Question 1 (12 marks) Marks

a) Differentiate the following;

i. e^{-3x} (1)

ii. $e^{\sin x}$ (1)

iii. $\log_e (4x+1)^2$ (2)

iv. $\dfrac{\log_e x}{x}$ (2)

b) Find the following;

i) $\int e^{-2x} dx$ (1)

ii) $\int_1^6 \dfrac{6}{2x+3} dx$ (2)

c) Find the equation of the normal to the curve $y = e^{x^2}$ at $(1, e)$. (3)

Question 2 (12 marks) (START A NEW PAGE)

a) Draw a sketch of $y = 3\cos 2x$ over $0 \le x \le 2\pi$ (2)

b) Differentiate the following;

i. $y = \tan^2 4x$ (2)

ii. $y = 2\sin x^2$ (2)

iii. $y = e^{2x} \cos 3x$ (2)

c) Find the area between the two curves $y = \cos x$ and $y = \sin x$ over the domain $0 \le x \le \dfrac{\pi}{4}$. (2)

d) Find the volume of the solid formed by $y = \sec x$ being rotated about the x axis between $x = 0$ and $x = \dfrac{\pi}{4}$. (2)

Question 3 ... page 2

Question 3 (11 marks) (START A NEW PAGE)

a) A piece of plyboard has many slits cut into it. Each pair of slits has a constant difference in length. The shortest slit is 30cm and the longest slit is 50cm.

i. If the total length of all the slits is 1240cm, find the number of slits. (2)

ii. Find the constant difference in length between the slits. (2)

iii. Find the length of the 11^{th} shortest slit. (1)

b) In a geometric sequence of numbers the third term is 72 and the eighth term is 17496.

i. Find the first term and the common ratio. (2)

ii. Which is the largest term of the sequence less than 500000? (2)

iii. What is the sum of the terms less than 500000? (2)

Question 4 (10 marks) (START A NEW PAGE)

a) Frank and Eddie shared a Lotto win. Their shares are \$10000 each.

i. Frank invested his \$10000 in an investment account on January 1^{st} 2000. It is invested at 9% per annum, compounded monthly. How much would be in the investment account on January 1^{st} 2010, if no other payments were made during this period? (2)

ii. Eddie invested his \$10000 in an investment account on the same day as Frank. The interest on his investment account was compounded annually, at 9% per annum. He decided to deposit an extra \$1000 on January 1st, each year from January 1^{st} 2001 to 1^{st} January 2010. How much would be in his account on January 1^{st} 2010, including the last deposit. (4)

b) Jan borrows \$8000 and agrees to repay it in equal instalments each year for 10 years. If the interest is charged at 7% per annum on any part of unpaid debt, calculate the amount of each yearly instalment. (4)

i. Let $\$A_n$ be the amount owed on the loan after n years and let M be the instalments.

Show that $A_n = 8000(1.07)^n - M\left(\dfrac{1.07^n - 1}{1.07 - 1}\right)$

ii. Hence, calculate the amount of each yearly instalment.

END OF TEST

Yr 12 Maths 2011
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Question 1 (12 marks)

(a)(i) $\frac{d}{dx} e^{-3x} = -3e^{-3x}$ ✓ (1)

(ii) $\frac{d}{dx} e^{\sin x} = \cos x \, e^{\sin x}$ ✓ (1)

(iii) $\frac{d}{dx} \log_e (4x+1)^2 = \frac{d}{dx} 2 \log_e (4x+1)$

$= 2 \times \frac{4}{4x+1}$ ✓

$= \frac{8}{4x+1}$ ✓ (2)

(iv) $\frac{d}{dx} \frac{\log_e x}{x} = \frac{x \times \frac{1}{x} - \log_e x \cdot 1}{x^2}$ ✓

$= \frac{1 - \log_e x}{x^2}$ ✓ (2)

(b) (i) $\int (e^{-x}) \, dx = -e^{-x} + C$ ✓ (1) (do not deduct marks for missing $+C$)

(ii) $\int_1^6 \frac{6}{2x+3} \, dx = \int_1^6 3 \times \frac{2}{2x+3} \, dx$

$= \left[3 \times \log_e (2x+3) \right]_1^6$ ✓

$= 3 (\log 15 - \log 5)$

$= 3 \log \frac{15}{5}$

$= 3 \log 3$ ✓ (2)

$= \log 27$

(c) $y = e^{x^2}$

$\frac{dy}{dx} = 2x e^{x^2}$ ✓

gradient of tangent at $(1, e)$

$\therefore m = 2e$

gradient of normal

$m = \frac{-1}{2e}$ ✓

equation of normal

$y - e = -\frac{1}{2e}(x-1)$ ✓ (3)

$2ey - 2e^2 = -x + 1$ $\therefore x + 2ey - 2e^2 - 1 = 0$

Question 2 (12 marks).

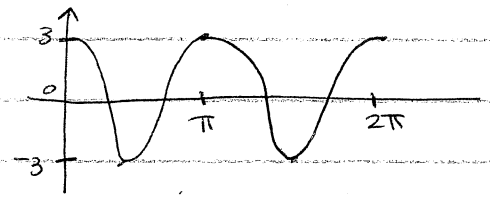
a) $y = 3\cos 2x \quad 0 \le x \le 2\pi$

amplitude $= 3$

period $= \pi$

1 mark correct curve

1 correct amplitude and period. (2)



b) (i) $y = \tan^2 4x$

$\frac{dy}{dx} = 2\tan 4x \cdot 4\sec^2 4x$ ✓ ✓

$= 8\tan 4x \cdot \sec^2 4x$ (2)

(ii) $y = 2\sin x^2$

$\frac{dy}{dx} = 2 \cdot 2x\cos x^2$ ✓

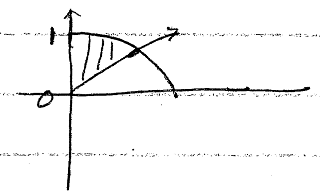
$= 4x\cos x^2$ ✓ (2)

(iii) $y = e^{2x} \cdot \cos 3x$

$\frac{dy}{dx} = e^{2x} \cdot -3\sin 3x + \cos 3x \cdot 2e^{2x}$ ✓

$= -3e^{2x}\sin 3x + 2e^{2x}\cos 3x$ ✓ (2)

c) $A = \int_0^{\frac{\pi}{4}} \cos x - \sin x \, dx$ ✓



$= \left[\sin x + \cos x\right]_0^{\pi/4}$

$= \left[\left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right) - (\sin 0 + \cos 0)\right]$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1$ ✓ (2)

$= \frac{2}{\sqrt{2}} - 1$

$= \sqrt{2} - 1$

d) $V = \pi \int_a^b y^2 \, dx$

$= \pi \int_0^{\frac{\pi}{4}} \sec^2 x \, dx$ ✓

$= \pi \left[\tan x\right]_0^{\pi/4}$

$= \pi\left(\tan\frac{\pi}{4} - \tan 0\right)$

$= \pi \; u^2$ ✓ (2)

Question 3

a) $a = 30cm \quad l = 50cm$

(i) $S_n = 1240cm$

$S_n = \frac{n}{2}(a+l)$

$1240 = \frac{n}{2}(30+50)$ ✓

$1240 = \frac{n}{2}(80)$

$1240 = n \times 40$

$n = 31$ ✓ (2)

∴ there are 31 slits.

(ii) $T_{31} = 50$

$a+(n-1)d = 50$

$30 + 30d = 50$ ✓

$30d = 20$

$d = \frac{2}{3}cm$ ✓ (2)

∴ the constant difference is $\frac{2}{3}cm$

(iii) $T_{11} = 30 + 10 \times \frac{2}{3}$

$= 36\frac{2}{3}cm$ ✓(1)

b) $T_3 = 72 = ar^2$ — ①

$T_8 = 17496 = ar^7$ — ②

$243 = r^5$

$3 = r$ ✓

$\therefore a \times 3^2 = 72$

$a = 8$ ✓ (2)

$\therefore a = 8$ and $r = 3$

c) $ar^{n-1} < 500000$

$8 \times 3^{n-1} < 500000$ ✓

$3^{n-1} < 62500$

$n-1 \log 3 < \log 62500$

$n-1 < \frac{\log 62500}{\log 3}$

$n-1 < 10.05170063$

$n-1 = 10$ ✓

$n = 11$ (2)

∴ the 11th term is the largest less than 500 000

Question 3

b)(iii) $S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{8(3^{11} - 1)}{(3 - 1)}$ ✓

$= 708587$ ✓ (2)

∴ The sum of terms 708 587

Question 4

(a) (i) $P = \$10000$

$r\% = 9\%$ p.a. $= 0.75\%$ per month $= 0.0075$

$n = 10$ years $= 120$ months

$A = 10000(1.0075)^{120}$ ✓

$= \$24513.57078$ ✓ (2)

$= \$24513.57$ (n.c).

(ii) $P = \$10000 + \$1000$ each year after.

$r\% = 9\%$ p.a $= 0.09$

$n = 10$ years

~~2000~~ $A_1 = 10000(1.09)^{10}$

2001 $A_2 = 1000(1.09)^9$

2002 $A_3 = 1000(1.09)^8$

2009 $A_{10} = 1000(1.09)^1$

2010 $A_{11} = 1000(1.09)^0$

✓

$$A_{TOTAL} = 10000(1.09)^{10} + 1000(1.09)^9 + 1000(1.09)^8 + \cdots + 1000(1.09)^1 + 1000$$

$$= 10000(1.09)^{10} + 1000(1 + 1.09 + 1.09^2 + \cdots + 1.09^9)$$

$$= 10000(1.09)^{10} + 1000\left(\frac{1((1.09)^{10} - 1)}{1.09 - 1}\right)$$ ✓

$= \$38866.56646$

$= \$38866.57$ (nc.) ✓ (3)

Alternative Method

2000 $A_1 = 10000(1.09)^1$

2001 $A_2 = (10000(1.09) + 1000)(1.09)^1$

$= 10000(1.09)^2 + 1000(1.09)^1$

2002 $A_3 = (10000(1.09)^2 + 1000(1.09)^1 + 1000)(1.09)$

$= (10000(1.09)^3 + 1000(1.09)^2 + 1000(1.09))$

2009 $A_{10} = 10000(1.09)^{10} + 1000(1.09)^9 + \cdots + 1000(1.09)^1$

2010 $A_T = 10000(1.09)^{10} + 1000(1.09)^9 + \cdots + 1000(1.09)^1 + 1000$ ✓

$$A_T = 10000(1.09)^{10} + 1000\left(\frac{1((1.09)^{10} - 1)}{1.09 - 1}\right)$$ ✓

$A_T = \$38866.57$ (nc) (3) ✓

(b) $P = \$8000$

$r = 7\%$ p.a. $= 0.07$

$n = 10$

$m =$ yearly instalment.

$A_1 = 8000(1.07)^1 - m$

$A_2 = (8000(1.07)^1 - M)(1.07) - m$

$= 8000(1.07)^2 - m(1.07) - m$

$A_3 = (8000(1.07)^2 - m(1.07) - m)(1.07)^1 - m$

$= 8000(1.07)^3 - m(1.07)^2 - m(1.07)^1 - m$

$\vdots$

$A_{10} = 8000(1.07)^{10} - m(1.07)^9 - m(1.07)^8 - m(1.07)^7 - \cdots m$ ✓

$= 8000(1.07)^{10} - m(1 + 1.07 + 1.07^2 + \cdots + 1.07^9)$.

$= 8000(1.07)^{10} - m\left(\frac{1(1.07^{10} - 1)}{1.07 - 1}\right)$ ✓

$= 8000(1.07)^{10} - m(13.81644796)$ ✓

$m = \frac{8000(1.07)^{10}}{13.81644796}$

(4)

$m = \$1139.020022$

$m = \$1139.02$ (nc) ✓