2004

HSC COURSE
ASSESSMENT TASK 3

Mathematics

General Instructions

- Time allowed – 50 minutes
- Write using blue or black pen
- Board-approved calculators and mathaids may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question
- Each question is to be answered on a separate answer page.

Topics:

- Trigononometric Functions
- Exponential and Logarithmic Functions

Total Marks – 34

- Attempt Questions 1 – 4

- Question 1 – 8 marks
  Question 2 – 9 marks
  Question 3 – 8 marks
  Question 4 – 9 marks

Question 1 (8 marks) Use a SEPARATE answer page.

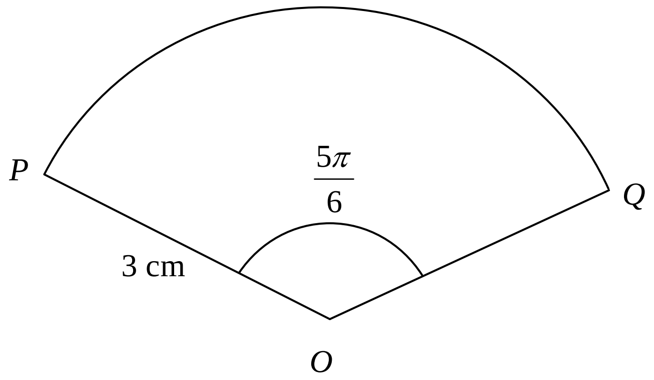
Marks

(a) Write down the exact value of $120°$ in radians. **1**

(b) Find the exact value of $\cos\frac{\pi}{4}$. **1**

(c) Solve $2\cos x + \sqrt{3} = 0$ for $0 \le x \le 2\pi$. **2**

(d)



In the diagram, PQ is an arc of a circle with centre O. The radius $OP = 3$ cm and the angle POQ is $\frac{5\pi}{6}$ radians. Find the exact area of the sector POQ. **2**

(e) Evaluate $\int_0^{\frac{\pi}{6}} 2\sin 2x \, dx$ **2**

Question 2 (9 marks) Use a SEPARATE answer page. **Marks**

(a) Differentiate $\dfrac{\sin x}{x}$. **2**

(b) Find $\int \sec^2(1+3x)\,dx$. **1**

(c) Find the equation of the tangent to the curve $y = 1 - 2\cos x$ at the point where $x = \dfrac{\pi}{2}$. **3**

(d)

(i) Sketch $y = 2\cos\dfrac{x}{2}$, for $0 \le x \le 2\pi$, showing all essential features and labelling the curve. **2**

(ii) Hence clearly sketch $y = 1 + 2\cos\dfrac{x}{2}$ on the same set of axes, labelling this curve also. **1**

Question 3 (8 marks) Use a SEPARATE answer page. **Marks**

(a) Evaluate, correct to three significant figures:

(i) $e^{-1.9}$ **1**

(ii) $\log_e 36$. **1**

(b) Solve $2\log_5 3 = \log_5 x - \log_5 9$. **2**

(c) Find $\int \frac{1}{5-2x}\, dx$. **1**

(d) Differentiate $3e^{x^2}$. **1**

(e)

(i) Show that the point $(e+2, 1)$ lies on the curve $y = \log_e(x-2)$. **1**

(ii) Sketch the graph of $y = \log_e(x-2)$, showing clearly any asymptotes. **1**

Question 4 (9 marks) Use a SEPARATE answer page. **Marks**

(a) A solid is formed by rotating the portion of the curve $y = e^x$ between $x = 0$ and $x = 2$ around the x-axis. **3**

Find the volume of this solid. Leave the answer in exact form.

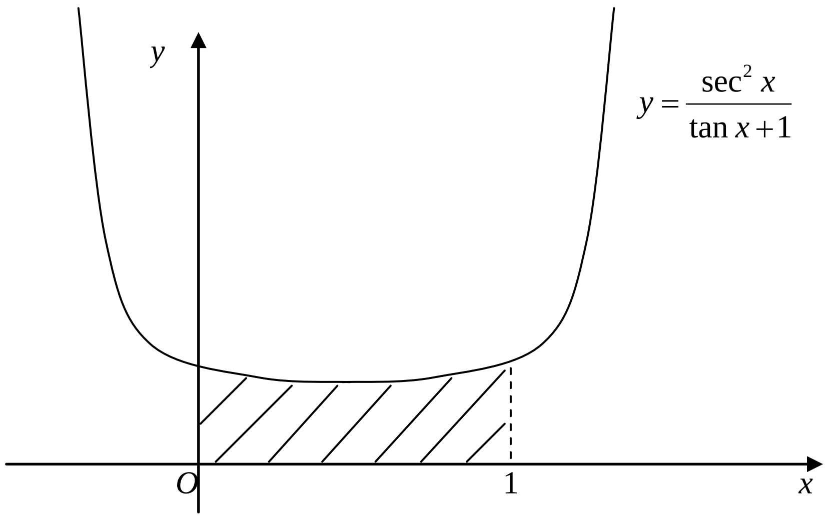
(b)

(i) Show that $\frac{d}{dx}(xe^{-x}) = (1-x)e^{-x}$. **1**

(ii) Find the stationary point on the curve $y = xe^{-x}$. **1**

(iii) Determine the nature of this stationary point. **1**

(c)



The shaded region in the diagram is bounded by the curve $y = \frac{\sec^2 x}{\tan x + 1}$, the x-axis, the y-axis, and the line $x = 1$. Find the area of the shaded region. **3**

End of Paper

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \, \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

HSC MATHS 2004 – T3

Q1

(a) $120^\circ = \frac{2\pi}{3}$

1

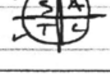
(b) $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

1

(c) $2\cos x + \sqrt{3} = 0 \qquad 0 \le x \le 2\pi$

$\cos x = -\sqrt{3}/2$

$x = \frac{5\pi}{6}, \frac{7\pi}{6}$



1 for one solution
2 · both " "

(d) $\text{Area} = \frac{1}{2}r^2\theta$

$= \frac{1}{2}\times 3^2 \times \frac{5\pi}{6}$

$= \frac{15\pi}{4}\ \text{cm}^2$

1 for correct substitution.
1 for correct answer with units.

(e) $\int_0^{\pi/6} 2\sin 2x\, dx = \left[-2\cos 2x \times \frac{1}{2}\right]_0^{\pi/6}$

$= \left[-\cos 2x\right]_0^{\pi/6}$

1 for correct primitive

$= -\cos\pi/3 - (-\cos 0)$

1 for correct answer.

$= -1/2 + 1$

$= 1/2.$

Q2.

(a) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\frac{d}{dx}\left(\frac{\sin x}{x}\right) = \frac{x\cos x - \sin x \times 1}{x^2}$

$= \frac{x\cos x - \sin x}{x^2}$

2 for correct answer
1 for correct application of rule except for 1 error.?

Q2 cont'd.

(b) $\int \sec^2(1-3x)\,dx = -\frac{1}{3}\tan(1-3x)+c$ — 1

(c) $y = 1 - 2\cos x$

$y' = 2\sin x$

at $x = \frac{\pi}{2}$, $y = 1 - \cos\frac{\pi}{2} = 1$

$y' = 2\sin\frac{\pi}{2} = 2 = m$

$y - y_1 = m(x - x_1)$

$y - 1 = 2(x - \frac{\pi}{2})$

$y = 2x + \pi + 1$

1 for correct gradient

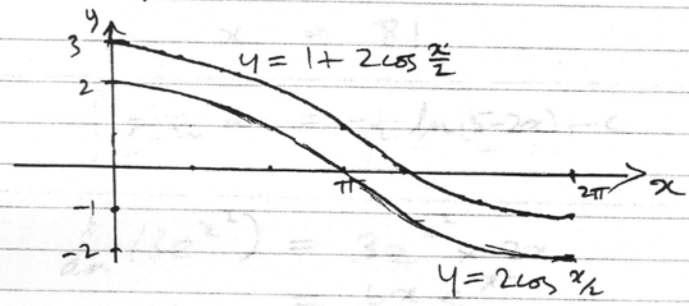
1 for correct subst'n into formula.

1 for correct answer

(d) $y = 2\cos\frac{x}{2}$ $\quad 0 \le x \le 2\pi$

(i) Period $= \frac{2\pi}{1/2} = 4\pi$

Amplitude $= 2$



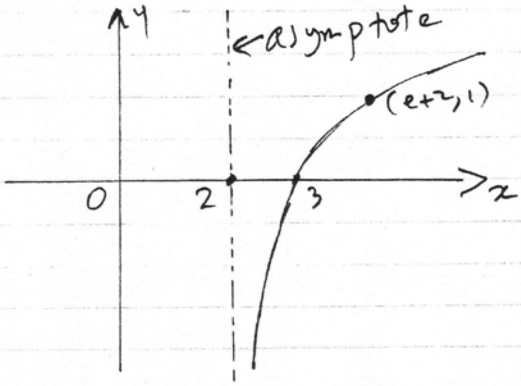
2 for correctly drawn & labelled graph

1 for correctly labelled graph showing either correct amplitude or correct period.

(ii) See graph.

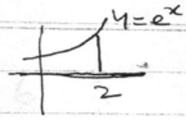
1 for correct graph, based on result of (i)

Q3.

(a) (i) $e^{-1.9} = 0.150$ (3.s.f.)	1
(ii) $\log_e 36 = 3.58$ (3.s.f.)	1
(b) $2\log_5 3 = \log_5 x - \log_5 9$ $\log_5 3^2 = \log_5 \frac{x}{9}$ $9 = \frac{x}{9}$ $x = 81$	2 for correct answer 1 for correct application of at least 1 log law.
(c) $\int \frac{1}{5-2x}\,dx = -\frac{1}{2}\ln(5-2x) + c$	1
(d). $\frac{d}{dx}(3e^{x^2}) = 3e^{x^2} \times 2x$ $= 6xe^{x^2}$	1
(e) (i) $y = \log_e(x-2)$ subst $(e+2, 1)$ $= \log_e(e+2-2)$ $= \log_e e$ $= 1$ $\therefore$ $(e+2, 1)$ lies on $y = \log_e(x-2)$	1
(ii) 	1 for showing correctly shaped curve & clearly indicating asymptote

Q4.

(a) $V = \pi \int_a^b y^2\, dx$



$= \pi \int_0^2 (e^x)^2\, dx$ — 1 for correct volume expression

$= \pi \int_0^2 e^{2x}\, dx$ — 1 for correct primitive

$= \frac{\pi}{2}\left[e^{2x}\right]_0^2$ — 1 for correct evaluation

$= \frac{\pi}{2}(e^4 - e^0) = \frac{\pi}{2}(e^4 - 1)$

(b) (i) $y = xe^{-x}$ $\quad uv' + vu'$

$y' = x \times (-e^{-x}) + e^{-x} \times 1$ — 1

$= e^{-x}(1-x)$

(ii) let $e^{-x}(1-x) = 0$

$x = 1 \quad (e^{-x} \neq 0)$ — 1

$y = 1 \times e^{-1}$

$= 1/e$

$\therefore$ Stationary Point $= (1, \frac{1}{e})$

(iii)

x	<1	1	>0
$e^{-x}(1-x)$	$(+)$	0	$(-)$
	/	—	\

— 1

$\therefore (1, \frac{1}{e})$ is MAXIMUM turning point — Must show working

---

(c) $A = \int_0^1 \frac{\sec^2 x}{\tan x + 1}\, dx$ — 1 for correct integral expression

$= \left[\ln(1 + \tan x)\right]_0^1$

$= (\ln(1 + \tan 1)) - \ln(1 + \tan 0)$ — 1 for correct primitive

$= \ln(1 + \tan 1) - 0$

$= 0.9389\ldots$ — 1 for correct evaluation

$= 0.94$ unit^2 (2 dec. pl).