NAME:____________________________

THE SCOTS COLLEGE

HSC 2 Unit MATHEMATICS
25% HSC ASSESSMENT
Wednesday 6th June 2007

INSTRUCTIONS:

* Time allowed: **50 minutes.**
* **Approved calculators may be used.**
* All questions are to be answered on the paper provided.
* Start each question on a new page.
* **All** necessary working must be shown.
* Marks will not be awarded for careless or poorly arranged work.

Outcomes Being Assessed.

H3 Manipulates algebraic expressions involving logarithmic and exponential functions.

H4 Expresses practical problems in mathematical terms based on simple given models.

H5 Applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems.

OUTCOME	QUESTION		MARKS
H3	Q2		/5
H4	Q1		/16
H5	Q3	/9	
	Q4	/11	/20
		TOTAL	/41

NAME:______________________________

QUESTION 1 **(16 Marks)**

a) The acceleration of a particle at any time, t seconds, is given by $a = -36t$. The particle starts from the origin with velocity 72m/s.

i) Find expressions for the velocity and the displacement of the particle.
ii) When is the particle at rest?
iii) Calculate the distance travelled between $t = 2$ and $t = 5$.

[8]

b) A particle moves along a straight line so that its displacement, x metres, from a point O is given by $x = 3 - 2\cos 2t$ where t is measured in seconds.

i) What is the initial displacement of the particle?
ii) Sketch the graph of x as a function of t for $0 \le t \le \pi$.
iii) Hence, or otherwise, find when **and** where the particle first comes to rest after $t = 0$.
iv) Find the time when the particle reaches its maximum speed.
v) What is the maximum speed?

[8]

QUESTION 2 **(5 Marks)** **START A NEW PAGE**

a) The number N of bacteria in a colony grows according to the law $N = N_0 e^{kt}$, where N_0 and k are positive constants, and where time, t, is measured in days.

i) Show that N satisfies the equation $\frac{dN}{dt} = kN$.
ii) Find the value of k if the original number increases from 5000 to 8400 in 4 days.
iii) How long will it take for the bacteria in the colony to triple?

[5]

NAME:______________________________

QUESTION 3 **(9 Marks)** **START A NEW PAGE**

a) Evaluate $\sum_{k=3}^{7}(10-2k)$ [1]

b) An arithmetic series is such that the seventh term is 19 and the fifteenth term is 35. Find

i) the common difference
ii) the first term
iii) the sum of the first 30 terms. [4]

c) The first 3 terms of a geometric sequence are 8, 4, 2, ...

i) Find an expression for the n^{th} term of the sequence.
ii) If the last term of the sequence is $\frac{1}{128}$, how many terms are in the sequence?
iii) Show that the sum of the sequence does not exceed 16.

[4]

QUESTION 4 **(11 Marks)** **START A NEW PAGE**

a) During the upcoming APEC summit in Sydney a limousine driver's route will require him to make a series of trips from the airport, to pick up visiting dignitaries from their hotels and take them back to the airport.
Each trip begins at the airport, then visits one hotel only at a time then returns to the airport.
The first hotel pick up point was 8km from the airport. The second hotel was 3km further away from the airport than the previous hotel. For each successive trip, the next hotel was 3km further away from the airport than the previous one.

i) How far away will the limousine travel on its 15^{th} trip?
ii) Calculate the total distance the limousine will travel if it makes 15 trips?

[3]

NAME:______________________________

b) A sum of money, $\$P$, was invested in a bank account that earned interest at a rate of 6% p.a. compounded annually. After 7 years the investment was worth \$50,000. Calculate the sum of money, $\$P$, that was originally invested. [2]

c) Guy borrows \$45 000 to purchase a new car. Interest is charged at the rate of 2% per month on the balance owing. Equal repayments of $\$M$ are made monthly.

i) Write an expression to represent how much Guy owes after his first repayment. [1]

ii) Show that the amount owing after the n^{th} month is given by

$$45000 \times 1.02^n - M\frac{(1.02^n - 1)}{0.02}$$ [2]

iii) Calculate the value of each monthly instalment, $\$M$, if Guy intends to repay the loan at the end of 5 years. [2]

iv) What is the total interest over the 5 years? [1]

END OF PAPER

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

HSC 2unit Maths Assessment Task #3

6th June 2007.

<u>Question 1</u> (16 marks)

a) i) $a = -36t$

$v = \int a \, dt$

$= \int -36t \, dt$

$v = -18t^2 + c$ 1

if $t=0$ $v=72$

$72 = -18(0)^2 + c$

$\therefore c = 72$

$\boxed{\therefore v = -18t^2 + 72}$ 1

$x = \int v \, dt$

$= \int -18t^2 + 72 \, dt$

$x = -6t^3 + 72t + k$ 1

at $x=0$ $t=0$

$0 = -6(0)^3 + 72(0) + k$

$k = 0$

$\boxed{\therefore x = -6t^3 + 72t}$ 1

ii) at rest when $v=0$

$0 = -18t^2 + 72$

$18t^2 = 72$

$t^2 = 4$ 1

$t = \pm\sqrt{4}$

$t = \pm 2$

$\therefore$ particle at rest at $t=2$ seconds. 1

iii) $x = -6t^3 + 72t$

at $t=2$

$x = -6(2)^3 + 72(2)$

$= -48 + 144$

$= 96$ m $\frac{1}{2}$

at $t=5$

$x = -6(5)^3 + 72(5)$

$= -750 + 360$ $\frac{1}{2}$

$= -390$ m

$\therefore$ distance travelled

$= 96 + 390$

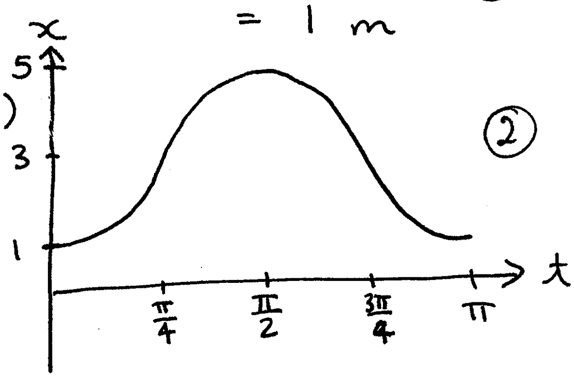
$= 486$ m 1

b) $x = 3 - 2\cos 2t$

i) $t=0$ $x = 3 - 2(1)$ 1

$= 1$ m

ii)



2

iii) $t = \frac{\pi}{2}$ 1, $x = 5$ m 1

iv) max speed when $a=0$

$v = 4\sin 2t$

$a = 8\cos 2t$

$8\cos 2t = 0$

$\cos 2t = 0$

$2t = \frac{\pi}{2}, \frac{3\pi}{2}$ 2

$\therefore t = \frac{\pi}{4}, \frac{3\pi}{4}$

v) max speed $= \cancel{8}$ 4 m/s 1

Question 2 (5 marks)

a) $N = N_0 e^{kt}$

i) $\frac{dN}{dt} = N_0 \times k e^{kt}$

$= k N_0 e^{kt}$

$= k N$ (1)

ii) $N = 8400$ $N_0 = 5000$ $k = 4$

$8400 = 5000 e^{4k}$

$\frac{8400}{5000} = e^{4k}$

$1.68 = e^{4k}$ (1)

$\ln 1.68 = 4k$

$\therefore k = \frac{\ln 1.68}{4}$

$k = 0.129698....$ (1)

iii) $N_0 = 5000$ $N = 15000$

$t = ?$ $k = 0.129698....$

$15\,000 = 5\,000 e^{0.129698....t}$

$3 = e^{0.1296...t}$

$\ln 3 = 0.1296...t$ (1)

$t = \frac{\ln 3}{0.1296...}$

$t = 8.4705...$

$t \doteqdot 8.5$ ~~yrs~~ days (1)

Question 3 (9 marks)

a) $\sum_{k=3}^{7} (10-2k)$

$= 4 + 2 + 0 + -2 + -4$

$= 0$ ①

b) i) $T_7 = 19$

$T_{15} = 35$

$a + 6d = 19$ — (1)

$a + 14d = 35$ — (2) ①

(2)−(1) $8d = 16$

$d = 2$ ①

ii) $a + 6(2) = 19$

$a + 12 = 19$

$a = 7$ ①

iii) $S_{30} = \frac{30}{2}(2(7) + 29\times2)$

$= 15(14 + 58)$

$= 1080$ ①

c) i) $a = 8$ $r = \frac{1}{2}$

$T_n = ar^{n-1}$

$T_n = 8\left(\frac{1}{2}\right)^{n-1}$ ①

ii) $\frac{1}{128} = 8\left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{1024} = \left(\frac{1}{2}\right)^{n-1}$ ①

$\frac{1}{2^{10}} = \frac{1}{2^{n-1}}$

$\therefore n - 1 = 10$ ①

$n = 11$ terms

iii) $S_\infty = \frac{a}{1-r}$

$= \frac{8}{1-\frac{1}{2}}$

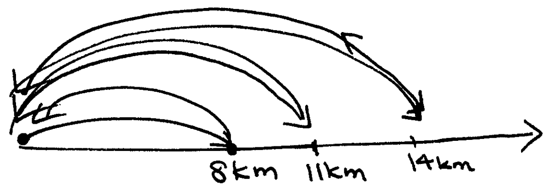
$= 16$ ①

since limiting sum = 16

$\therefore$ sum of series cannot exceed 16.

Question 4 (11 marks)

a)



i) $16 + 22 + 28 + \dots$

$a = 16 \quad d = 6$

$T_{15} = 16 + 14 \times 6$

$= 100$ km

limo will travel 100 km on its 15th trip.

ii) $S_{15} = \frac{15}{2}\left(2\times 16 + (14)(6)\right)$

$= 870$ km (3)

total distance travelled = 870km

b) $50\,000 = P(1.06)^7$

$P = \frac{50\,000}{(1.06)^7}$ (1)

$P = \$33\,252.86$ (1)

\$33 252.86 was originally invested.

c) i) $A_1 = 45\,000(1.02) - M$ (1)

ii) $A_2 = A_1(1.02) - M$

$= [45\,000(1.02) - M](1.02) - M$

$= 45\,000(1.02)^2 - M(1.02) - M$

$= 45\,000(1.02)^2 - M(1 + 1.02)$ (1)

$A_3 = A_2(1.02) - M$

$= [45000(1.02) - M(1 + 1.02)](1.02) - M$

$= 45000(1.02)^3 - M(1 + 1.02 + 1.02^2)$

$\vdots$

$A_n = 45\,000(1.02)^n - M\left(\frac{1(1.02^n - 1)}{1.02 - 1}\right)$

$= 45000(1.02)^n - M\left(\frac{1.02^n - 1}{0.02}\right)$ (1)

iii) 5yrs = 60mths = n

$A_{60} = 0$

$0 = 45\,000(1.02)^n - M\left(\frac{1.02^{60} - 1}{0.02}\right)$

$45000(1.02)^{60} = M\left(\frac{1.02^{60} - 1}{0.02}\right)$ (1)

$M = \frac{45\,000(1.02)^{60}}{\left(\frac{1.02^{60} - 1}{0.02}\right)}$

$= \frac{147\,646.3855\ldots}{114.05153\ldots}$

$= 1294.5584\ldots$ (1)

$\therefore$ mthly repaymt = \$1294.56

iv) $I = (1294.56 \times 60) - 45\,000$

$= 77\,673.60 - 45\,000$

$= \$32\,673.60$ (1)

HSC 2unit Maths Assessment Task
6th June 2007.

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at $x=0$ $t=0$

$0 = -6(0)^3 + 72(0) + k$

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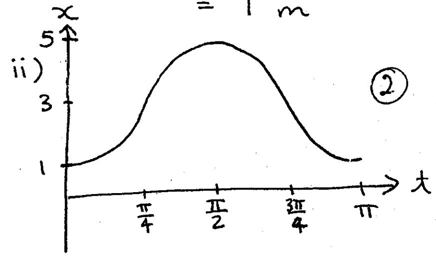
$= 1$ m

ii) (2)



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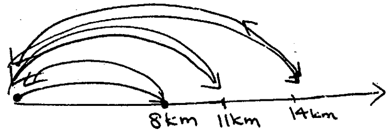

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