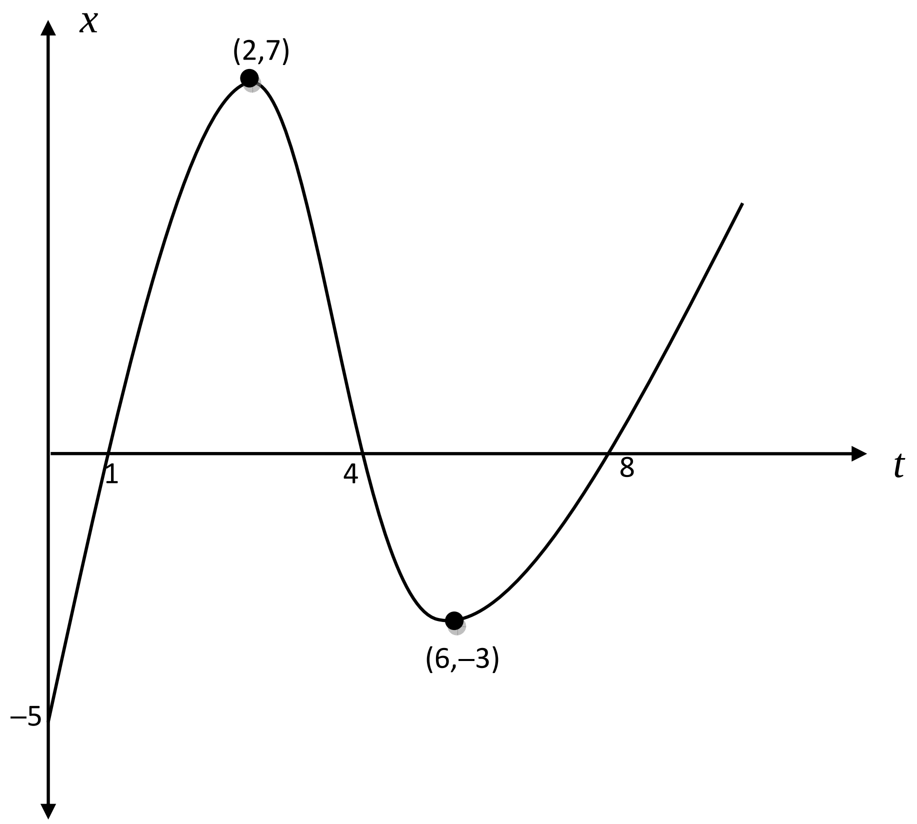
Question 1 *(10 marks)* **Marks**

a) The graph shows displacement, x metres from the origin, at any time t seconds, of a particle moving in a straight line. **5**



i) Where was the particle initially?

ii) When was the particle at the origin?

iii) When was the particle at rest?

iv) Estimate the time when the acceleration was zero.

v) How far did the particle travel in the first 8 seconds.

Question 1 continued. **Marks**

b) The acceleration of a particle, at time t seconds, is given by $\frac{d^2x}{dt^2} = 6 + \frac{2}{t^2}\ cm/s^2$. **5**

When $t = 1$, $\frac{dx}{dt} = -3$ and $x = 2$. Find correct to 4 significant figures, the value of x when $t = 2$.

Question 2 *(10 marks)* Start this question on a new page.

a) A full water tank is being drained and the number of litres in the tank, L, at time t minutes is given by the equation $L = 100(50 - t)^2$. **4**

i) At what rate is the water draining out of the tank when $t = 6$ minutes?

ii) What is the capacity of the tank?

iii) How long will it take to completely empty the tank?

b) The population of rabbits, N, on an island was estimated to satisfy the equation $N = N_0 e^{kt}$ where N_0 and k are constants and t is measured in months. **6**

i) Initially the rabbit population was 500 and at the end of 8 months there were 2200 rabbits. Find the value of k to 3 decimal places.

ii) Find the number of rabbits on the island after 1 year. (Give your answer correct to the nearest hundred).

iii) How long will it take for the rabbit population to exceed 20 000?

iv) Show that $\frac{dN}{dt} = kN$.

Question 3 *(10 marks)* Start this question on a new page. **Marks**

a) The third term of an arithmetic series is 7 and the tenth term is 42. **3**

i) Find the first term and the common difference.

ii) Find the sum of the first 12 terms.

b) The limiting sum of a geometric series is 4 and the first term is 1. **3**

i) Determine the common ratio

ii) Determine the fourth term of the sequence in rational form.

c) A person invests \$2 000 at the beginning of each year in a superannuation fund. Compound interest is paid at 8% per annum on the investment. The first \$2 000 is invested at the beginning of 2009 and the last is to be invested at the beginning of 2038. Calculate to the nearest dollar: **4**

i) the amount to which the 2009 investment will have grown by the beginning of 2039.

ii) the total value of the person's investment by the beginning of 2039.

Question 4 on page 4.

Question 4 *(10 marks)* Start this question on a new page. **Marks**

a) A particle moves so that its distance $\boldsymbol{x}$ cm from a fixed point O at time t seconds is given by $x = 2\cos 3t$. Determine when the particle first comes to rest after $t = 0$. **2**

b) When Jack left school he borrowed \$25 000 to buy his first car. The interest rate on the loan was 9% per annum and Jack planned to pay back the loan in 60 equal monthly repayments of \$M. **8**

i) Show that the amount owing immediately after making his first monthly repayment was $\$[25000 \times 1.0075 - M]$.

ii) Show that the amount owing immediately after making his third monthly repayment was

$$\$[25000 \times 1.0075^3 - M(1 + 1.0075 + 1.0075^2)].$$

iii) Calculate the value of M.

iv) If Jack could afford to make monthly repayments of \$1000, how long would it take him to repay the loan?

End of paper.

Question 1

ii) Particle initially 5 units left of origin ①

i) At the origin when $t = 1, 4, 8$ ①

iii) At rest when $t = 2$ and $t = 6$ ①

iv) Acceleration zero when $t \doteq 3$ ①

v) Distance $= 5+7+7+3+3$
$= 25m$ ①

b) $a = 6 + \frac{2}{t^2}$

$v = \int a \, dt$

$= \int (6 + 2t^{-2}) \, dt$

$= 6t - 2t^{-1} + c$ ①

$= 6t - \frac{2}{t} + c$

When $t = 1$ $v = -3$

$\therefore -3 = 6 \times 1 - \frac{2}{1} + c$

$-3 = 6 - 2 + c$

$\therefore c = -7$ ①

$\therefore \frac{dx}{dt} = 6t - 2t^{-1} - 7$

$x = \int \frac{dx}{dt} \, dt$

$x = \int (6t - 2t^{-1} - 7) \, dt$

$= \int 6t -$

$= 3t^2 - \frac{2}{t} - 7$ ①

$x = 3t^2 - 2\ln t - 7t + k$

When $t = 1$ $x = 2$

$\therefore 2 = 3 \times 1^2 - 2\ln 1 - 7 \times 1 + k$

$2 = 3 - 0 - 7 + k$ ①

$\therefore k = 6$

$\therefore x = 3t^2 - 2\ln t - 7t + 6$

$t = 2$ $x = 2.614$ ①

Question 2

a i) $L = 100(50-t)^2$

Rate Implies $\frac{dL}{dt}$

$$\frac{dL}{dt} = 2 \times 100(500 - t) \times -1$$

$$= -200(500 - t) \quad (1)$$

When $t = 6$ $\quad \frac{dL}{dt} = -200(500 - 6)$

$$= -8800 \quad (1)$$

$\therefore$ Draining at a rate of 8800 L/Min

a ii) Capacity when $t = 0$

$$\text{Capacity} = 100(50-0)^2$$
$$= 250000\ L \quad (1)$$

iii) Empty when $L = 0$

$$\therefore 100(50-t)^2 = 0$$
$$50 - t = 0 \quad (1)$$
$$t = 50$$

$\therefore$ Empty after 50 minutes

b i) $N = N_0 e^{kt}$

When $t = 8$ $N = 2200$, $N_0 = 500$

$$\therefore 2200 = 500 e^{k \times 8}$$
$$e^{8k} = \frac{2200}{500}$$
$$e^{8k} = 4.4$$
$$k = \frac{\ln 4.4}{8} \quad (2)$$
$$k = 0.1852 \ldots \text{(Calc)}$$
$$k = 0.185$$

ii) $N = N_0 e^{kt}$

$$= 500 e^{0.185 \times 12} \quad (1)$$
$$= 4603.6 \ldots \text{(Calc)}$$

Population after 1 year = 4600 rabbits

iii) Find t when $N > 20000$

$$20000 = 500 e^{0.185t}$$
$$40 = e^{0.185t}$$
$$t = \frac{\ln 40}{0.185} \quad (2)$$
$$\therefore t > 19.93 \ldots \text{(Calc)}$$

$\therefore$ Population > 20000 after 20 months

iv) $N = N_0 e^{kt}$

$$\frac{dN}{dt} = kN_0 e^{kt} \quad (1)$$

Since $N = N_0 e^{kt}$

$$\frac{dN}{dt} = kN \qquad \text{Q.E.D}$$

Q3 (a)(i) $T_3 = 7 \quad T_{10} = 42.$

$a + 2d = 7. \quad ①$

$a + 9d = 42 \quad ②$ ✓

② − ① $\quad 7d = 35$

$d = 5.$

$a = 7 - 10 = -3$, First term -3, common diff 5.] both ✓ (2).

(ii) $S_{12} = \frac{12}{2}(2\times -3 + 11\times 5).$

$= 294.$ (1) ✓

(b)(i) $S_\infty = \frac{a}{1-r}$

$4 = \frac{1}{1-r}$ ✓

$4 - 4r = 1$

$4r = 3$

$r = \frac{3}{4}.$ ✓ (2)

(ii) $T_4 = 1 \times \left(\frac{3}{4}\right)^3$

$T_4 = \frac{27}{64}.$ ✓ (1)

(c)(i) $A_1 = 2000 \times 1{\cdot}08^{30}$

$= 20125{\cdot}31$ ✓ (1).

(ii) $A_2 = 2000 \times 1{\cdot}08^{29}$

$\vdots$

$A_{30} = 2000 \times 1{\cdot}08.$ ✓

Total $= A_1 + A_2 + \ldots + A_{30}$

$= 2000\times 1{\cdot}08^{30} + 2000 \times 1{\cdot}08^{29} + \ldots + 2000 \times 1{\cdot}08.$

$= 2000(1{\cdot}08^{30} + 1{\cdot}08^{29} + \ldots + 1{\cdot}08)$

ie GP $a = 1{\cdot}08 \quad r = 1{\cdot}08 \quad n = 30.$

$= 2000 \times \frac{1{\cdot}08(1{\cdot}08^{30} - 1)}{1{\cdot}08 - 1}$ ✓

$= \$\,244\,692$ (nearest dollar) ✓ (3) (224 566. (if 29))

8/6.

Q4 a) $x = 2\cos 3t$ $\quad -6\sin 3t = 0$

$\dot{x} = -6\sin 3t$ ✓ $\quad \sin 3t = 0$

$3t = 0, \pi, \ldots$

$t = \pi/3$ ✓

b) i) $A_1 = 25000 + \frac{9/12}{100} \times 25000 - m$ ✓

$= 25000(1.0075) - m$

ii) $A_2 = A_1(1.0075) - m$

$= (25000(1.0075) - m)(1.0075) - m$

$= 25000(1.0075)^2 - m(1.0075) - m$ ✓

$A_3 = A_2(1.0075) - m$

$= (25000(1.0075)^2 - m(1.0075) - m)(1.0075) - m$ ✓

$= 25000(1.0075)^3 - m(1.0075)^2 - m(1.0075) - m$

$= 25000(1.0075)^3 - m(1 + (1.0075) + 1.0075^2)$ ✓

iii) $A_{60} = 0$

$25000(1.0075)^{60} = m(1 + (1.0075) + \ldots 1.0075^{59})$

$m = \dfrac{25000(1.0075)^{60}}{\dfrac{1(1.0075^{60} - 1)}{1.0075 - 1}}$ ✓

$= \$518.96$ ✓

iv) $25000(1.0075)^n = 1000(1 + (1.0075) + \ldots 1.0075^{n-1})$

$= 1000\left(\dfrac{1(1.0075^n - 1)}{1.0075 - 1}\right)$ ✓

$= 133333.33(1.0075^n) - 133333.33$

$108333.33(1.0075)^n = 133333.33$

$1.0075^n = 1.230769223$

$n = \dfrac{\log 1.23076923}{\log 1.0075}$

$n = 27.8$ m ✓