SYDNEY BOYS HIGH
MOORE PARK, SURRY HILLS

JUNE 2006
TASK #3
YEAR 12

Mathematics

General Instructions:

- Reading time—5 minutes.
- Working time—2 hours.
- Write using black or blue pen.
- Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may NOT be awarded for messy or badly arranged work.
- Start each NEW section in a separate answer booklet.
- Hand in your answer booklets in 3 sections:
  Section A(Questions 1 and 2),
  Section B(Questions 3 and 4),
  Section C(Questions 5 and 6),

Total marks—100 Marks

- Attempt questions 1–6.
- All questions are NOT of equal value.

Examiner: Mr R. Boros

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate.

STANDARD INTEGRALS

$$
\begin{aligned}
\int x^n dx &= \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0 \\
\int \frac{1}{x} dx &= \ln x, \quad x > 0 \\
\int e^{ax} dx &= \frac{1}{a}e^{ax}, \quad a \neq 0 \\
\int \cos ax dx &= \frac{1}{a}\sin ax, \quad a \neq 0 \\
\int \sin ax dx &= -\frac{1}{a}\cos ax, \quad a \neq 0 \\
\int \sec^2 ax dx &= \frac{1}{a}\tan ax, \quad a \neq 0 \\
\int \sec ax \tan ax dx &= \frac{1}{a}\sec ax, \quad a \neq 0 \\
\int \frac{1}{a^2+x^2} dx &= \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0 \\
\int \frac{1}{\sqrt{a^2-x^2}} dx &= \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a \\
\int \frac{1}{\sqrt{x^2-a^2}} dx &= \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0 \\
\int \frac{1}{\sqrt{x^2+a^2}} dx &= \ln\left(x+\sqrt{x^2+a^2}\right)
\end{aligned}
$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Section A

Marks

Question 1 (20 marks)

(a) Differentiate with respect to x:

(i) $x^4 - \sqrt{x} + 1$ $\boxed{1}$

(ii) $x \tan x$ $\boxed{1}$

(iii) $\cos^2 3x$ $\boxed{1}$

(iv) xe^{2x} $\boxed{1}$

(v) $(x+2)\ln(x+2)$ $\boxed{2}$

(b) Find a primitive of:

(i) $2x^2 + 4x - 1$ $\boxed{1}$

(ii) $\dfrac{2}{2x+3}$ $\boxed{1}$

(iii) $3\sin 2x$ $\boxed{1}$

(iv) $\sec^2(3x+1)$ $\boxed{1}$

(c) If $\frac{dy}{dx} = 6x - 1$ and the function passes through $\left(1,\ 2\frac{1}{2}\right)$, find y as a function of x. $\boxed{2}$

(d) Consider the curve $y = x\left(x^2 - 12x + 45\right)$.

(i) Find the coördinates of all the stationary points and determine their nature. $\boxed{3}$

(ii) Find the coördinates of the points of inflexion. $\boxed{2}$

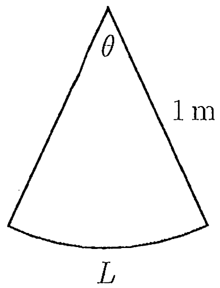
(iii) Sketch the curve, showing all important features. $\boxed{1}$

(iv) Find the equation (written in general form) of the tangent to the curve at the origin. $\boxed{2}$

Marks

Question 2 (18 marks)

(a)

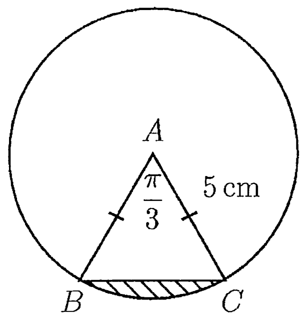


A pendulum of length 1 m takes one second to swing from one side to the other. It travels at 180 cm/min.

(i) Find the distance L through which it swings. $\boxed{2}$

(ii) Calculate the angle θ (in degrees and minutes) through which it swings each second. $\boxed{2}$

(b)



Consider the above diagram, not drawn to scale. Give answers correct to the nearest cm^2.

(i) Find the area of the sector ABC. $\boxed{1}$

(ii) Find the area of the shaded segment. $\boxed{2}$

(c) State the largest possible domain of the function $y = \ln\left(1 - 4x^2\right)$. $\boxed{1}$

(d) (i) Sketch the parabola $y = 6x - 3x^2$, showing the x-intercepts and the vertex. $\boxed{2}$

(ii) Find the area enclosed by the parabola in (d)(i) and the x-axis from $x = -1$ to $x = 1$. $\boxed{3}$

(e) Find, correct to 2 decimal places,

(i) $\displaystyle\int_{-1}^{0} e^{5-2x}\,dx$, $\boxed{2}$

(ii) $\displaystyle\int_{0}^{2} \tan^2\left(\frac{x}{2}\right)dx$. $\boxed{2}$

(f) Solve, correct to 3 decimal places, the equation $e^{2x} = \ln 1994$. $\boxed{1}$

Section B

(Use a separate writing booklet.)

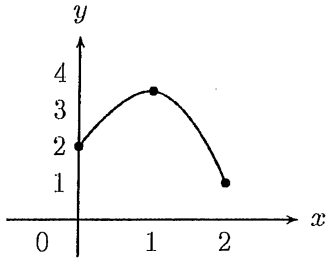
Marks

Question 3 (15 marks)

(a) (i) Sketch the curves $y = \sin x$ and $y = 1 + \sin x$ on the same axes for $0 \leq x \leq \pi$. **2**

(ii) The region between the curves in 3(a)(i) is rotated about the x-axis, $0 \leq x \leq \pi$. Find the volume of the solid of revolution in exact form. **3**

(b) A curve $y = f(x)$ is known to pass through the points $(0, 2)$, $(1, 3\frac{1}{2})$, $(2, 1)$. **2**



The region bounded by the curve and the x-axis from $x = 0$ to $x = 2$ is rotated about the x-axis. Use Simpson's Rule to approximate the volume of the solid of revolution.

(c) Find the equation (written in general form) of the tangent to the curve $y = e^{x^2}$ at the point $(1, e)$ on it. **2**

(d) A pool is being drained and the number of litres of water, L, in the pool at time t minutes is given by the equation

$$L = 120(40 - t)^2.$$

(i) At what rate is the water draining out of the pool when $t = 6$ minutes? **2**

(ii) How long will it take for the pool to be completely empty? **1**

(e) Show that the curve $y = x^3 + 3x + 1$ is increasing for all values of x. **2**

(f) Sketch the graph of the function which has all these features: **1**

$$\begin{array}{ll} f(2) = 0, & f'(2) = 0, \\ f'(x) < 0 & \text{for all } x < 2, \\ f'(x) > 0 & \text{for all } x > 2. \end{array}$$

Marks

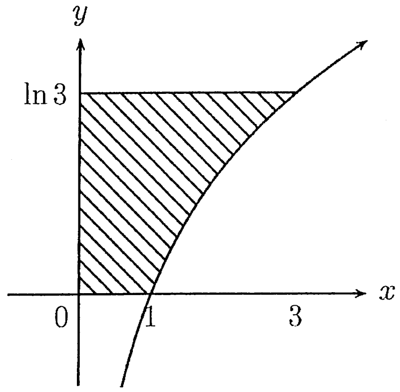
Question 4 (17 marks)

(a) A spherical bubble is being inflated so that the rate of change, R, of the volume, V, in cm^3/s, at any instant of time, t, in seconds, is given by $\boxed{4}$

$$R = \frac{4t}{t^2+1}, \qquad t \geq 0.$$

Initially the volume of the bubble was $40\,\text{cm}^3$. Given that the volume of a sphere with radius r is $\frac{4}{3}\pi r^3$, find the radius of the bubble after 100 seconds, correct to 2 significant figures.

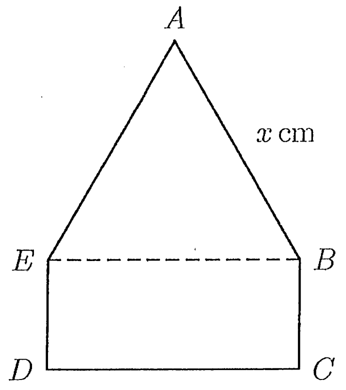
(b)



The diagram shows the area bounded by the graph $y = \ln x$, the coördinate axes, and the line $y = \ln 3$.

(i) Find the size of the shaded area. $\boxed{2}$

(ii) Hence or otherwise find the exact value of $\displaystyle\int_1^3 \ln x \, dx$. $\boxed{2}$

(c) 

$ABCDE$ is a pentagon of fixed perimeter P cm. In the figure, triangle ABE is equilateral and $BCDE$ is a rectangle. The length of AB is x cm.

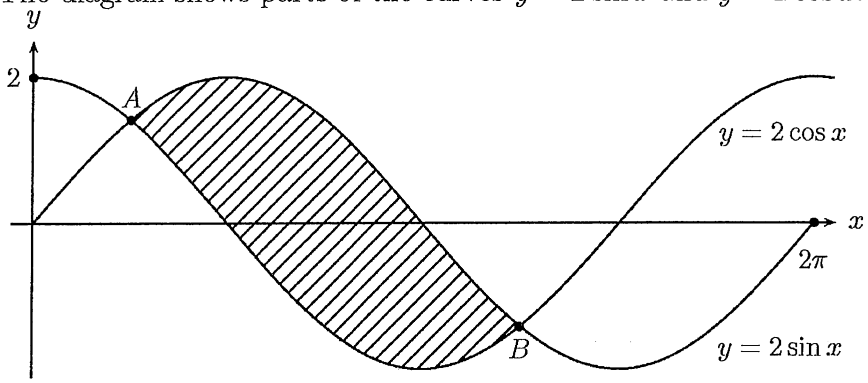
(i) Show that the length of BC is $\dfrac{P-3x}{2}$ cm. $\boxed{1}$

(ii) Show that the area of the pentagon is given by $\boxed{2}$

$$A = \frac{1}{4}\left[2Px - (6-\sqrt{3})x^2\right] \text{ cm}^2.$$

(iii) Find the value of $\dfrac{P}{x}$ for which the area of the pentagon is a maximum. $\boxed{2}$

(d) The diagram shows parts of the curves $y = 2\sin x$ and $y = 2\cos x$.



(i) Find the points A and B. $\boxed{2}$

(ii) Hence calculate the shaded area (leaving your answer in exact form). $\boxed{2}$

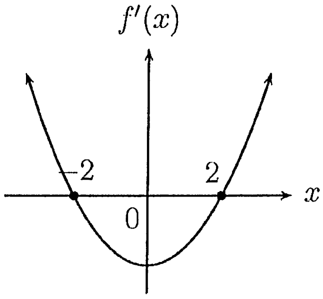
Section C

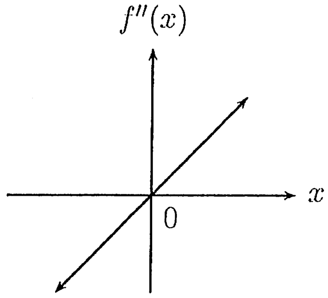
(Use a separate writing booklet.)

Marks

Question 5 (15 marks)

(a) The graphs below show the first and second derivatives of a curve $y = f(x)$.





(i) For which values of x is the function **2**

(α) increasing,

(β) concave down?

(ii) Give the x-coördinate for the maximum turning point. **2**

(b) (i) Sketch the graph of $y = 3\cos 2x$ in the range $0 \leq x \leq 2\pi$. **2**

(ii) Using your graph of 5(b)(i), find how many solutions there are to the equation $\cos 2x = \frac{1}{3}$ in the range $0 \leq x \leq 2\pi$. **1**

(c) The number of vehicles N, at time t, in the City of Sydney has been graphed over a period of 10 years. It was found that over the whole period

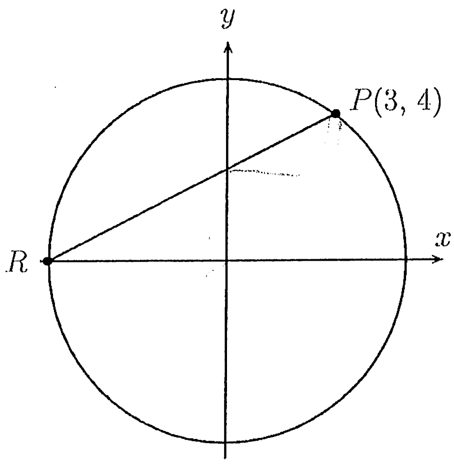
$$\frac{dN}{dt} > O \text{ while } \frac{d^2N}{dt^2} < O.$$

(i) What does this tell you about the number of vehicles in Sydney over this time? **1**

(ii) What can be said about the rate of change in the number of cars in Sydney over this period of time? **1**

(iii) Make a neat sketch of N against t. **1**

(d)

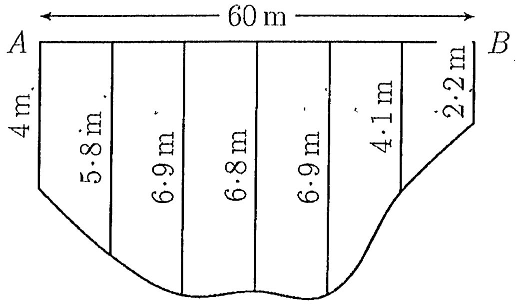


$P(3,\ 4)$ is a point on the circle $x^2 + y^2 = 25$. Find the length of the minor arc PR correct to 3 significant figures. 3

(e) Given the function $f(x) = xe^{-x^2}$, show that this is an odd function. 2

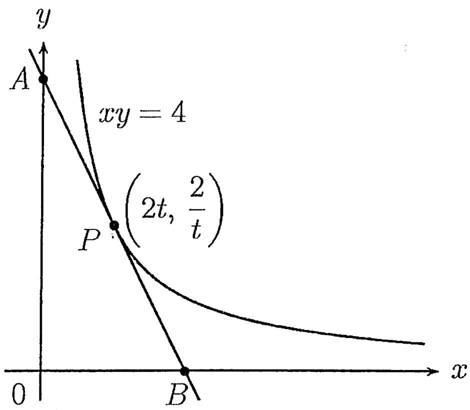
Marks

Question 6 (15 marks)

(a) 

$\boxed{2}$

The diagram represents the cross section of a stream which is 60 m wide. The depth of the stream was measured at 10 m intervals from bank A to bank B. Use the trapezoidal rule to approximate the area of the cross-section of the river.

(b) 

In the diagram, $P\left(2t, \frac{2}{t}\right)$ is a variable point on the branch of the hyperbola $xy = 4$ in the first quadrant. The tangent at P meets the y-axis at A and the x-axis at B.

(i) Show that the tangent at P is $t^2 y = 4t - x$. $\boxed{3}$

(ii) Let the square of the length of AB, *i.e.* $(AB)^2$, be denoted by W. Find the minimum value of W. $\boxed{4}$

(c) A patient sick in hospital is being treated with a new medicine to fight bacteria present in the bloodstream. The number of bacteria present, N, t hours after the medicine is administered is given by the equation $N = Be^{-0{\cdot}04t}$, where B is a constant.

(i) Show that the rate at which the number of bacteria decreases is proportional to the number of bacteria present. $\boxed{1}$

(ii) After h hours the number of bacteria has halved. Find the value of h (to one decimal place). $\boxed{2}$

(iii) After 36 hours the number of bacteria is estimated at 5×10^4. Find the value of B (to the nearest whole number). $\boxed{1}$

(iv) The patient is discharged when the number of bacteria reduces to 10 000. When can the patient leave hospital? (Use the value of B found in part (iii) and answer to the nearest whole hour) $\boxed{2}$

End of Paper

SECTION A, 2006 HSC TASK 5, MATHEMATICS

QUESTION ONE

a) i) $\frac{d}{dx}(x^4-\sqrt{x}+1)$

$= 4x^3 - \frac{1}{2\sqrt{x}}$.

ii) $\frac{d}{dx}(x\tan x)$

$= \tan x + x\sec^2 x$.

iii) $\frac{d}{dx}(\cos^2 3x)$

$= -6\cos 3x \sin 3x$.

iv) $\frac{d}{dx}(x e^{2x})$

$= e^{2x} + 2xe^{2x}$.

$= e^{2x}(1+2x)$.

v) $\frac{d}{dx}((x+2)\ln(x+2))$

$= \ln(x+2) + x+2 \times \frac{1}{x+2}$.

$= \ln(x+2) + 1$.

b) i) $\int 2x^2+4x-1 \, .dx$.

$= \frac{2}{3}x^3 + 2x^2 - x + C$.

ii) $\int \frac{2}{(2x+3)} \, .dx = \ln(2x+3)+C$

iii) $\int 3\sin 2x \, .dx = 3\int \sin 2x \, .dx$

$= -\frac{3}{2}\cos 2x + C$

iv) $\int \sec^2(3x+1) \, .dx$.

$= \frac{1}{3}\tan(3x+1) + C$.

c) $\frac{dy}{dx} = 6x-1$ thro' $(1, 2\frac{1}{2})$

$y = \int 6x-1 \, .dx$.

$= 3x^2 - x + C$.

When $y = 2\frac{1}{2}$, $x=1$.

$\therefore$ $2\frac{1}{2} = 3(1) - 1 + C$

$\therefore$ $C = \frac{1}{2}$.

QUESTION ONE (CONT).

d) Stationary points when $\frac{dy}{dx}=0$

$y = x^3 - 12x^2 + 45x.$

$\frac{dy}{dx} = 3x^2 - 24x + 45.$

$\therefore \quad 3x^2 - 24x + 45 = 0.$

$3(x-5)(x-3) = 0.$

$x = 5$ or $x = 3.$

$y = 50$ $\quad y = 54.$

Turning points occur at.

A(5,50) & B(3,54)

NATURE

A $\frac{d^2y}{dx^2} = 6x - 24$

$= 6(5) - 24.$

> 0

$\therefore$ Minimum at A(5,50).

B// $\frac{d^2y}{dx^2} = 6(3) - 24$

< 0

$\therefore$ maximum at B(3,54)

i) P.O.I when $\frac{d^2y}{dx^2} = 0.$

$\therefore \quad 6x - 24 = 0$

$x = 4$

$y = 52.$

$\therefore$ possible P.O.I @ (4,52)

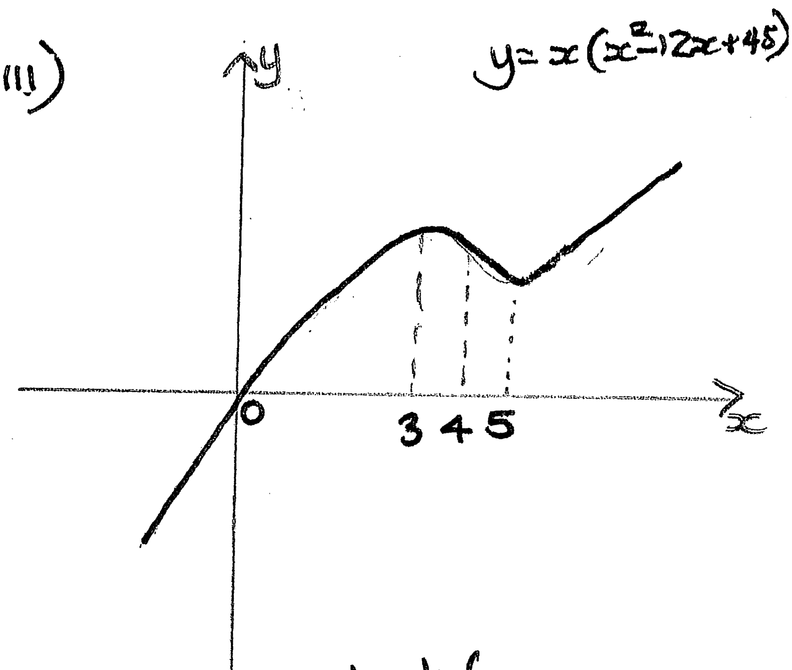
CONCAVITY.

$x > 4 \quad \frac{d^2y}{dx^2} > 0$ concave up.

$x < 4 \quad \frac{d^2y}{dx^2} < 0$ concave down.

Since there is a change in concavity there is a P.O.I at (4,52).

iii)



iv) at origin gradient of tangent given by $\frac{dy}{dx}$.

$\frac{dy}{dx} = 3x^2 - 24x + 45.$

when $x = 0.$

$\frac{dy}{dx} = 45$

$\therefore$ pt gradient formula gives

$y - 0 = 45(x - 0).$

$45x - y = 0$

is eqtn of tangent at (0,0)

SECTION A, 2006, HSC TASK #3 MATHEMATICS.

QUESTION TWO.

a) i) Travels @ 180 cm/min.

$\therefore$ travels 3 cm/sec.

$\therefore$ length of swing = 3 cm.

ii) $l = r\theta$ $\quad l = 3\text{cm} \quad r = 100\text{cm}$.

$3\text{cm} = 100\,\theta$

$\theta = \frac{3}{100}^{c}$

$\underline{0.03^c = 1^\circ 43' 8''}$

b) i) Area of Sector ABC $= \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 25 \times \frac{\pi}{3}$.

$\underline{= 13.09\text{ cm}^2}$.

ii) AREA OF SEGMENT = Area of sector − Area $\triangle ABC$.

Area of $\triangle ABC = \frac{1}{2}bc\sin A$.

$= \frac{1}{2}(25)\sin\frac{\pi}{3}$.

$= 10.83\text{ cm}^2$.

AREA OF SEGMENT $= 13.09 - 10.83$.

$\underline{= 2.26\text{ cm}^2}$.

QUESTION TWO (CONT)

c) $y = \ln(1-4x^2)$

$1-4x^2 > 0$

$x^2 < \frac{1}{4}.$

$-\frac{1}{2} < x < \frac{1}{2}.$

d) i) $y = 6x - 3x^2$

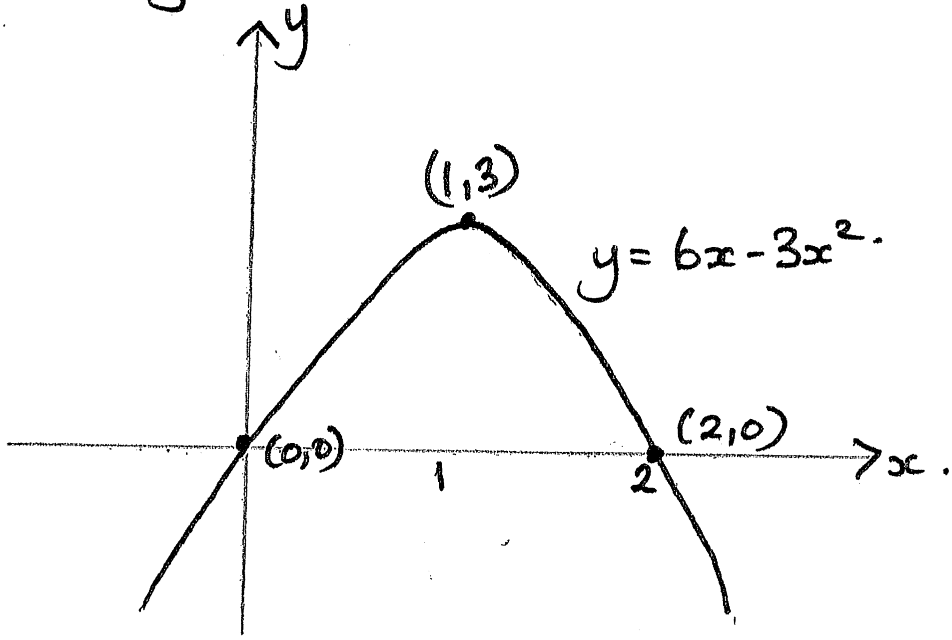
$\therefore\ y = 3x(2-x).$

x intercepts at $x=0$ & $x=2$.
$y=0$ $y=0$

x. Vertex $= \frac{-b}{2a}.$

$x = 1$

$y = 3.$



QUESTION TWO (CONT)

d) ii) $\text{Area} = \left|\int_{-1}^{0} y.\,dx\right| + \int_{0}^{1} y.\,dx.$

$= \left|\left[3x^2 - x^3\right]_{-1}^{0}\right| + \left[3x^2 - x^3\right]_{0}^{1}$

$= |-2| + 4$

$= 6 \text{ units}^2.$

e) i) $\int_{-1}^{0} e^{5-2x}.\,dx = \left[\frac{-1}{2}e^{5-2x}\right]_{-1}^{0}$

$= \frac{-1}{2}e^{5} + \frac{1}{2}e^{7}$

$= 474 \cdot 11 \text{ units}^2.$ (2 dp)

ii) $\int_{0}^{2} \tan^2\left(\frac{x}{2}\right).\,dx = \int_{0}^{2} \sec^2\left(\frac{x}{2}\right) - 1.\,dx.$

$= \left[2\tan\left(\frac{x}{2}\right) - x\right]_{0}^{2}.$

$= (2\tan 1 - 2) - (2\tan 0 - 0)$

$= 1 \cdot 1148$

$= 1 \cdot 11$ (2 dp)

f) $e^{2x} = \ln 1994.$

$\ln e^{2x} = \ln(\ln 1994).$

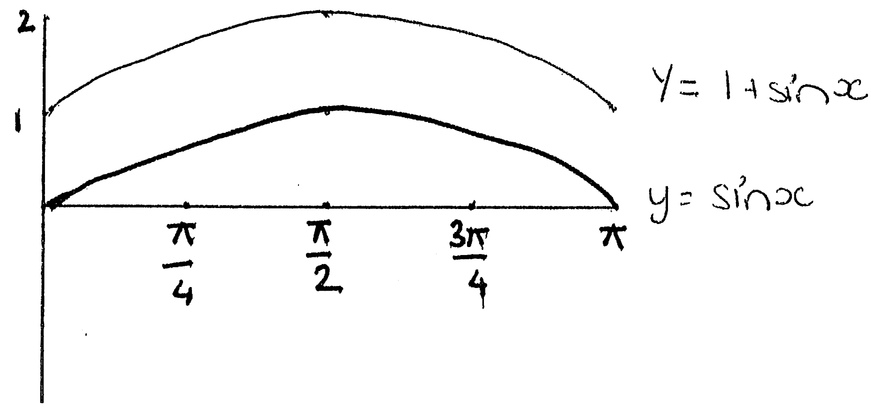
$2x = \ln(\ln 1994)$

$x = 1 \cdot 014$ (3dp).

Section 'B':

Question 3:

a) i)



1

1

/2

b) ii) $V = \pi \int_0^{\pi} (1+\sin x)^2 dx - \pi \int_0^{\pi} (\sin x)^2 \, dx$ 1/2

$= \pi \int_0^{\pi} \left[(1+\sin x)^2 - (\sin x)^2 \right] dx$

$= \pi \int_0^{\pi} \left[1 + 2\sin x + \cancel{\sin^2 x} - \cancel{\sin^2 x} \right] dx$

$= \pi \int_0^{\pi} \left[1 + 2\sin x \right] dx$

$= \pi \left[x + - 2\cos x \right]_0^{\pi}$ 1

$= \pi \left[(\pi - 2\cos\pi) - (0 - 2\cos 0) \right]$ 1/2

$= \pi \left[\pi + 2 - 0 + 2 \right]$

$= \pi \left[\pi + 4 \right]$ 1

/3

1 mark for

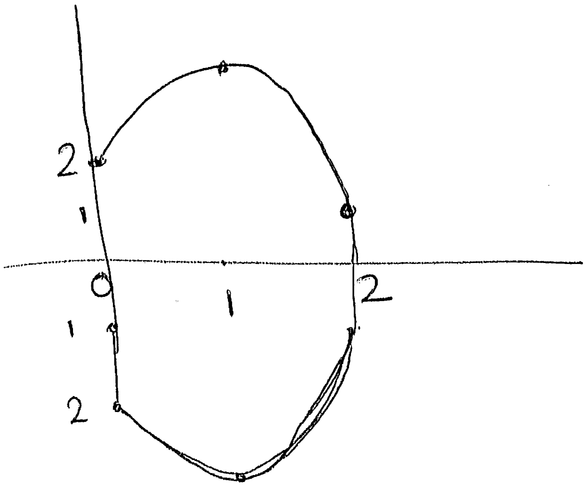
$y^2 = \cancel{\sin}(1 + \sin x - \sin x)^2$

$= 1^2$

$= 1.$

Question 3

a)



x	0	1	2
$f(x)$	2	$3\frac{1}{2}$	1
$f(x)^2$	4	$12\frac{1}{4}$	1

$$V = \pi \int_0^2 f(x)^2 \, dx.$$

Simpsons Rule $= \frac{1}{3}\left[4 + 1 + 4(12\tfrac{1}{4})\right]$

$= 18.$ (1)

1 for $\pi \times$ SR for $f(x)$

$V = \pi \times 18$

$= 18\pi u^3$ (1)

1 for SR $f(x)^2$

/2

1/2 only for simpsons rule $f(x)$

$\frac{2}{6}\left[2 + 4(3.5) + 1\right]$

Question 3.

i) $y = e^{x^2}$ point $(1, e)$

$\frac{dy}{dx} = e^{x^2} \times 2x$

$= 2xe^{x^2}$ 1

when $x = 1$ $\frac{dy}{dx} = 2 \cdot 1 \cdot e^{1^2}$

$= 2e$ 1/2

$\therefore$ T: $\frac{y-e}{x-1} = 2e$

-1/2 for not gen form.

$y - e = 2e(x-1)$

$y - e = 2ex - 2e$

$2ex - y - e = 0$ 1/2

/2

d) $L = 120(40-t)^2$

i) $\frac{dL}{dt} = 240(40-t) \times -1$

$= -240(40-t)$ 1

1/2 for dL/dt if whole answer wrong.

when $t = 6$

$\frac{dL}{dt} = -240(40-6)$

$= -240(34)$

$= -8160 \ Lm^{-1}$

$\therefore$ draining out at $8160 \ Lm^{-1}$. 1

/2

d) ii) $L = 0$ $\quad t = ?$

$$0 = 120(40 - t)^2$$

$$(40 - t)^2 = 0$$

$$40 - t = 0$$

$$t = 40 \quad 1$$

$\therefore$ The pool will be empty after 40 minutes.

T.

e). $y = x^3 + 3x + 1.$

$$y' = 3x^2 + 3 \quad 1/2$$

for increasing curve $y' > 0.$

$$3x^2 + 3 > 0 \quad 1/2$$

$\cancel{3x^2 > -3}$

$\cancel{x^2 > -1}$

$\cancel{x > \pm\sqrt{-1}}$

x^2 will alway be positive.

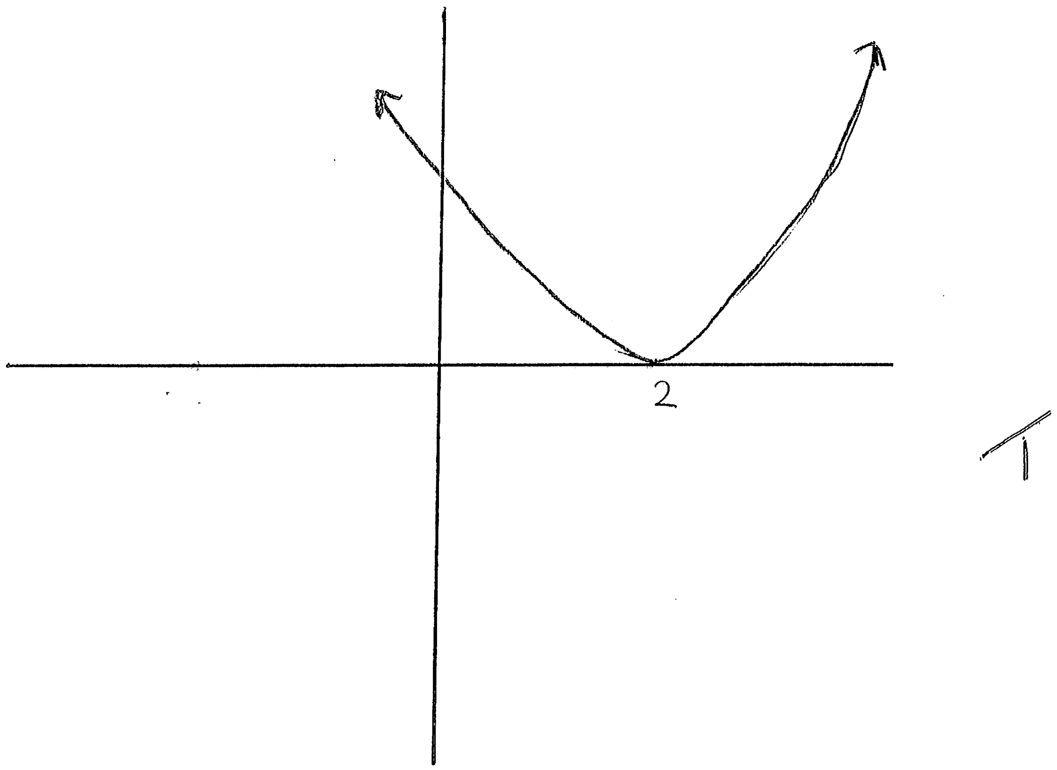
$\therefore$ $3x^2 + 3$ will always be positive 1

$\therefore$ $f'(x) > 0$ for all x

$\therefore$ $f(x)$ is increasing for all x.

/2

f)



Question 4

i) $R = \frac{4t}{t^2+1} = \frac{dv}{dt}$

$V = \int \frac{4t}{t^2+1}\, dt$

$= 2\int \frac{2t}{t^2+1}\, dt$

$= 2\ln(t^2+1) + c$. ①

when $t=0$, $V=40$

$40 = 2\ln(0^2+1) + c$

$40 = 0 + c$

$c = 40$

$\therefore V = 2\ln(t^2+1) + 40$ ①

when $t=100$

$V = 2\ln(100^2+1) + 40$

$= 2\ln(10001) + 40$

$= 58.42088073$ ①

$V = \frac{4}{3}\pi r^3$

$58.42088073 = \frac{4}{3}\pi r^3$

$r^3 = 58.42088073 \times \frac{3}{4\pi}$

$r = 2.407094627$

$= 2.4\text{cm}$ (2 sig fig) ①

/4

Question 4

b) i) $y = \ln x$

$\therefore x = e^y$ 1

$A = \int_0^{\ln 3} e^y$

$= \left[e^y\right]_0^{\ln 3}$ ½

$= 3 - 1$

$= 2$ ½

/2

ii) $\int_1^3 \ln x \, dx$

A = area of rectangle − shaded region. ½

$= 3 \times \ln 3$ ½ $- 2$

$= 3\ln 3 - 2$. 1

/2

Question 4

i) i) $P = AE + AB + BC + CD + DE.$

$= x + x + BC + x + DE$

$= 3x + BC + DE$

$P - 3x = BC + DE$

AS BCDE is a rectangle $BC = DE$ ⇒ opposite sides of rectangle BCDE.

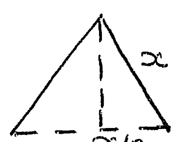
$\therefore P - 3x = BC + BC$

$P - 3x = 2BC$

$\therefore BC = \dfrac{P - 3x}{2}.$ 1

ii) $A =$ area of triangle + area of rectangle.

area of triangle $= \frac{1}{2} b h$



$h^2 = x^2 - \left(\frac{x}{2}\right)^2$

$= x^2 - \dfrac{x^2}{4}$

$h^2 = \dfrac{3x^2}{4}$

$h = \dfrac{\sqrt{3}x}{2}$ ½

$= \dfrac{1}{2} \times x \times \dfrac{\sqrt{3}x}{2}$

$= \dfrac{\sqrt{3}x^2}{4}$ ½

area of rectangle $= \dfrac{P - 3x}{2} \times x$

$= \dfrac{Px - 3x^2}{2}$ ½

$A = \dfrac{\sqrt{3}x^2}{4} + \dfrac{Px - 3x^2}{2}$

$$A = \frac{\sqrt{3}x^2 + 2Px - 6x^2}{4}$$

$$A = \frac{1}{4}\left[\sqrt{3}x^2 + 2Px - 6x^2\right]$$

$$A = \frac{1}{4}\left[2Px - (6-\sqrt{3})x^2\right] cm^2 \quad (1/2)$$

/2

ii) $A = \frac{Px}{2} - \frac{6x^2}{4} + \frac{\sqrt{3}x^2}{4}$

$$\frac{dA}{dp} = \frac{P}{2} - \frac{12x}{4} + \frac{2\sqrt{3}x}{4}$$

let $dA/dp = 0$ to find stationary point.

$$\therefore \; 0 = \frac{P}{2} - \frac{12x}{4} + \frac{2\sqrt{3}x}{4} \quad (1/2)$$

$$0 = 2P - 12x + 2\sqrt{3}x$$

$12x - 2\sqrt{3}x = 2P$

$6x - \sqrt{3}x = P$

$(6-\sqrt{3})x = P$

$6-\sqrt{3} = P/x$. (1)

$$\frac{d^2A}{dx^2} = -3 + \frac{\sqrt{3}}{2} \quad (1/2)$$

$= -2.133974596$

$\therefore$ $6-\sqrt{3}$ is a max. turning point.

$\therefore$ when $P/x = 6-\sqrt{3}$ the area of the pentagon is a maximum.

/2

Question 4

1) i) $y = 2\sin x, \quad y = 2\cos x$

$\therefore \quad 2\sin x = 2\cos x$

$\therefore \quad \sin x = \cos x$ (1)

$\sin x = \cos x$ between $0 \leq x \leq 2\pi$ at

$\pi/4$ & $5\pi/4$ (1)

/2

ii) $A = \int_{\pi/4}^{5\pi/4} [2\sin x - 2\cos x] \, dx$

$= 2\int_{\pi/4}^{5\pi/4} [\sin x - \cos x] \, dx$

$= 2\left[-\cos x - \sin x\right]_{\pi/4}^{5\pi/4}$ (1)

$= 2\left[\left(-\cos 5\pi/4 - \sin 5\pi/4\right) - \left(-\cos \pi/4 - \sin \pi/4\right)\right]$

$= 2\left[\left(1/\sqrt{2} - - 1/\sqrt{2}\right) - \left(-1/\sqrt{2} - \frac{1}{\sqrt{2}}\right)\right]$

$= 2\left[2/\sqrt{2} + \frac{2}{\sqrt{2}}\right]$

$= 4\sqrt{2} \, u^2$ (1)

/2

Section C

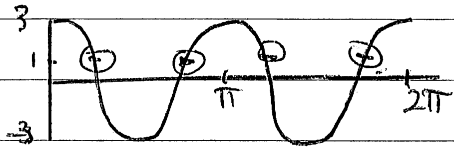
Question 5.

a i α) $x<-2$, $x>2$.

β) $x<0$

ii) $x=-2$.

b i)



Amplitude $=3$

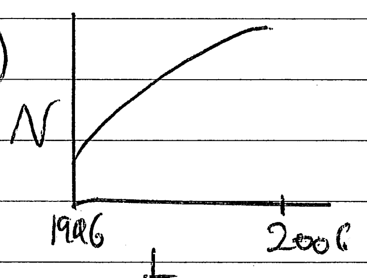
Period $=\pi$

ii) 4 solutions

C i) The number of vehicles is increasing.

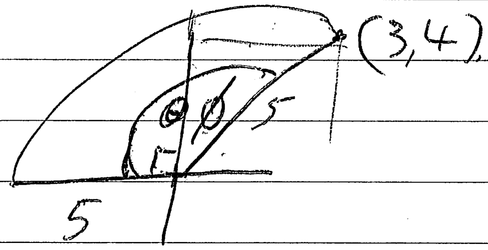
ii) The rate of increase is decreasing.

iii)



d) $\tan\phi=\left(\frac{3}{4}\right)$

$\cancel{\theta}=\tan^{-1}\left(\frac{3}{4}\right)$.



$\theta=\phi+\frac{\pi}{2}$.

$l=r\theta$

$=5\times\left(\tan^{-1}\left(\frac{3}{4}\right)+\frac{\pi}{2}\right)$

$=11.1$

e) $f(-a) = -ae^{-(-a)^2}$

$= -ae^{-a^2}$

$= -f(a).$

QUESTION 6.

a) $A \doteq \frac{10}{2}(4+2.2+2(5.8+6.9+6.8+9.6+4.1))$

$= 336 m^2.$

bi) $y = 4x^{-1}$

$\frac{dy}{dx} = -4x^{-2}$

At $x = 2t$

$m = -\frac{1}{t^2}$

$\therefore y - \frac{2}{t} = -\frac{1}{t^2}(x-2t)$

$t^2 y = 4t - x.$

ii) A when $x=0$.

$y = \frac{4}{t}$

B when $y=0$

$x = 4t.$

By Pythagoras.

$$W = (4t)^2 + \left(\frac{4}{t}\right)^2$$

$$\therefore W = 16t^2 + \frac{16}{t^2}$$

$$\frac{dW}{dt} = 32t - 32t^{-3}.$$

$$\frac{d^2W}{dt^2} = 32 + 96t^{-4}.$$

Stat pts when $\frac{dW}{dt} = 0$.

$$0 = 32t - 32t^{-3}$$

$$t^4 - 1 = 0.$$

$$t = 1, -1.$$

$$\frac{d^2W}{dt^2} = 32 + 96(1)^{-4}$$

$$= 128 > 0 \quad \text{minima}$$

$$\frac{d^2W}{dt^2} = 32 + 96(-1)^{-4}$$

$$= 128 > 0 \quad \text{minimum}$$

$\therefore$ minimum at $t = 1, -1$.

i.e. $W = 32$.

c i) $\frac{dN}{dt} = -0.04 Be^{-0.04t}$

$= -0.04N.$

ii) $\frac{1}{2}B = Be^{-0.04h.}$

$h = \frac{\ln \frac{1}{2}}{-0.04}$

$= 17.3$ hours.

iii) $5 \times 10^4 = Be^{-0.04(36)}$

$B = 211035$ bacteria

iv) $10\,000 = 211035\ e^{-0.04t}$

$t = \frac{\ln\left(\frac{10000}{211035}\right)}{-0.04}$

$= 76$ hours.