NORTH SYDNEY GIRLS HIGH SCHOOL

Assessment 3
6 June
2007

Mathematics

General Instructions

- Reading Time – 2 minutes
- Working Time – 1 hour 10 min
- Write using black or blue pen
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this page
- All necessary working should be shown in every question

Total Marks – 50

Attempt Questions 1–5

All questions are of equal value

Begin each question on a NEW PAGE.

NAME:____________________________ **TEACHER:**________________________

Question	H3	H5	H6	H8	Total
1	/1	/9			/10
2	/3	/2		/5	/10
3		/3		/7	/10
4	/2	/4		/4	/10
5			/10		/10
TOTAL	/6	/18	/10	/16	/50

Question 1 (10 marks) **Marks**

(a) Differentiate with respect to x

(i) $y = (e^x + 4)^7$ **2**

(ii) $f(x) = \ln(x^2 + 1)$ **1**

(b) Find

(i) $\int \sqrt{x}\,dx$ **1**

(ii) $\int (4x+2)^5\,dx$ **2**

(c) Evaluate $\int_{-1}^{2} (5 + e^{3x})\,dx$ correct to 1 decimal place. **3**

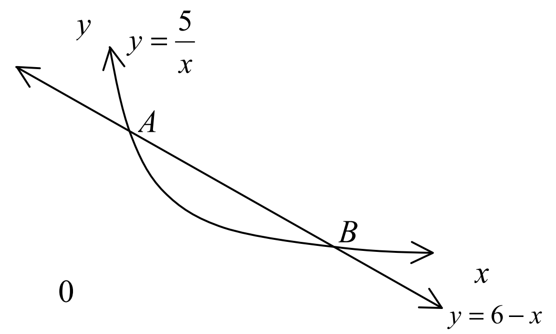
(d) Use the change of base formula to evaluate $\log_5 9$ correct to 3 decimal places. **1**

Question 2 (10 marks) Start a new page.

(a) Differentiate $\log_e \dfrac{x+7}{x-3}$ **2**

(b) Solve $2\log_e x = \log_e (6-5x)$. **3**

(c) The line $y = 6 - x$ meets the curve $y = \dfrac{5}{x}$ at points A and B as shown in the diagram.



(i) Find the co-ordinates of A and B. **2**

(ii) Find the shaded area between the curves. **3**

Question 3 (10 marks) Start a new page. **Marks**

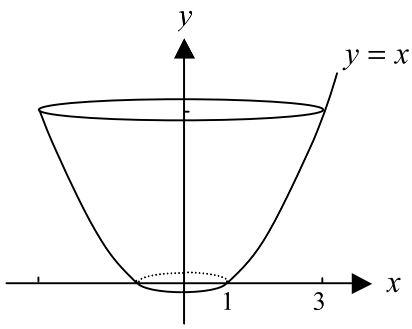
Continued overleaf

(a) Evaluate $\int_{-2}^{1}\left(x^3-4\right)dx$. **3**

(b) (i) Use the trapezoidal rule with 4 function values to find an approximation to the area under the curve $y=\ln x$ between $x=e$ and $x=4e$. **2**

(ii) Is the approximation found in (i) an overestimate or an underestimate of the exact area? Justify your answer. **1**

(c) A bowl is designed by rotating the portion of the curve $y=x^2-1$ between $x=1$ and $x=3$ around the y-axis as illustrated in the diagram below. **4**
Find the volume of the solid formed.



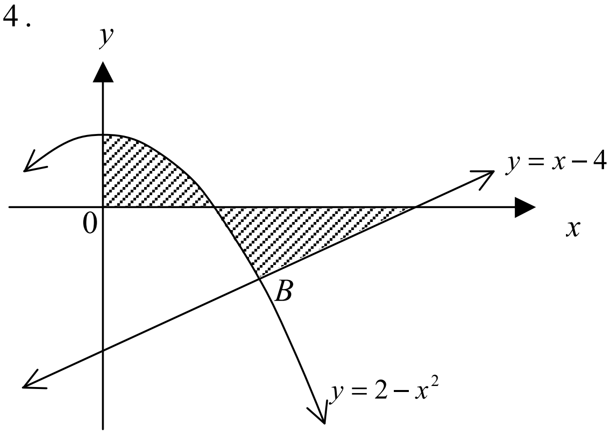
Question 4 (10 marks) Start a new page.

(a) If $\log_a p=1.18$ and $\log_a q=1.54$ find the value of $\log_a(apq)$ **2**

(b) (i) Differentiate $\sqrt{2x^2+3}$ with respect to x. **2**

(ii) Hence find $\int\frac{8x}{\sqrt{2x^2+3}}dx$. **2**

(c) In the diagram the shaded region is bounded by the parabola $y=2-x^2$, the axes and the line $y=x-4$.



(i) Show that the co-ordinates of the point of intersection B are (2, -2). **1**

(ii) Write an expression for the shaded area. Do NOT find the area.

Question 5 (10 marks) Start a new page. Continued overleaf **Marks**

The function $f(x) = xe^{-2x} + 3$ has a first derivative $f'(x) = e^{-2x} - 2xe^{-2x}$ and second derivative $f''(x) = 4xe^{-2x} - 4e^{-2x}$.

(a) Find the stationary point on $y = f(x)$. **2**

(b) For what values of x is the function increasing? **1**

(c) Show that the co-ordinates of the point of inflexion are $\left(1,\ 3 + \frac{1}{e^2}\right)$. **2**

(d) Determine when the curve is concave up. **1**

(e) Sketch $y = f(x)$ for $-\frac{1}{2} \le x \le 2$. **3**

(f) What value does $f(x)$ approach as x gets very large? **1**

End of paper

SOLUTIONS - 2007 MATHEMATICS Task 3 - YR 12

Q1 a) $y = (e^x + 4)^7$

$\frac{dy}{dx} = 7e^x (e^x + 4)^6$ ②

$f'(x) = \frac{2x}{x^2+1}$ ①

b)(i) $\int \sqrt{x}\, dx = \frac{2}{3} x^{3/2} + C$

$= \frac{2}{3}\sqrt{x^3} + C$ ①

(ii) $\int (4x+2)^5 dx = \frac{(4x+2)^6}{24} + C$ ②

c) $\int_{-1}^{2} (5 + e^{3x}) dx = \left[5x + \frac{1}{3}e^{3x}\right]_{-1}^{2}$ ③

$= \left(10 + \frac{e^6}{3}\right) - \left(-5 + \frac{e^{-3}}{3}\right)$

$= 149.5$

d) $\log_5 9 = \frac{\log 9}{\log 5} = 1.365$ ①

Question 2

a) $\frac{d}{dx}\left(\log_e \frac{x+7}{x-3}\right) = \frac{d}{dx}\left(\log_e(x+7) - \log_e(x-3)\right)$

$= \frac{1}{x+7} - \frac{1}{x-3}$ ②

b) $2\log_e x = \log_e(6-5x)$

$x^2 = 6 - 5x$ where $x > 0$ and $6 - 5x > 0$

$x^2 + 5x - 6 = 0$ $\therefore 0 < x < \frac{6}{5}$

$(x+6)(x-1) = 0$

$\therefore x = 1$ or -6 ③

$\therefore x = 1$ is the only solution

c)(i) $\frac{5}{x} = 6 - x$ at A and B

$x^2 - 6x + 5 = 0$

$(x-5)(x-1) = 0$

$x = 1, 5$ at A $x = 1, y = 5$

at B $x = 5$ $y = 1$

$\therefore A = (1,5)$ and $B = (5,1)$ ②

(ii) Area $= \int_1^5 6 - x - \frac{5}{x}\, dx$

$= \left[6x - \frac{x^2}{2} - 5\log_e x\right]_1^5$

$= 12 - 5\ln 5$

$= 3.95$ (2 d.p)

Question 3

a) $\int_{-2}^{1} (x^3 - 4)\, dx = \left[\frac{x^4}{4} - 4x\right]_{-2}^{1}$

$= \left(\frac{1}{4} - 4\right) - (4 + 8)$

$= -15\frac{3}{4}$

b)(i)

x	e	$2e$	$3e$	$4e$
$\ln x$	$\ln e$	$\ln 2e$	$\ln 3e$	$\ln 4e$
$=$	1	$1+\ln 2$	$1+\ln 3$	$1+\ln 4$

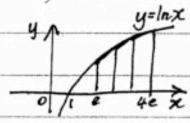
Calculator: $\ln(2e)$

Area $\doteqdot \frac{e}{2}(1 + 1 + \ln 2) + \frac{e}{2}(1 + \ln 2 + 1 + \ln 3) + \frac{e}{2}(1 + \ln 3 + 1 + \ln 4)$

$= \frac{e}{2}\{6 + 2\ln 2 + 2\ln 3 + \ln 4\}$ (exact)

$= 14.91$ (2 d.p)

(ii) The curve is concave down and the trapezia are under the curve at all times. $\therefore$ This estimate underestimates the true area.



c) $V = \pi \int x^2\, dy$ $x = 1, y = 0$

$= \pi \int_0^8 (y+1)\, dy$ $x = 3, y = 8$

$y = x^2 - 1$

$= \pi\left[\frac{y^2}{2} + y\right]_0^8$ $x^2 = y + 1$

$= \pi\left[\frac{64}{2} + 8 - 0\right]$

$= 40\pi$ units^3

Question 4

a) $\log(apq) = \log_a a + \log_a p + \log_a q$

$= 1 + 1.18 + 1.54$

$= 3.72$

b) (i) $f'(x) = \frac{1}{2}(2x^2 + 3)^{-1/2} \cdot 4x$

$= \frac{2x}{\sqrt{2x^2+3}}$

(ii) $\int \frac{8x}{\sqrt{2x^2+3}} = 4 \cdot \int \frac{2x}{\sqrt{2x^2+3}}\, dx$

$= 4\sqrt{2x^2+3} + C$

Question 4 (cont)

c) (i) At $x=2$ $\quad y=x-4 = -2$ $\quad$ and $\quad y=2-x^2 = 2-2^2 = -2$

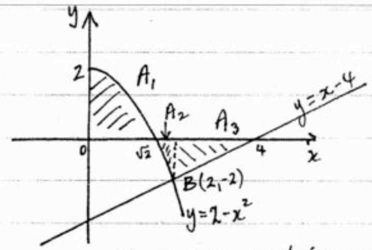
$\therefore$ B has co-ordinates $(2,-2)$

(ii) $\text{Area} = \int_0^{\sqrt{2}}(2-x^2)\,dx - \int_{\sqrt{2}}^{2}(2-x^2)\,dx - \int_2^4 (x-4)\,dx$

OR

$\text{Area} = \int_0^{\sqrt{2}}(2-x^2)\,dx + \left|\int_{\sqrt{2}}^{2}(2-x^2)\,dx\right| + \left|\int_2^4 (x-4)\,dx\right|$

or area of $\Delta = \frac{1}{2}\times 2\times 2$



A_2 and A_3 are below x axis

$\therefore \int$ will be negative for each

Question 5

$f(x) = xe^{-2x}+3$

$f'(x) = e^{-2x}-2xe^{-2x}$

$f''(x) = 4xe^{-2x}-4e^{-x}$

a) Stat pt if $f'(x)=0$

$\therefore e^{-2x}(1-2x)=0$

As $e^{-2x}>0$ for all x, then $x=\frac{1}{2}$ is only sol^n

then $y = 3+\frac{1}{2e}$

$\therefore$ Stat pt at $\left(\frac{1}{2}, 3+\frac{1}{2e}\right)$

b) Increasing if $f'(x)>0$

$e^{-2x}(1-2x)>0$

$\therefore 1-2x>0$ as $e^{-2x}>0$

$\therefore x<\frac{1}{2}$

c) $f''(x)=0$ for pt of inflexion

$4e^{-2x}(x-1)=0$

$\therefore x=1$ as $e^{-2x}>0$

then $y = e^{-2}+3 = 3+\frac{1}{e^2}$

Check concavity

x	$\frac{1}{2}$	1	$1\frac{1}{2}$
$f''(x)$	$-$	0	$+$

$\therefore$ Concavity changes

$\therefore$ Inflexion at $\left(1, 3+\frac{1}{e^2}\right)$

d) Concave up if $f''(x)>0$

ie $4e^{-2x}(x-1)>0$

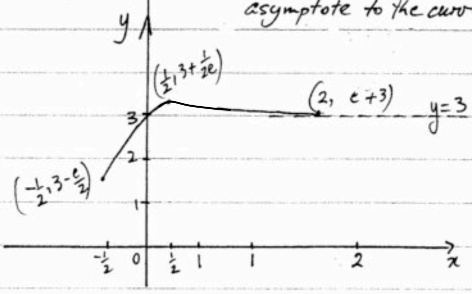
$\therefore x>1$ since $4e^{-2x}>0$

f) As $x\to\infty$, $e^{-2x}\to 0$

$\therefore f(x)\to 3$

ie $f(x)$ approaches 3

Note: The curve exists at $x=0$ and so there is NOT an open circle at $(0,3)$. A curve may pass through an horizontal asymptote. It is only as becomes very large in either direction, that the line becomes an asymptote to the curve



c) When $x=0$, $f(x)=3$

$x=-\frac{1}{2}$ $\quad f(x) = 3-\frac{e}{2} \doteqdot 1.64$

$x=\frac{1}{2}$ $\quad f(x) = 3+\frac{1}{2e} \doteqdot 3.18$

$x=1$ $\quad f(x) = 3+\frac{1}{e^2} \doteqdot 3.14$

$x=2$ $\quad f(x) = 3+\frac{2}{e^4} \doteqdot 3.037$