\beta} = \frac{16}{5}$ (2)

(iv) $\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta$

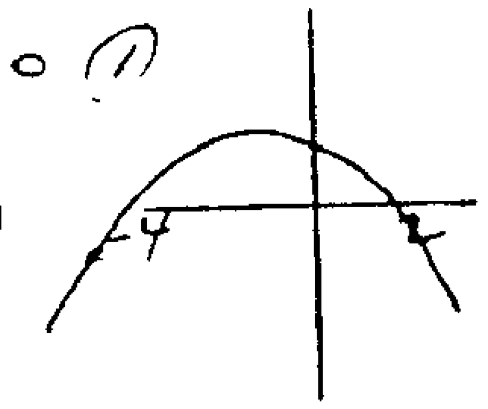
$= 64-2.5$

$= 54$ (2)

(c) $-(x^2+2x-1)>0$

$-(x+4)(x-2)>0$ (1)

$-4<x<2$. (1)



(d) $2x^2-3x+2$.

$\Delta = 9-4.2.2$ (1)

$= -7$

<0 & $a>0$

∴ positive definite. (1)

Section C.

(a) (i) $N = N_0 e^{kt}$ (2)

$\frac{dN}{dt} = kN_0e^{kt}$

$= kN$ ... since $N = N_0e^{kt}$

(ii)(α) $N_0 = 1000000$

$1.04N_0 = N_0e^{k}$ (1)

$k = 0.0392$ or 4%

(β) $t=10$

$N = 1000000\,e^{10k}$ $10.\ln 1.04$ (1)

$= 1000000\,e^{...}$

$\doteqdot 1480000$ (nearest thousand) (1)

(γ) $2000000 = 1000000\,e^{\ln 1.04 . t}$ (1)

$t \doteqdot 17.7$ (1)

$1997+17 = \underline{2014}$ (1)

(b)

P(S1 open, S2 open, S3 open)

+

P(S1 open, S2 closed, S3 open) (2)

+

P(S1 open S2 open S3 closed)

$= \frac{1}{64}+2(\frac{1}{4}\times\frac{1}{4}\times\frac{3}{4})$

$= \frac{7}{64}$.

Section D

a (i) Initial displacement = 0.8 m (1)

Initial vel = 0 m/s. (1)

(ii) $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \frac{11}{2}, \frac{13}{2}$ seconds. (1)

(iii) $x = -0.8$ m. (1)

(iv) same as (ii). (1)

(b).

	2	2	4	5	6
2	NA	2,2	2,4	2,5	2,6
2	2,2	NA	2,4	2,5	2,6
4	4,2	4,2	NA	4,5	4,6
5	5,2	5,2	5,4	NA	5,6
6	6,2	6,2	6,4	6,5	NA

P(different) $= \frac{18}{20} = \frac{9}{10}$ 1

P(Sum of 9) $= \frac{2}{20} = \frac{1}{10}$ 1

P(Differ by 1) $= \frac{4}{20} = \frac{1}{5}$ 2

(a) $a = \frac{-1}{t^2}$

$V = t^{-1}+c$ (1)

$V = \frac{1}{t}+5$ (1)

as $t\to\infty$ $V\to 5$ m/s which is the limiting velocity

(b) $x^2+(x+k)^2=16$

$2x^2+2kx+k^2-16=0$

$\Delta = 4k^2-4.2.(k^2-16)$

$= -4k^2+128$

(i) touches when $\Delta=0$. (1)

∴ $k = \pm 4\sqrt{2}$ (1)

(ii) Intersects when $\Delta>0$

$-4k^2+128>0$

$-4\sqrt{2}<k<4\sqrt{2}$. (1)

(c) $\frac{3-x}{2} = x^2-2$

$0 = 2x^2+x-7$

$\alpha+\beta = -\frac{1}{2}$. (1)

∴ x coord of M point $= -\frac{1}{4}$ (1)

∴ $(-\frac{1}{4}, \frac{13}{8})$ (1)