Number:	
Class:	

YEAR 12

ASSESSMENT TEST 3
TERM 2, 2015

MATHEMATICS

***Time Allowed** – 90 Minutes*
(Plus 5 minutes Reading Time)

Total marks - 60

- *All* four questions may be attempted
- Department of Education approved calculators are permitted
- All necessary working should be shown in al questions.
- Marks may not be awarded for careless or badly arranged work.
- This is an open book test. You may take one A4 sheet of paper into the examination with any required notes or formulas written on the sheet.
- Write your student number at the top of every page of your answer sheets.

Question 1

Marks

(a) Two cards are chosen at random, without replacement from the seven cards below.

2	2	3	4	6	6	6

What is the probability that:

(i) both cards are 2? 1

(ii) the sum of the two numbers on the cards chosen, is greater than 9? 2

(b) A function $f(x)$ has the following information supplied about its derivative $f'(x)$; 4

- $f'(x)=0$ at $x=$-1, 1 and 3
- $f'(x)>0$ for $x<$-1 and $x>3$
- $f'(x)<0$ for -1$<x<$1 and 1$<x<$3

Sketch the shape of a possible graph of $f(x)$. Include all stationary points.

(c) Consider the function $y=2x\log_e x$.

(i) Find $\frac{dy}{dx}$. 1

(ii) Hence find the minimum value of $2x\log_e x$ and justify your answer. 3

(d) A bottle of solvent is open and the solvent evaporates in such a way that the amount remaining, $V\ ml$, in the bottle is given by $V=4000e^{-0.005t}$, where t is time in hours.

(i) How much solvent is in the bottle initially? 1

(ii) How much solvent has evaporated out of the bottle after 20 hours? Answer to 2 decmal places. 1

(iii) How long is it before half of the initial amount of solvent has evaporated from the bottle? Answer to 2 decimal places. 1

(iv) If the solvent continues to evaporate, will the bottle ever become empty? Explain. 1

Question 2 (Start a new page)

(a) Farmer Joe had discovered that a certain type of mustard seedling has a $\frac{2}{3}$ probability of germination when planted in pots.

(i) If farmer Joe plants 3 seeds, calculate the probability that;

(α) all seeds germinate. 1

(β) at least one seed germinates. 2

(ii) Farmer Joe wishes to know the number of plants which must be planted so as to ensure that the probability of at least one successful germination is greater than 99%. Let n be the required number of plants. Calculate n. 3

(b) A particle P moves along a line so that at time t its displacement from the origin o is given by

$$x = \frac{6t^2}{8+2t^3}$$

(i) Find the velocity of the particle at time t. 2

(ii) Find the times when the particle is momentarily at rest. 2

(iii) Find the greatest displacement of the particle from the origin. 3

(iv) Describe the motion of the particle, paying particular attention to what happens for very large values of t. Do not consider describing acceleration. 2

Question 3 (Start a new page)

(a) Consider the curve given by $y = \frac{1}{4}x^4 - x^3$.

(i) Find any turning points and determine their nature. 4

(ii) Find any points of inflexion. 2

(iii) Sketch the curve for $-1.5 \le x \le 4.5$, indicating all the stationary points. 2

(iv) For what values of x is the curve concave down. 2

(b) The acceleration a, metre per second square of a moving object is given at time t seconds $(t \ge 0)$ by

$$a = 2\pi^2 \cos \pi t$$

At time $t = 0$, the object is at the point $x = 0$, and travelling with velocity $v = \pi$ metres per second.

(i) Find the velocity v and the displacement x as a function of t, for $t \ge 0$. 3

(ii) Find, for t in the range $0 \le t \le 4$, the values of t for which the object is stationary. 2

Question 4 (Start a new page)

(a) In Carlingford suburb, data has been collected on the chances of getting a red or green light at the traffic lights along a street. In this suburb, the traffic lights are either red or green.

If a driver gets a red light at one intersection there is a 70% chance they will get a red light again at the next intersection. However, if they get a green light at one intersection, there is a 40% chance of a red light at the next intersection.

Suppose at the first intersection the light is green.

(i) Draw a tree diagram showing all the information given above. 1

(ii) What is the probability of a red light at the third intersection? 2

(b) A police patrol officer while doing his duty on Friday night found a man unconscious on the roadside from a drug overdose. On arrival at the hospital, the doctor or the police did not know how much of the drug the unconscious man had taken.

The rate of change of concentration of the drug (C) in the blood is proportional to the concentration,

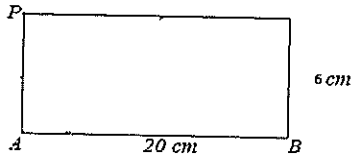
$$\text{i.e. } \frac{dC}{dt} = kC$$

(i) Prove that $C = C_o e^{kt}$ is a solution to $\frac{dC}{dt} = kC$ 1

(ii) Two hours after the unconscious man has taken the overdose, the blood concentration of the drug was 2.25 mg/L. An hour later the concentration dropped to 1.54 mg/L. Determine the initial concentration of the drug in the man's blood. Give your answer correct to one decimal place. 3

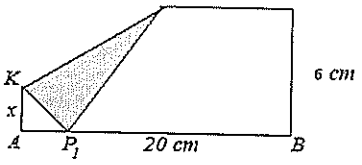
(iii) If the doctor did not give any further medication to the man, when would the drug concentration fall below the critical value of 0.4 mg/L? Answer to one decimal place. 2

(c) Shown below is a rectangular piece of paper 6 cm high by 20 cm long.



Not to scale

The vertex labelled P was placed on the side AB. Vertex P now lies on top of P_1.



Not to scale

At the bottom left of the rectangle, there is a small triangle AKP_1. Let the length of KA be x cm.

(i) Explain why KP_1 is $(6-x)$ cm long. 1

(ii) Show that the area of ΔAKP_1 is given by $A = x\sqrt{9-3x}$. 2

(iii) Hence show that when x is one-third the length of PA, the area of ΔAKP_1 is a maximum. 3

$\lambda = 2$

End of Examination