SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2004

YEAR 12

HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK # 3

Mathematics

General Instructions

- Working time – 2 hours.
- Reading Time – 5 minutes.
- Write using black or blue pen.
- Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may not be awarded for messy or badly arranged work.
- Hand in your answer booklets in **3** sections.
- Start each **NEW** section in a separate answer booklet.

Total Marks - 90

- Attempt Sections A – C
- All sections are of *equal* value.

Examiner: *P. Parker*

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate.

Total marks – 90
Attempt Questions 1 – 6
All questions are of EQUAL value

Answer each section in a SEPARATE writing booklet. Extra writing booklets are available.

SECTION A (Use a SEPARATE writing booklet)

Question 1 (15 marks) **Marks**

(a) Write down the value of $\cos 1\cdot 5$, correct to 3 decimal places. **1**

(b) Find $\int \left(3x^2+1\right) dx$ **1**

(c) Sketch, showing intercepts with the coordinate axes, **2**

$$y = 2\cos x \text{ for } 0 \le x \le 2\pi$$

(d) If $f(x) = 3\cos 2x$, find the exact values of

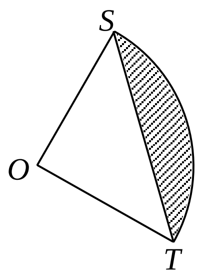
(i) $f\left(\frac{\pi}{6}\right)$ **2**

(ii) $f'\left(\frac{\pi}{6}\right)$ **2**

(iii) $f''\left(\frac{\pi}{6}\right)$ **2**

(iv) $\int_0^{\frac{\pi}{6}} f(x)\, dx$ **2**

(e) The diagram below shows a sector of a circle with centre *O* and radius 4 cm and $\angle TOS = \frac{\pi}{6}$.



(i) Find the length of the arc *ST*. **1**

(ii) Find the area of sector *OST*, leaving your answer in terms of π. **1**

(iii) Hence, or otherwise, calculate the shaded area correct to 2 decimal places. **1**

Question 2 (15 marks) **Marks**

(a) Find the equation of the normal to the curve $y = 4\ln x$ at the point where $x = 1$. **2**

(b) Simplify $e^{2\ln 4}$. **1**

(c) The displacement (x m) of an object (at time t secs) is given by the equation

$$x = t^3 - 15t^2 + 48t - 25$$

(i) Find expressions for the object's velocity and acceleration. **2**

(ii) Find its initial displacement, velocity and acceleration. **3**

(iii) When is the object at rest? **2**

(d) Using the Table of Integrals provided, find the exact value of **2**

$$\int_0^{\frac{\pi}{8}} \sec 2x \tan 2x \, dx$$

(e) Which of the following areas is larger? Justify your answer **3**

Area 1 is bounded by $y = 1\cdot 5 + \sin x$, x axis, $x = 0$ and $x = \pi$.

Area 2 is bounded by $y = 1\cdot 5 + x^2$, x axis, $x = 0$ and $x = \pi$.

SECTION B (Use a SEPARATE writing booklet)

Question 3 (15 marks) **Marks**

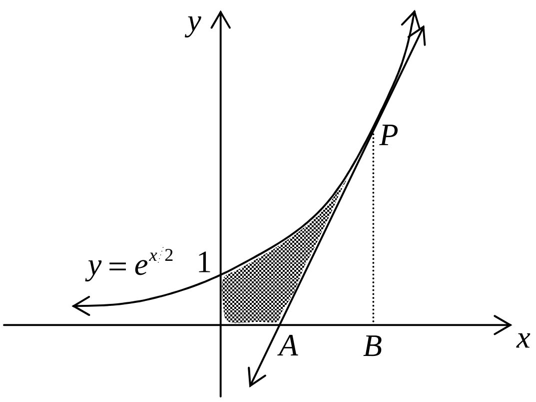
(a) Find $\frac{dy}{dx}$ given

(i) $y = e^{-x} \tan x$ **2**

(ii) $y = \ln\left(x^4\right) + \left(\ln x\right)^4$ **2**

(iii) $y = \frac{x^2}{\ln x}$ **2**

(b) The diagram below shows the graph of $y = e^{x/2}$, together with a tangent line *AP*.
B is the point $\left(3, 0\right)$ and *P* is the point $\left(3, e^{3/2}\right)$



(i) Find the equation of the tangent *AP*. **2**

(ii) Show that *A* has coordinates $(1, 0)$. **1**

(iii) Hence, or otherwise, find the shaded area in terms of e. **2**

(c) Find $\int \left(\frac{e}{x} + \frac{x}{e}\right) dx$ **2**

(d) Differentiate $\ln\left(e^2 x^2\right)$ **2**

Question 4 (15 marks) **Marks**

(a) The height of the tide at Sunset Beach can be modelled by the equation

$$H = 4 + 2\cos\frac{\pi t}{6},\ t > 0$$

where H is the height of the tide in metres after time t hours.

(i) Find the rate of change of height of the tide after 10 hours. **2**

(ii) State whether the tide was an incoming or outgoing tide after 10 hours (give reasons). **1**

(b) "*Whilst the price of houses continues to increase, recent moves by the State Government aimed at reducing the price of houses seem to be taking effect.*"

Given that P is the price of houses in Australian dollars, what does the above statement say about:

(i) $\frac{dP}{dt}$? **1**

(ii) $\frac{d^2P}{dt^2}$? **1**

(c) The rate of decay of a radioactive element is proportional to the mass M of the radioactive element present. The process is described by the differential equation

$$\frac{dM}{dt} = -kM \text{ , where } k > 0$$

(i) Show that $M = M_0e^{-kt}$ satisfies this differential equation where M_0 is a constant. **1**

(ii) Fifty percent of the original mass of the radioactive element Strontium 90 remains after 28 years. Show that $k \cong 0 \cdot 02476$. **2**

(iii) How long will it take for Strontium 90 to decay to 1% of its original mass? **2**

(d) The function $f(x) = x^{-e}e^x$, where $x > 0$

(i) Show that $f'(x) = e^x x^{-(e+1)}\left(x - e\right)$. **3**

(ii) Classify the turning point at $x = e$. Justify your answer. **2**

SECTION C (Use a SEPARATE writing booklet)

Question 5 (15 marks) **Marks**

(a) The velocity of a particle traveling in a straight line is given by

$$v = e^{3t} - 4e^{t} \text{ m/s}$$

where t is in seconds and $t \geq 0$.

(i) Find the initial velocity and acceleration. **2**

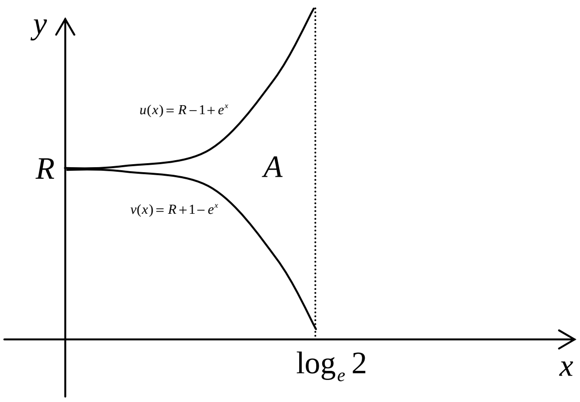
(ii) Find the value of t when the particle is stationary. Give your answer in the form $\ln k$, where k is a constant. **2**

(iii) If initially the particle is at $x = 0$, find an expression for the position x in terms of t. **2**

(iv) Show that at $t = \ln 4$ seconds the particle is 9 metres to the right of the origin. **1**

(v) Find the distance travelled by the particle in the first $\ln 4$ seconds **2**

(b) In the diagram below $u(x) = R - 1 + e^{x}$ and $v(x) = R + 1 - e^{x}$



[Note: Diagram is not drawn to scale, $R > 1$]

(i) Show that A, the area between the curves $u(x)$ and $v(x)$ where $0 \leq x \leq \log_e 2$ is given by **3**

$$A = 2\left(1 - \log_e 2\right)$$

(ii) A solid is formed by rotating the area between the curves $u(x)$ and $v(x)$ where $0 \leq x \leq \log_e 2$, about the x axis. **3**
Show that the volume of the solid generated is $2\pi RA$ where A is the area between the curves.

Question 6 (15 marks) **Marks**

(a) Evaluate **3**

$$\int_{-1}^{1}\left(2x-\frac{1}{x-2}\right)dx$$

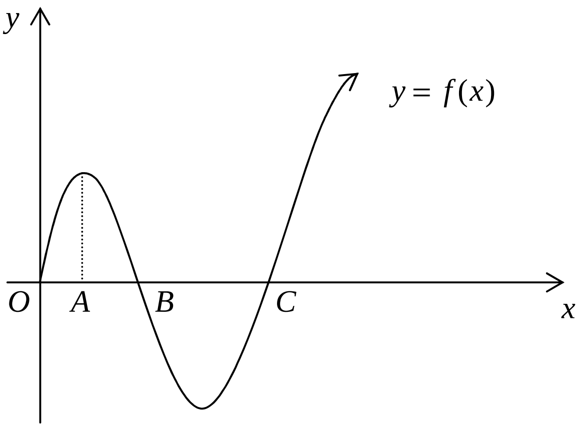
Express your answer in the form $\ln k$, where k is a real constant.

(b)

(i) If $f(x)=e^{3x}\sin 2x$ show that $f'(x)=0$ when $\tan 2x=-\frac{2}{3}$ **2**

The diagram below shows the graph of $f(x)=e^{3x}\sin 2x$ for $x\geq 0$.

This graph intersects the x axis at the points B and C. The point A is a local maximum.



(ii) (α) Show that the x coordinates of B and C are $\frac{\pi}{2}$ and π respectively. **2**

(β) Find the x coordinate of A, correct to 2 decimal places. **2**

(iii) (α) If $g(x)=\frac{1}{13}\left(3e^{3x}\sin 2x-2e^{3x}\cos 2x\right)$ show that **3**

$$g'(x)=e^{3x}\sin 2x$$

(β) Hence or otherwise, find the area between the graph of $f(x)=e^{3x}\sin 2x$ and the x axis from O to B. **3**

End of paper

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x,\ x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax},\ a \neq 0$$

$$\int \cos ax\, dx = \frac{1}{a} \sin ax,\ \ a \neq 0$$

$$\int \sin ax\, dx = -\frac{1}{a} \cos ax,\ \ a \neq 0$$

$$\int \sec^2 ax\, dx = \frac{1}{a} \tan ax,$$

$$\int \sec ax \tan ax\, dx = \frac{1}{a} \sec ax,\ \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a},\ \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a},\ \ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x,\ \ x > 0$

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Mathematics

Sample Solutions

SECTION	MARKER
A	Mr Choy
B	Mr Gainford
C	Mr Fuller

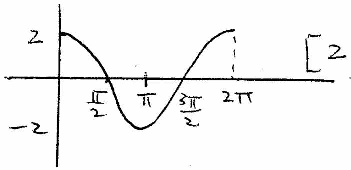
Question (1).

(a) $\cos 1.5^c = 0.071$ [1]

(b) $\int (3x^2+1)\,dx$

$= x^3 + x + c$ [1]

(c)



[2]

$y = 2\cos x, \quad 0 \le x \le 2\pi$

(d) $f(x) = 3\cos 2x,$

(i) $f(\frac{\pi}{6}) = 3\cos\frac{\pi}{3}$

$= \frac{3}{2}$ [2]

8

(ii) $f'(\frac{\pi}{6}) = -6\sin\frac{\pi}{3}$

$= -3\sqrt{3}$ [2]

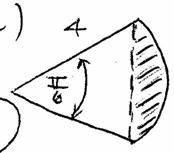
(iii) $f''(\frac{\pi}{6}) = -12\cos\frac{\pi}{3}$

$= -6$ [2]

(iv) $\int_0^{\pi/6} f(x)\,dx = 3\int_0^{\pi/6} \cos 2x\,dx$

$= \left[\frac{3\sin 2x}{2}\right]_0^{\pi/6} = \frac{3\sqrt{3}}{4}$ [2]

(e)



3

(i) $l = 4 \times \frac{\pi}{6}$

$= \frac{2\pi}{3}$ [1]

(ii) $A = \frac{1}{2} \times 16 \times \frac{\pi}{6}$

$= \frac{4\pi}{3}$ [1]

(iii) Area of minor segment

$= \frac{4\pi}{3} - 4 = 4(\frac{\pi}{3} - 1)$

$= 0.19$ (2 d.p.) [1]

Question (2).

(a) $y = 4\ln x$

$\frac{dy}{dx} = \frac{4}{x}, \quad \frac{dy}{dx}\Big|_{x=1} = 4$

$x = 1, \ y = 0 \quad (1, 0) \quad m' = -\frac{1}{4}$

$y - 0 = -\frac{1}{4}(x - 1)$

$4y = -x + 1$ [2]

$\therefore x + 4y - 1 = 0$

$y = -\frac{x}{4} + \frac{1}{4}$

(b) $e^{2\ln 4} = e^{\ln 16} = 16$ [1]

(c) $x = t^3 - 15t^2 + 48t - 25$

(i) $\frac{dx}{dt} = 3t^2 - 30t + 48$ ✓ [2]

$\frac{d^2x}{dt^2} = 6t - 30$ ✓

(ii) When $t = 0$, $x = -25$ ✓

[3] $v = 48$ ✓, $a = -30$ ✓

(iii) $v = 0$ when $\frac{dx}{dt} = 0$

$3(t^2 - 10t + 16) = 0$

$(t - 8)(t - 2) = 0$

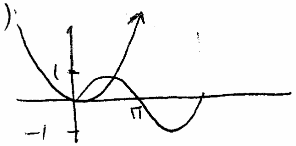
$\Longrightarrow t = 2, 8$ [2]

(d) $\int_0^{\pi/8} \sec 2x \tan 2x\,dx$ [2]

$= \frac{1}{2}\left[\sec 2x\right]_0^{\pi/8}$

$= \frac{1}{2}(\sec\frac{\pi}{4} - \sec 0) = \frac{\sqrt{2} - 1}{2}$

(e)



3

Area is bounded by $y = 1.5 + x^2$

SECTION B

Question 3

(a) (i) $y = e^{-x} \tan x$

$$\frac{dy}{dx} = e^{-x} \sec^2 x + \tan x(-e^{-x})$$

$$= e^{-x}(\sec^2 x - \tan x)$$ [2]

(ii) $y = \ln(x^4) + (\ln x)^4$

$$= 4\ln x + (\ln x)^4$$

$$\frac{dy}{dx} = \frac{4}{x} + 4 \cdot (\ln x)^3 \cdot \frac{1}{x}$$

$$= \frac{4}{x}\left(1 + (\ln x)^3\right)$$ [2]

(iii) $y = \dfrac{x^2}{\ln x}$

$$\frac{dy}{dx} = \frac{\ln x \cdot 2x - x^2 \cdot \frac{1}{x}}{(\ln x)^2}$$

$$= \frac{2x\ln x - x}{(\ln x)^2}$$ [2]

(b) (i) $y = e^{x/2}$

$$y' = \frac{1}{2}e^{x/2}$$

$\therefore m(3) = \frac{1}{2}e^{-3/2}$

$\therefore$ Tangent is

$y - y_1 = m(x - x_1)$

$$y - e^{3/2} = \frac{1}{2}e^{3/2}(x - 3)$$

$$2y - 2e^{3/2} = xe^{3/2} - 3e^{3/2}$$

$$\therefore xe^{3/2} - 2y - e^{3/2} = 0$$

or $y = \frac{1}{2}e^{3/2}(x - 1)$ [2]

(ii) When $x = 1$, $y = 0$

$\therefore$ A is $(1, 0)$. [1]

(iii) Area $= \int_0^3 e^{x/2}\,dx - \Delta ABP$

$$= \left[2e^{x/2}\right]_0^3 - \frac{1}{2} \times 2 \times e^{3/2}$$

$$= 2e^{3/2} - 2 - e^{3/2}$$

$$= e^{3/2} - 2 \text{ units}^2$$

[2]

(c) $\displaystyle\int\left(\frac{e}{x} + \frac{x}{e}\right)dx$

$$= e\ln x + \frac{x^2}{2e} + C$$ [2]

(d) $\dfrac{d}{dx}\ln(e^2x^2)$

$$= \frac{d}{dx}\left[\ln e^2 + 2\ln x\right]$$

$$= \frac{2}{x}$$ [2]

Question 4

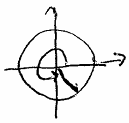
(a) $H = 4 + 2\cos\frac{\pi t}{6}, \ t > 0$

(i) $\frac{dH}{dt} = 0 + \left(-2\sin\frac{\pi t}{6}\right) \times \frac{\pi}{6}$

$= -\frac{\pi}{3}\sin\frac{\pi t}{6}$

When $t = 10$

$\frac{dH}{dt} = -\frac{\pi}{3}\sin\frac{10\pi}{6}$



$= -\frac{\pi}{3}\left(-\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{3}\pi}{6} = \frac{\pi}{2\sqrt{3}}$

∴ Rate of change is $\frac{\pi}{2\sqrt{3}}$ m/hr
[≑ 0.9069] [2]

(ii) Incoming, because rate is positive, indicating tide rising. [1]

(b) (i) $\frac{dP}{dt} > 0$ [1] (ii) $\frac{d^2P}{dt^2} < 0$ [1]

(c) (i) Given $\frac{dM}{dt} = -kM \quad k > 0$

Consider $M = M_0 e^{-kt}$

$\frac{dM}{dt} = M_0(-ke^{-kt})$

$= -k(M_0 e^{-kt})$

$= -kM$ as req'd [1]

(ii) $\frac{1}{2}M_0 = M_0 e^{-k \times 28}$

∴ $\ln\frac{1}{2} = -28k$

∴ $k = \frac{\ln\frac{1}{2}}{-28}$

$≑ 0.024755...$

$≑ 0.02476$ [2]

(iii) $\frac{M_0}{100} = M_0 e^{-kt}$

∴ $\ln\frac{1}{100} = -kt$

$t = \frac{\ln\frac{1}{100}}{-k}$

$= \frac{\ln 100}{k}$ [2]

$≑ 186.03$

$≑ 186$ yrs 10 days

(d) $f(x) = x^{-e}e^{x}, \ x > 0$

(i) $f'(x) = x^{-e}.e^{x} + e^{x}.(-ex^{-e-1})$

$= e^{x}(x^{-e} - ex^{-(e+1)})$

$= e^{x}x^{-(e+1)}(x - e)$ [3]

(ii) $f'(e^-) = (+)(+)(-)$
< 0

$f'(e^+) = (+)(+)(+)$
> 0

∴ Turning Point is a Relative Minimum. [2]

SECTION C

Question 5

(a) $v = e^{3t} - 4e^{t}$

(i) initial velocity $= e^{3(0)} - 4e^{(0)}$

$= 1 - 4$

$= -3 \text{ m/s}$

$= 3 \text{ m/s}$ (1) in the left direction

$a = \frac{dv}{dt}$

$= 3e^{3t} - 4e^{t}$

initial acceleration $= 3e^{3(0)} - 4e^{(0)}$

$= 3 - 4$

$= -1 \text{ m/s}^2$

$= 1 \text{ m/s}^2$ to the left. (1)

(ii) particle is stationary when $v = 0$.

$0 = e^{3t} - 4e^{t}$

$e^{3t} = 4e^{t}$

$e^{2t} = 4$ (take lns of both sides)

$2t = \ln 4$

$t = \frac{1}{2} \ln 4$

$t = \ln 4^{\frac{1}{2}}$

$t = \ln 2$ seconds.

(2)

(iii) $v = \frac{dx}{dt} = e^{3t} - 4e^{t}$

$x = \int (e^{3t} - 4e^{t})\, dt$

$x = \frac{1}{3} e^{3t} - 4e^{t} + c$ when $t = 0$, $x = 0$

$0 = \frac{1}{3} e^{3(0)} - 4e^{(0)} + c$

$0 = \frac{1}{3} - 4 + c$

$c = \frac{11}{3}$

(2)

$\therefore x = \frac{1}{3} e^{3t} - 4e^{t} + \frac{11}{3}$ ($3\frac{2}{3}$)

(iv) when $t = \ln 4$

$x = \frac{1}{3} e^{3(\ln 4)} - 4e^{(\ln 4)} + \frac{11}{3}$

$x = \frac{1}{3} e^{\ln 4^3} - 4e^{\ln 4} + \frac{11}{3}$

$x = \frac{1}{3} \cdot 64 - 4 \cdot 4 + \frac{11}{3}$

$x = 9$. metres to the right of the origin (1)

(v) Find displacement after $t = \ln 2$ seconds

$x = \frac{1}{3} e^{3(\ln 2)} - 4e^{(\ln 2)} + \frac{11}{3}$

$x = \frac{1}{3} e^{\ln 2^3} - 4e^{\ln 2} + \frac{11}{3}$

$x = \frac{1}{3}.8 - 4.2 + \frac{11}{3}$

$x = -\frac{5}{3}$ m

$x = \frac{5}{3}$ m to the left of the origin.

$\therefore$ distance travelled in the first ln 4 seconds

$= \frac{5}{3} + (9 - -\frac{5}{3})$

$= \frac{37}{3}$ m.

$= 12\frac{1}{3}$ m.

(2)

(b) $u(x) = R - 1 + e^x$
$v(x) = R + 1 - e^x$

(i) $A = \int_0^{\ln 2} [u(x) - v(x)]\, dx$

$A = \int_0^{\ln 2} (R - 1 + e^x - (R + 1 - e^x))\, dx$

$A = \int_0^{\ln 2} (-2 + 2e^x)\, dx$

$A = 2\int_0^{\ln 2} (e^x - 1)\, dx$

$A = 2\left[e^x - x\right]_0^{\ln 2}$

$A = 2\left[(e^{\ln 2} - \ln 2) - (e^0 - 0)\right]$

$A = 2\left[2 - \ln 2 - 1\right]$

$A = 2(1 - \log_e 2)$

(3)

(ii) $V = \pi \int_0^{\ln 2} \left((R-1+e^x)^2 - (R+1-e^x)^2 \right) dx$

$= \pi \int_0^{\ln 2} \left((R-1+e^x)(R-1+e^x) - (R+1-e^x)(R+1-e^x) \right) dx$

$= \pi \int_0^{\ln 2} \left(R^2 - R + Re^x - R + 1 - e^x + Re^x - e^x + e^{2x} \right.$
$\left. - (R^2 + R - Re^x + R + 1 - e^x - Re^x - e^x + e^{2x}) \right) dx$

$= \pi \int_0^{\ln 2} (-4R + 4Re^x)\, dx$

$= 4\pi R \int_0^{\ln 2} (e^x - 1)\, dx$

$= 2\pi R \cdot 2 \int_0^{\ln 2} (e^x - 1)\, dx$

from (i) $A = 2\int_0^{\ln 2} (e^x - 1)\, dx$.

$= 2\pi RA$.

(3)

Section C

Question 6

(a) $$\int_{-1}^{1}\left(2x-\frac{1}{x-2}\right)dx$$

> **NB** $\ln x$ is **ONLY** defined for $x > 0$

$$=\left[x^2-\ln\left|x-2\right|\right]_{-1}^{1}$$
$$=\left(1^2-\ln\left|-1\right|\right)-\left(\left(-1\right)^2-\ln\left|-1-2\right|\right)$$
$$=1-0-\left(1-\ln 3\right)$$
$$=\ln 3$$

ALTERNATIVELY using point symmetry about x = 2. Draw a diagram!

$$\int_{-1}^{1}\left(2x-\frac{1}{x-2}\right)dx=\int_{-1}^{1}2x\,dx-\int_{-1}^{1}\frac{dx}{x-2}$$
$$=\left[x^2\right]_{-1}^{1}+\int_{3}^{5}\frac{dx}{x-2}$$
$$=0+\left[\ln(x-2)\right]_{3}^{5}$$
$$=\ln 3-\ln 1$$
$$=\ln 3$$

> **NB**
> $$-\int_{-1}^{1}\frac{dx}{x-2} = +\int_{3}^{5}\frac{dx}{x-2}$$

(b)(i) $f(x) = e^{3x} \sin 2x$

$f'(x) = e^{3x} . 2\cos 2x + 3e^{3x} . \sin 2x$

$f'(x) = 2e^{3x}\cos 2x + 3e^{3x}\sin 2x$

let $f'(x) = 0$

$2e^{3x}\cos 2x + 3e^{3x}\sin 2x = 0$

$e^{3x}(2\cos 2x + 3\sin 2x) = 0$

$e^{3x} \neq 0.$ $\quad 2\cos 2x + 3\sin 2x = 0$

$3\sin 2x = -2\cos 2x$

$\frac{3\sin 2x}{\cos 2x} = -2$

$3\tan 2x = -2$

$\tan 2x = \frac{-2}{3}.$

$\therefore f'(x) = 0$ when $\tan 2x = -\frac{2}{3}.$ (2)

(ii)(α) let $f(x) = 0$

$e^{3x}\sin 2x = 0$

$e^{3x} \neq 0$ $\quad \sin 2x = 0$

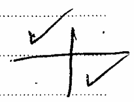
$2x = 0, \pi, 2\pi, \ldots$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots$

$\therefore B = \frac{\pi}{2}$ and $C = \pi$ (2)

(β) from (i) $f'(x) = 0$ when $\tan 2x = -\frac{2}{3}$

acute angle



$\tan \alpha = \frac{2}{3}$

$\alpha = 0.588$

$2x = \pi - \alpha, 2\pi - \alpha, \ldots$

$x = \frac{\pi - \alpha}{2}, \frac{2\pi - \alpha}{2}, \ldots$ (2)

$x = 1.28$ the x - coordinate of A, correct to 2 dec. places.

(iii) (α) $g(x) = \frac{1}{13}\left(3e^{3x}\sin 2x - 2e^{3x}\cos 2x\right)$

$g'(x) = \frac{1}{13}\left(3e^{3x}.2\cos 2x + 9e^{3x}.\sin 2x - (2e^{3x}.(-2\sin 2x) + 6e^{3x}.\cos 2x)\right)$

$= \frac{1}{13}\left(6e^{3x}\cos 2x + 9e^{3x}\sin 2x + 4e^{3x}\sin 2x - 6e^{3x}\cos 2x\right)$

$= \frac{1}{13}\left(13e^{3x}\sin 2x\right)$

$= e^{3x}\sin 2x$ (3)

(β) $A = \int_0^{\frac{\pi}{2}} e^{3x}\sin 2x \, dx$

$= \frac{1}{13}\left[3e^{3x}\sin 2x - 2e^{3x}\cos 2x\right]_0^{\frac{\pi}{2}}$

$= \frac{1}{13}\left[3e^{\frac{3\pi}{2}}\sin 2(\frac{\pi}{2}) - 2e^{\frac{3\pi}{2}}\cos 2(\frac{\pi}{2}) - (3e^{3(0)}\sin 2(0) - 2e^{3(0)}\cos 2(0)\right.$

$= \frac{1}{13}\left[3e^{\frac{3\pi}{2}}\sin\pi - 2e^{\frac{3\pi}{2}}\cos\pi - (3e^{0}\sin 0 - 2e^{0}\cos 0)\right]$

$= \frac{1}{13}\left[3e^{\frac{3\pi}{2}}(0) - 2e^{\frac{3\pi}{2}}(-1) - (3(1)(0) - 2(1)(1))\right]$

$= \frac{1}{13}\left(2e^{\frac{3\pi}{2}} + 2\right)$

$= \frac{2}{13}\left(e^{\frac{3\pi}{2}} + 1\right)$ units^2. (3)