JAMES RUSE AGRICULTURAL HIGH SCHOOL

Year 12 Term 2 2000
2 Unit Mathematics

INSTRUCTIONS: Time Allowed 85 MINUTES

- **This is an open book exam**
- **Show all necessary working**
- **Marks may be deducted for poorly set out work**
- **Start each question on a new page**

QUESTON 1 (12 Marks)

1. Solve the inequality: $(x+2)(3-2x)>0$

2. If α and β are the roots of $2x^2+5x-1=0$, find the value of:
   a) $\alpha+\beta$
   b) $\alpha\beta$
   c) $\alpha^2+\beta^2$
   d) $\alpha-\beta$

3. Of two uniform cubes, one has on its faces the numerals 1, 2, 2, 3, 4, 5 and the other 2, 3, 4, 5, 6, 6. Find the probability that if both are thrown, the upper most numbers
   a) are both the same numbers
   b) have a total of 7
   c) are both the same numbers, if it is known that at least one dice shows a 2.

QUESTON 2 (12 Marks) Start a new page

1. Show that $f(x)=5x^2+4mx+m^2$ is positive definite irrespective of the value of m.

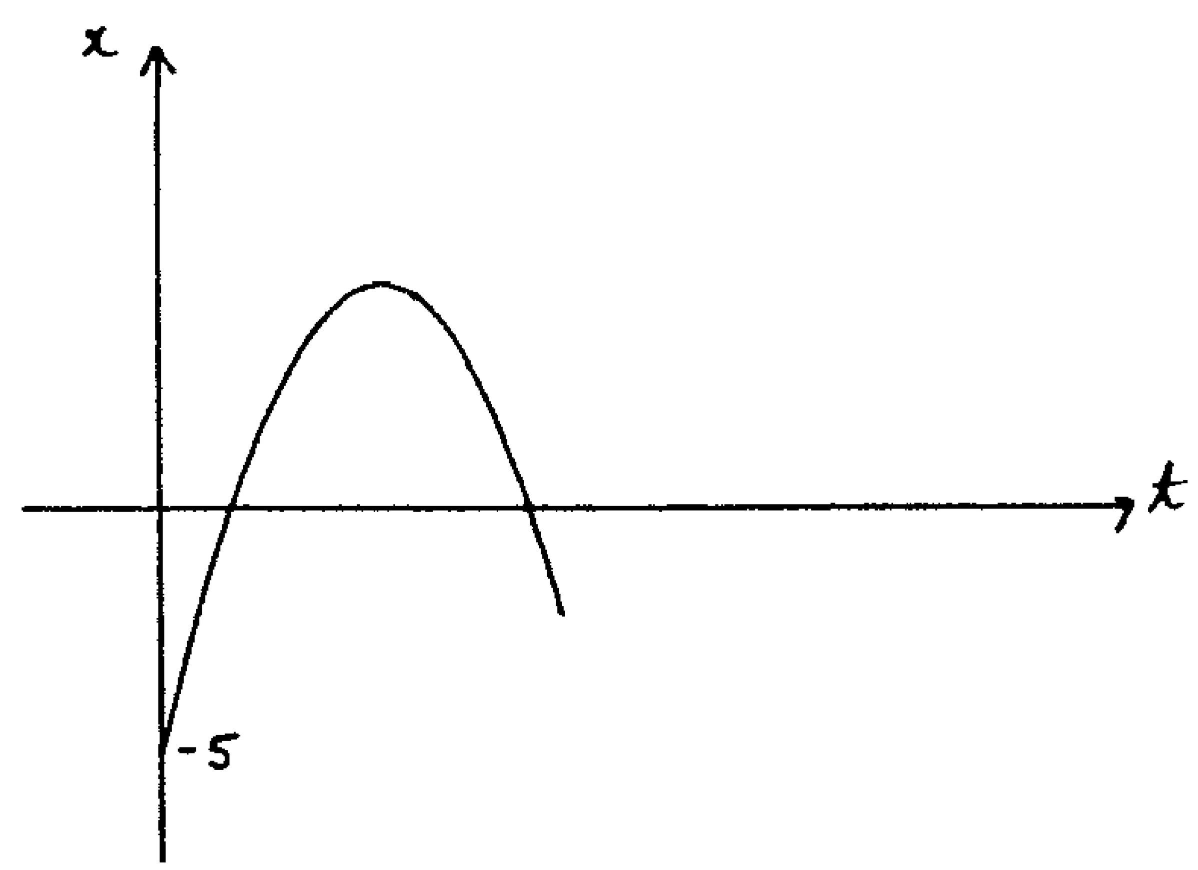
2. The roots of the quadratic equation $px^2-x+q=0$ are -1 and 3. Find the value of p and q.

3. The number of bacteria in a colony grows according to the law $N(t)=2500e^{kt}$, where N(t) is the number of bacteria after t days and k is a constant. At the end of 10 days, the initial number of bacteria has doubled.
   a) Show that initially there were 2500 bacteria.
   b) Find the exact value of the growth constant k.
   c) How many bacteria would you expect after another 10 days?
   d) How long would it take the number of bacteria to treble? (Answer to 3 significant figures)

QUESTON 3 (12 Marks) Start a new page

1. A bag contains 8 marbles of which 3 are red and 5 blue. From it, three are drawn without replacement. Find the probability that:
   a) the first is red and the second blue
   b) of the first two drawn, one is red
   c) at least one of three is blue

2. Two particles A and B start moving along the x-axis at t=0. Particle A has a velocity in metres per second given by $v=2t-2$. Initially it is at $x=4$. Particle B has a position function given by $x=6t-t^2-5$. This function is shown on the diagram below.



   a) Find the position function for A at time t.
   b) Show the graphs of both position functions on the same diagram.
   c) Show that the distance D, between A and B at time t is given by D$=2t^2-8t+9$.
   d) Find when the particles are closest together.
   e) Find the least distance between the particles.

QUESTON 4 (12 Marks) Start a new page

1. A block of ice originally of mass 84kg is melting at a rate equal to 3% of its mass. Assuming that at any time t hours, its mass M is given by $M = M_o e^{-kt}$,
   a) What will its mass be after 12 hours? (answer to 3 sig. figures).
   b) What time will elapse before 80% of its mass has melted? (answer to 3 sig. figures).

2. The probability that it will rain on Tuesday is 0.75. If it rains, the probability that an employee will be absent is 0.3. If it is fine, the probability that the employee will be at work is 0.9. Find the probability that the employee is absent on Tuesday.

3. a) Show that the line L, which contains the point (1, -3) and having gradient m has equation $y = mx - m - 3$.
   b) Show that, where L meets the hyperbola $y = \frac{1}{x}$ then $mx^2 - (m+3)x - 1 = 0$
   c) Determine the value(s) of m for which the line L is a tangent to the hyperbola $y = \frac{1}{x}$.

QUESTON 5 (12 Marks) Start a new page

1. The quadratic equation $x^2 + Lx + M = 0$ has one root which is twice the other. Prove that $2L^2 = 9M$.

2. A particle P moves along a straight line so that at time t, its displacement from a fixed point O on that line is given by $x(t) = \frac{3t^2}{4+t^3}$
   a) Find the velocity at any time t.
   b) Find the times when the particle is momentarily at rest.
   c) With the use of a calculator, or otherwise find the exact position at $t_2 = 2 + 2\sqrt{2}$. Also find the position when $t_1 = 1$.
   d) Give a brief description of the way the displacement changes over the time period from t_1 to t_2.
   e) Describe the motion of the particle as t increases without bound.
   f) Find the maximum displacement from O.

STANDARD INTEGRALS

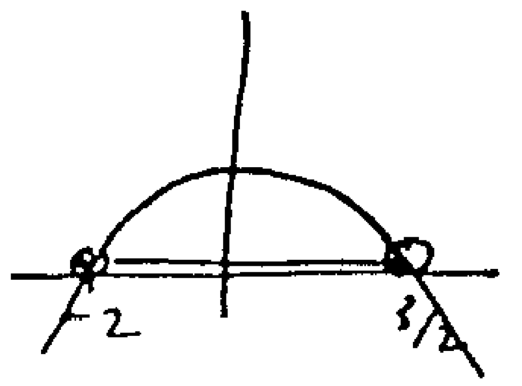
$\int x^n\,dx$	$= \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$
$\int \frac{1}{x}\,dx$	$= \ln x, \quad x > 0$
$\int e^{ax}\,dx$	$= \frac{1}{a}e^{ax}, \quad a \neq 0$
$\int \cos ax\,dx$	$= \frac{1}{a}\sin ax, \quad a \neq 0$
$\int \sin ax\,dx$	$= -\frac{1}{a}\cos ax, \quad a \neq 0$
$\int \sec^2 ax\,dx$	$= \frac{1}{a}\tan ax, \quad a \neq 0$
$\int \sec ax \tan ax\,dx$	$= \frac{1}{a}\sec ax, \quad a \neq 0$
$\int \frac{1}{a^2+x^2}\,dx$	$= \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$
$\int \frac{1}{\sqrt{a^2-x^2}}\,dx$	$= \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$
$\int \frac{1}{\sqrt{x^2-a^2}}\,dx$	$= \ln\left(x + \sqrt{x^2-a^2}\right), \quad x > a > 0$
$\int \frac{1}{\sqrt{x^2+a^2}}\,dx$	$= \ln\left(x + \sqrt{x^2+a^2}\right)$

NOTE: $\ln x = \log_e x, \quad x > 0$

Solutions YR12 T2 2U Assessment Task (2000)

Question 1 (12 marks)

1. $(x+2)(3-2x) > 0$



$-2 < x < \frac{3}{2}$

2. $2x^2+5x-1=0$

a) $\alpha+\beta = -\frac{5}{2}$

b) $\alpha\beta = -\frac{1}{2}$

c) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(\frac{-5}{2}\right)^2 - 2\left(\frac{-1}{2}\right)$

$= \frac{29}{4}$

d) $(\alpha-\beta)^2 = \alpha^2+\beta^2-2\alpha\beta$

$= \frac{29}{4}$

$\alpha-\beta = \pm\sqrt{\frac{29}{4}}$ or $\pm\frac{\sqrt{29}}{2}$

3. a) $P(\text{both same})$

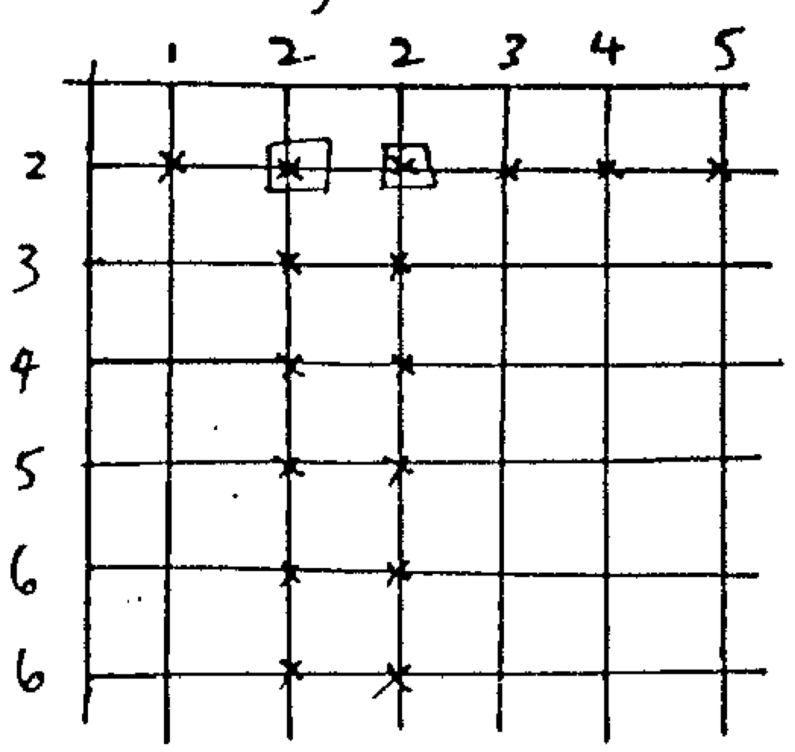
$= P(2,2) + P(3,3) + P(4,4) + P(5,5)$

$= \frac{2}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36}$

$= \frac{5}{36}$

b) $P(\text{total } 7) = \frac{7}{36}$

c)



$P(2 \text{ same, if 1 is a } 2) = \frac{2}{16} = \frac{1}{8}$

Question 2 (12 marks)

1. For positive definite:

(i) $a>0$ Here $a=5$ so $a>0$ ✓

(ii) $\Delta<0$

Here $\Delta = (4m)^2 - 4.5.m^2$

$= 4m^2 - 20m^2$

$= -16m^2$

<0 as $m^2>0$ and

$-16m^2<0$ always

irrespective of m ✓

2. Roots are $-1, 3$.

$px^2 - x + q = 0$

$x^2 - \frac{1}{p}x + \frac{q}{p} = 0$

$\therefore \alpha+\beta = \frac{1}{p}$ $\alpha\beta = \frac{q}{p}$

ie $-1+3 = \frac{1}{p}$ $-1\times 3 = \frac{q}{p}$

$2 = \frac{1}{p}$ $\therefore q = -3\times\frac{1}{2}$

$p = \frac{1}{2}$ $q = -\frac{3}{2}$

3. a) $N(t) = 2500e^{kt}$

$N(0) = 2500e^0 = 2500$

b) When $t=10$, $N(10) = 5000$

$5000 = 2500e^{10k}$

$2 = e^{10k}$

$\ln 2 = 10k$

$k = \frac{\ln 2}{10}$

c) $N(20) = 2500e^{\frac{\ln 2}{10}\times 20}$

$= 10000$

d) $3\times 2500 = 2500e^{\frac{\ln 2}{10}t}$

$t = 15.8$

$\therefore 15.8$ yrs

Question 3 (12 marks)

1. a) $\frac{3}{8}\times\frac{5}{7}\times 1 = \frac{15}{56} = P(RB)$

b) $P(RB) + P(BR)$

$= \frac{15}{56} + \frac{5}{8}\times\frac{3}{7}$

$= \frac{15}{28}$

c) $P(\text{at least 1 blue})$

$= 1 - P(\text{no blue})$

$= 1 - P(\text{all red})$

$= 1 - \frac{3}{8}\times\frac{2}{7}\times\frac{1}{6}$

$= \frac{55}{56}$

2. A: $v = 2t-2$, $t=0$, $x=4$

B: $x = 6t - t^2 - 5$

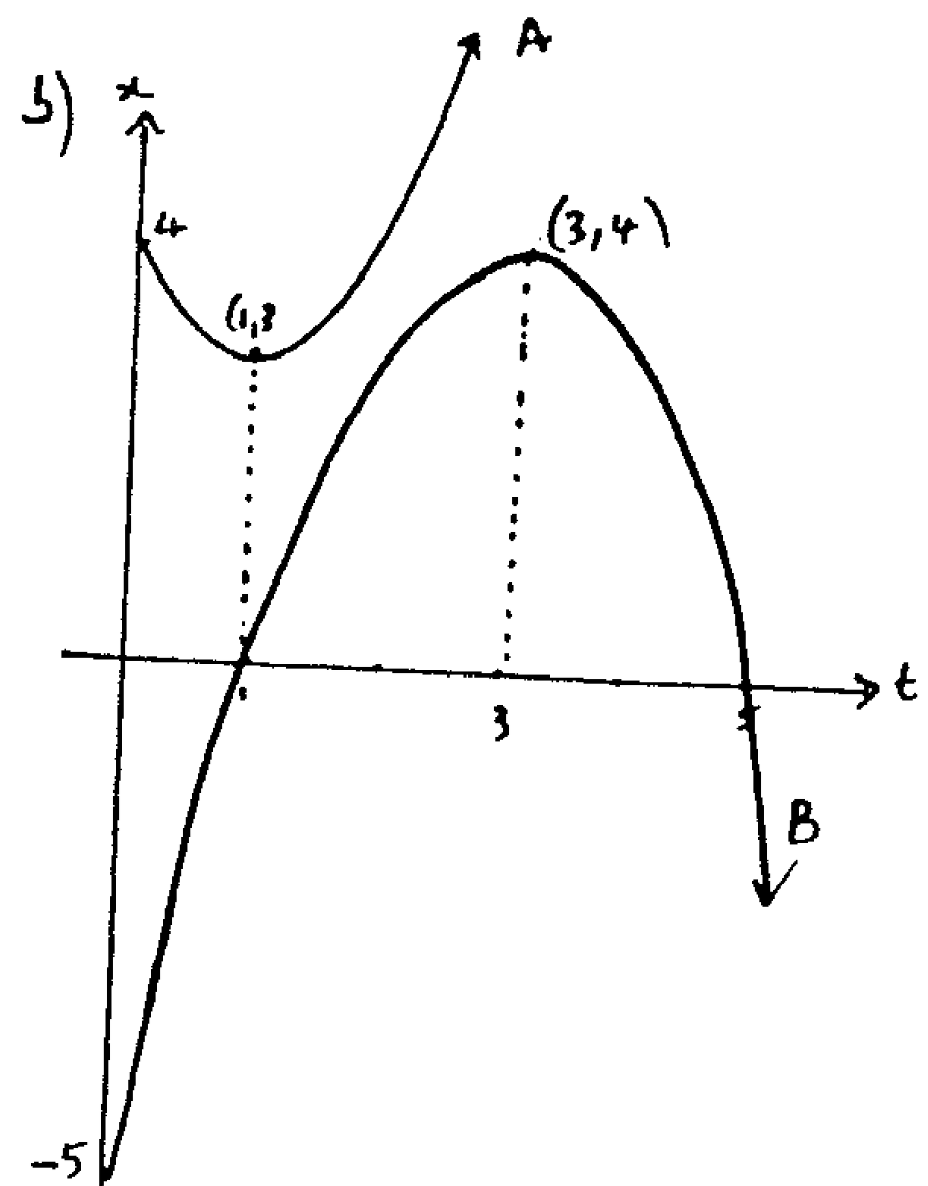
a) $v = 2t-2$

$x = t^2 - 2t + c$

at $x=4$, $t=0 \Rightarrow c=4$.

$x = t^2 - 2t + 4$

b)



c) Since t is the same unit for both particles, the required distance D, is the length of the vertical line b/w the curves.

$D = (t^2-2t+4) - (6t-t^2-5)$.

$D = 2t^2-8t+9$

d) Particles are closest when D is least

$\frac{dD}{dt} = 4t-8 = 0$

$\Rightarrow t = 2$

$\frac{d^2D}{dt^2} = 4 > 0 \Rightarrow$ concave up

$\therefore$ local min at $t=2$

Since this is a continuous function and there is only 1 min turning point, then $t=2$ is the absolute minimum.

$\therefore$ Closest together when $t=2$.

e) min. distance $= 2(2)^2 - 8(2) + 9$

$= 1\text{m}$

Question 4 (12 marks)

1. a) $\frac{dM}{dt} = -kM$ where $k = 0.03$

$\therefore M = M_0 e^{-kt}$

ie $M = 84 e^{-0.03t}$

When $t = 12$: $M = 84 e^{-0.03 \times 12}$

$= \underline{58.6 \text{ kg}}$

b) $0.2 \times 84 = 84 e^{-0.03 \times t}$

$0.2 = e^{-0.03t}$

$\underline{t = 53.6 \text{ hrs.}}$

2. $P(\text{Rain}) = 0.75$ $P(\text{Fine}) = 0.25$

$P(R \,\&\, \text{absent}) = 0.3$

$P(F \,\&\, \text{absent}) = 0.1$

$\therefore P(\text{absent Tuesday})$

$= P(\text{Rain absent})$ $P(\text{Fine \& absent})$

$= 0.75 \times 0.3 + 0.25 \times 0.1$

$= \underline{0.25}$

3. a) $y + 3 = m(x-1)$

$y = \underline{mx - m - 3}$

b) $y = \frac{1}{x}$
$y = mx - m - 3$

$\frac{1}{x} = mx - m - 3$

$mx^2 - mx - 3x = 1$

$m^2 - (m+3)x - 1 = 0$

c) For a tangent, $\Delta = 0$

$(m+3)^2 - 4m(-1) = 0$

$(m+3)^2 + 4m = 0$

$m^2 + m + 9 = 0$

$(m+9)(m+1) = 0$

$\underline{m = -9 \text{ or } -1}$

Question 5 (12 marks)

1. Let roots be α & 2α

$\alpha + 2\alpha = -L$

$3\alpha = -L$

$\alpha = -\frac{L}{3}$ - - - ①

$\alpha(2\alpha) = M$

$2\alpha^2 = M$ - - - ②

Solving ① & ②:

$2\left(-\frac{L}{3}\right)^2 = M$

$\frac{2L^2}{9} = M$

$\underline{2L^2 = 9M}$

2. $x(t) = \frac{3t^2}{4+t^3}$

$\therefore v(t) = \frac{(4+t^3).6t - 3t^2.3t^2}{(4+t^3)^2}$

$= \frac{24t + 6t^4 - 9t^4}{(4+t^3)^2}$

$v(t) = \frac{24t - 3t^4}{(4+t^3)^2}$

b) at rest when $v(t) = 0$

$24t - 3t^4 = 0$

$t(8 - t^3) = 0$

$\underline{t = 0 \text{ or } t = 2}$

c) at $t_2 = 2 + 2\sqrt{2}$: $x = \frac{3}{5}$

at $t_1 = 1$: $x = \frac{3}{5}$

d) The particle moves right from $t = 1$ $(x = \frac{3}{5})$ coming to rest when $t = 2$ (at $x = 1$). It then changes direction moving left to $x = \frac{3}{5}$ at $t = 2 + 2\sqrt{2}$.

e) as $t \to \infty$, $x \to 0$ and $v \to 0$.

So particle is slowing down but never comes to rest and never reaching $x = 0$.

f) Max. displacement when $v = 0$. ie When $t = 2$ and is 1 unit right of origin.

$v(t) = \frac{24t - 3t^4}{(4+t^3)^2}$

$= \frac{3t(8 - t^3)}{(4+t^3)^2}$