Name: ____________________________

Teacher: __________________________

The Scots College

2nd June 2009

Year 12 Mathematics

Weighting: 20%

Time allowed: 45 minutes

General Instructions

- *Working time – 45 minutes*
- *Show all necessary working out*
- *Please attempt all questions*
- *Board approved calculators may be used in this assessment task*
- *Attempt each Question on a new page*

Questions	Outcomes	Marks
1,2	***H9*** ***communicates using mathematical language, notation, diagrams and graphs***	**/18**
3,4	***H5*** ***applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems***	**/15**
5	***H4*** ***expresses practical problems in mathematical terms based on simple given models***	**/11**
	Total:	**/44**

Question 1 (9 marks)

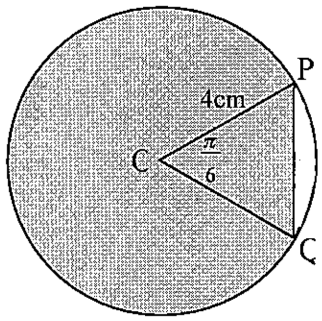
a) Express the following angles in radians **(2 marks)**

i) $150°$ ii) $45°$

b) Find the exact value of the following trigonometric ratios. **(3 marks)**

i) $\cos\frac{\pi}{6}$ ii) $\tan\frac{3\pi}{4}$

c) Find the area of the shaded region. Express your answer in exact form. **(3 marks)**



d) State the period of the graph of $y = 3\tan\frac{x}{2}$. **(1 mark)**

Question 2 (9 marks)

a) Find the gradient of the tangent to the curve $f(x) = \cos(2x - \pi)$ when $x = \frac{\pi}{3}$. **(2 marks)**

b) Clearly sketch the graph of $y = 4\sin(2x) + 1$ in the domain $0 \le x \le 2\pi$. **(4 marks)**

c) The gradient function of a curve is given by the equation $\frac{dy}{dx} = 2\sin x + \cos x$. The curve passes through the point (0, 2). Find the equation of the curve. **(3 marks)**

Question 3 (7 marks)

a) Differentiate the following functions. **(7 marks)**

i) $\cos(x^2)$

ii) $3x\tan(3x-1)$

iii) $(\sin x)^{\frac{1}{2}}$

Question 4 (8 marks)

a) Integrate the following function. **(2 marks)**

$$\int 4\cos\left(\tfrac{x}{4}\right)dx$$

b) Evaluate the following definite integrals. Leave your answer in exact form. **(6 marks)**

i) $\int_0^{\frac{1}{6}} \sec^2(2\pi x)$ ii) $\int_0^{\frac{\pi}{4}} \sqrt{x} + \sin(2x)\, dx$

Question 5 (11 marks)

a) Charlie throws a ball straight up in the air with an initial velocity of 20m/s. Let the acceleration due to gravity be -9.8m/s.

i) Write an equation for the velocity of the ball at a given time, t. **(2 marks)**

ii) How long does the ball stay in the air if it is travelling at -20m/s when it hits the ground? **(2 marks)**

iii) Find an expression for the height above the ground, h, as a function of time. **(2 marks)**

iv) The ball takes the same amount of time to reach its maximum height as it does to come back down to earth. What is the maximum height that the ball reaches? **(1 mark)**

b) An archaeological sample is found in Egypt. Scientists looking to determine the age of the sample decide to examine the levels of Carbon 14 that still remain. The quantity of Carbon 14 that is being lost at any time is proportional to the amount that still remains, Q,

$$\text{i.e. } \frac{dQ}{dt} = -kQ$$

i) The scientists know that the half life of Carbon 14 is 5370 years. Find the decay rate, k, of this substance correct to 3 significant figures. **(2 marks)**

ii) It is discovered that only 15% of the original Carbon 14 remains. How old is the sample to the nearest hundred years? **(2 marks)**

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END OF TEST

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	Total:	**/44**

Solutions

Question 1 (9 marks)

a) Express the following angles in radians **(2 marks)**

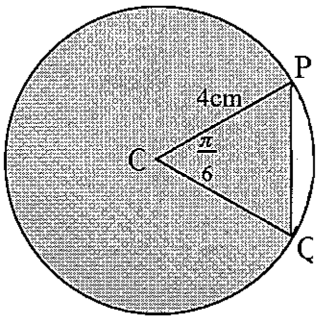
i) $150^\circ \times \frac{\pi}{180^\circ} = \frac{5\pi}{8}$ ii) $45^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{4}$

b) Find the exact value of the following trigonometric ratios. **(3 marks)**

i) $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ ii) $\tan\frac{3\pi}{4} = -\tan\frac{\pi}{4}$

$= -1$

c) Find the area of the shaded region. Express your answer in exact form. **(3 marks)**



$$\textit{Area of the minor segment} = \tfrac{1}{2}r^2(\theta - \sin\theta)$$
$$= \tfrac{1}{2}\times 4^2(\tfrac{\pi}{6} - \sin\tfrac{\pi}{6})$$
$$= 8(\tfrac{\pi}{6} - \sin\tfrac{\pi}{6})$$
$$= 8(\tfrac{\pi}{6} - \tfrac{1}{2})$$
$$= \tfrac{4\pi}{3}$$

$$\textit{Area of the circle} = \pi r^2$$
$$= \pi \times 4^2$$
$$= 16\pi$$

$$\textit{Area of the maj orsegment} = \textit{Area of the circle - Area of the minor segment}$$
$$= 16\pi - \left(\tfrac{4\pi}{3} - 4\right)$$
$$= \tfrac{48\pi}{3} - \tfrac{4\pi}{3} + \tfrac{12}{3}$$
$$= \tfrac{44\pi + 12}{3}$$
$$= \tfrac{4(11\pi + 3)}{3}\,cm^2$$

d) State the period of the graph of $y = 3\tan\frac{x}{2}$. **(1 mark)**

Period $= 2\pi$

Question 2 (9 marks)

a) Find the gradient of the tangent to the curve $f(x)=\cos(2x-\pi)$ when $x=\frac{\pi}{3}$. **(2 marks)**

$f(x)=\cos(2x-\pi)$

$f'(x)=-2\sin(2x-\pi)$

$f'(\frac{\pi}{3})=-2\sin(2\times\frac{\pi}{3}-\pi)$

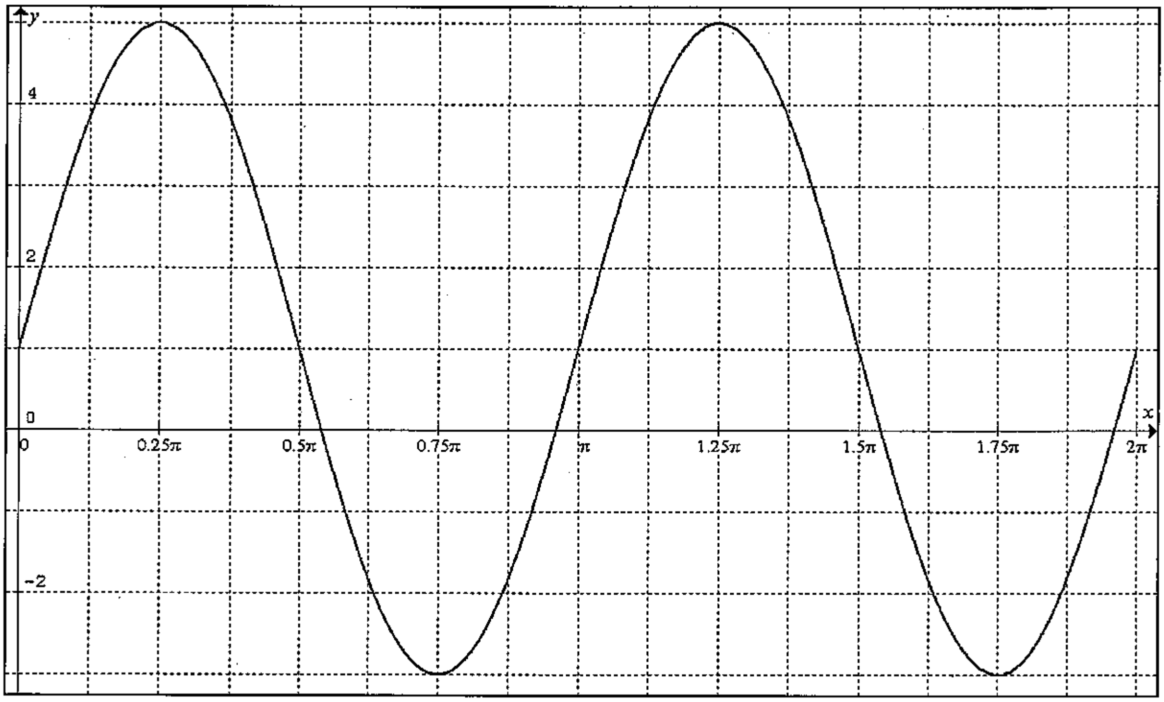
$f'(\frac{\pi}{3})=-2\sin(-\frac{\pi}{3})$

$f'(\frac{\pi}{3})=-2\times(-\sin(\frac{\pi}{3}))$

$f'(\frac{\pi}{3})=-2\times(-\frac{\sqrt{3}}{2})$

$f'(\frac{\pi}{3})=\frac{\sqrt{3}}{2}$

b) Clearly sketch the graph of $y=4\sin(2x)+1$ in the domain $0\le x\le 2\pi$. **(4 marks)**



c) The gradient function of a curve is given by the equation $\frac{dy}{dx}=2\sin x+\cos x$. The curve passes through the point (0, 2). Find the equation of the curve. **(3 marks)**

$\frac{dy}{dx}=2\sin x+\cos x$

$y=-2\cos x+\sin x+c$

When $x=0, y=2$

$2=-2\cos 0+\sin 0+c$

$2=-2+0+c$

$c=4$

$y=-2\cos x+\sin x+4$

Question 3 (7 marks)

a) Differentiate the following functions. **(7 marks)**

i) $\cos(x^2)$

$$\frac{d}{dx}\left(\cos(x^2)\right) = -2x\sin x$$

ii) $3x\tan(3x-1)$

$$\frac{d}{dx}\left(3x\tan(3x-1)\right) = 3\times 3\sec^2(3x-1) + 3\times\tan(3x-1)$$

$$= 9\sec^2(3x-1) + 3\tan(3x-1)$$

iii) $(\sin x)^{\frac{1}{2}}$

$$\frac{d}{dx}\left(\sin(x)^{\frac{1}{2}}\right) = \tfrac{1}{2}\times\cos x\times(\sin x)^{-\frac{1}{2}}$$

$$= \frac{\cos x}{2\sqrt{\sin x}}$$

Question 4 (8 marks)

a) Integrate the following function. **(2 marks)**

$$\int 4\cos(\tfrac{x}{4})dx$$

$$= \frac{4\sin(\frac{x}{4})}{(\frac{1}{4})} + c$$

$$= 16\sin(\tfrac{x}{4}) + c$$

b) Evaluate the following definite integrals. Leave your answer in exact form. **(6 marks)**

i) $\int_0^{\frac{1}{6}} \sec^2(2\pi x) = \left[\frac{1}{2\pi}\tan 2\pi x\right]_0^{\frac{1}{6}}$

$$= \tfrac{1}{2\pi}\tan\tfrac{\pi}{3} - \tfrac{1}{2\pi}\tan 0$$

$$= \tfrac{\sqrt{3}}{2\pi}$$

ii) $\int_0^{\frac{\pi}{4}} \sqrt{x} + \sin(2x)\,dx = \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{\cos(2x)}{2}\right]_0^{\frac{\pi}{4}}$

$$= \left(\frac{2(\frac{\pi}{4})^{\frac{3}{2}}}{3} - \frac{\cos\frac{\pi}{2}}{3}\right) - \left(0 - \frac{\cos 0}{2}\right)$$

$$= \left(\frac{2\sqrt{\pi}^3}{24} - 0\right) - \left(0 - \frac{1}{2}\right)$$

$$= \frac{\sqrt{\pi}^3}{12} + \frac{1}{2}$$

$$= \frac{\sqrt{\pi}^3 + 6}{12}$$

Question 5 (11 marks)

a) Charlie throws a ball straight up in the air with an initial velocity of 20m/s. Let the acceleration due to gravity be -9.8m/s.

i) Write an equation for the velocity of the ball at a given time, *t*. **(2 marks)**

$a = -9.8$

$v = \int a\,dt$

$v = -9.8t + c$

$t = 0, v = 20$

$20 = -9.8(0) + c$

$c = 20$

$v = -9.8t + 20$

ii) How long does the ball stay in the air if it is travelling at -20m/s when it hits the ground? **(2 marks)**

$v = -9.8t + 20$

$-20 = -9.8t + 20$

$-40 = -9.8t$

$t = 4.0816s$

iii) Find an expression for the height above the ground, h, as a function of time.

$h = \int v dt = \frac{-9.8t^2}{2} + 20t + k$

$h = -4.9t^2 + 20t + k$

$t = 0, h = 0$

$0 = -4.9(0)^2 + 20(0) + k$

$k = 0$

$h = -4.9t^2 + 20t$

(2 marks)

iv) The ball takes the same amount of time to reach its maximum height as it does to come back down to earth. What is the maximum height that the ball reaches?

$t = \frac{4.0816}{2} = 2.0408s$

$h = -4.9t^2 + 20t$

$= -4.9(2.0408)^2 + 20(2.0408)$

$= 61.63265306$

$= 61.63s$ **(1 mark)**

b) An archaeological sample is found in Egypt. Scientists looking to determine the age of the sample decide to examine the levels of Carbon 14 that still remain. The quantity of Carbon 14 that is being lost at any time is proportional to the amount that still remains, Q,

i.e. $\frac{dQ}{dt} = -kQ$

i) The scientists know that the half life of Carbon 14 is 5370 years. Find the decay rate, k, of this substance correct to 3 significant figures. **(2 marks)**

$\frac{dQ}{dt} = -kQ$

$Q = Q_0 e^{-kt}$

$\frac{Q_0}{2} = Q_0 e^{-5370k}$

$\frac{1}{2} = e^{-5370k}$

$\log_e(\frac{1}{2}) = -5370k$

$k = \frac{\log_e \frac{1}{2}}{-5370}$

$k = 0.0001290776873$

$k = 1.29 \times 10^{-4}$

ii) It is discovered that only 15% of the original Carbon 14 remains. How old is the sample to the nearest hundred years? **(2 marks)**

$0.15Q_0 = Q_0 e^{-0.000129t}$

$0.15 = e^{-0.000129t}$

$\log_e(0.15) = -0.000129t$

$t = \frac{\log_e 0.15}{-0.000129}$

$t = 14,697.50524$

$\approx 14,700 years$

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END OF TEST

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