



SYDNEY BOYS HIGH
MOORE PARK, SURRY HILLS

JULY 2009
TASK #3
YEAR 12

Mathematics

General Instructions:

- Reading time—5 minutes.
- Working time—2 hours.
- Write using black or blue pen.
- Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may NOT be awarded for messy or badly arranged work.
- Answer in simplest exact form unless otherwise stated.

Total marks—89 Marks

- Attempt questions 1–6.
- The mark-value of each question is boxed in the right margin.
- Start each NEW section in a separate answer booklet.
- Hand in your answer booklets in 3 sections:
Section A(Questions 1 and 2),
Section B(Questions 3 and 4),
Section C(Questions 5 and 6),

Examiner: Mr P. Bigelow

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate.

Section A

Marks

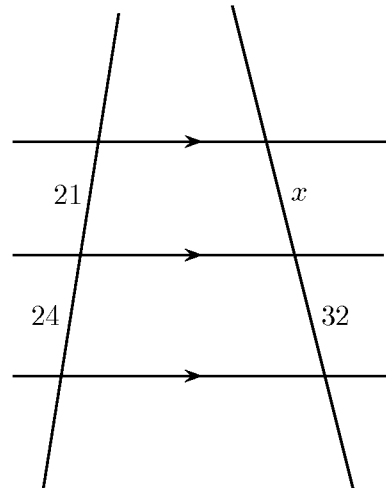
Question 1 (15 marks)

(a) Find a primitive of $1 - 2x$.

1

(b)

2



Find the value of x , stating reasons.

(c) Differentiate

4

(i) $\cos 4x$,

(ii) $\ln(4x + 5)$,

(iii) $\frac{\sin x}{x}$,

(iv) $\sqrt{e^x}$.

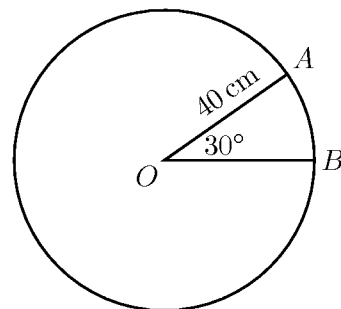
(d) Find x , correct to three significant figures, if $e^x = 4$.

1

(e) Write down the exact value of $2 \cos \pi/4$.

1

(f)



O is the centre of the circle with radius 40 cm. Find the length of the minor arc AB which subtends 30° at O .

2

(g) Sketch $y = 3 \cos 2x$ for $0 \leq x \leq 2\pi$.

2

(h) Find correct to two decimal places

2

(i) $\log_e \frac{11}{4}$,

(ii) $\tan 4^\circ$.

Question 2 (14 marks)

(a) Evaluate

(i) $\int_0^{16} \sqrt{x} dx,$

1

(ii) $\int_0^9 e^{\frac{x}{3}} dx,$

1

(iii) $\int_{\frac{1}{6}}^{\frac{1}{2}} \cos \pi x dx.$

2

(b) The gradient of any point on the curve $y = f(x)$ is given by $f'(x) = 2x - 5$.
Given that the point $(2, -3)$ lies on the curve, find its equation.

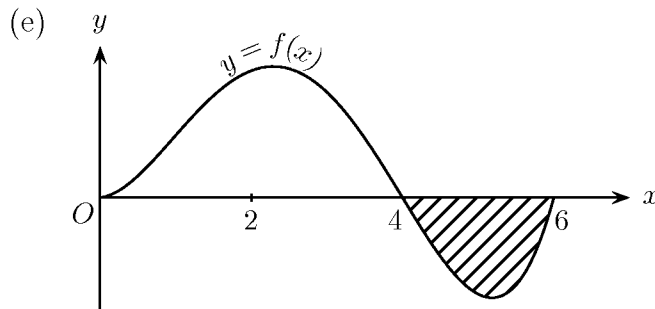
2

(c) Find the equation of the normal to $y = \ln x$ at the point $(1, 0)$.

2

(d) If $f(x) = \frac{3}{1+3x}$, find $\int_0^1 f(x) dx.$

2



Given that

$$\int_0^4 f(x) dx = 15,$$

and that

$$\int_0^6 f(x) dx = 9;$$

(i) what is the area of the shaded region,

1

(ii) and what is the value of $\int_4^6 f(x) dx$?

1

(f) A doctor reports that:

2

“Although the patient’s blood-pressure is rising,
attempts to bring it back to normal are taking effect.”

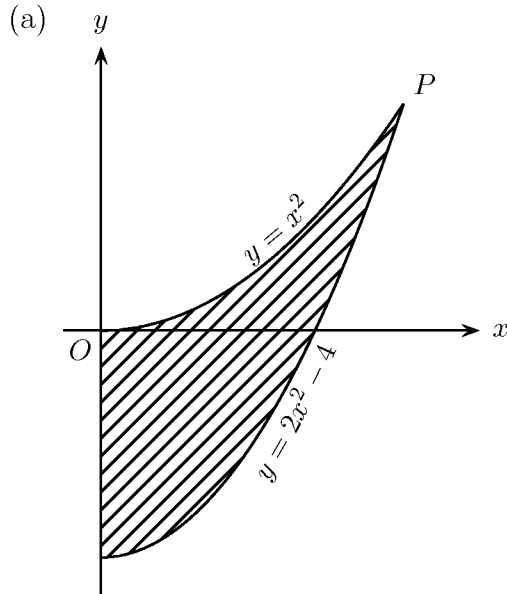
If the patient’s blood-pressure is given by B ,
what can be said about $\frac{dB}{dt}$ and $\frac{d^2B}{dt^2}$?

Section B

(Use a separate writing booklet.)

Marks

Question 3 (15 marks)



P is the point of intersection of $y = x^2$ and $y = 2x^2 - 4$.

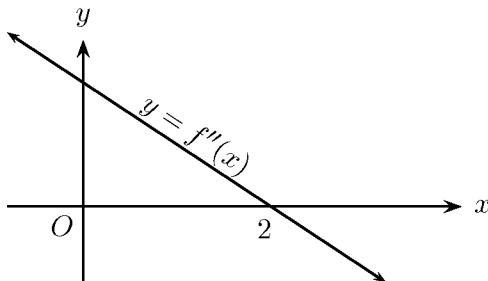
- (i) Find the coördinates of P . 1
- (ii) Find the area of the shaded region. 2
- (b) Find $\lim_{x \rightarrow 0} \frac{\sin 2x}{3x}$. 1
- (c) Find the slope of the tangent to $y = 4 \tan 2x$ at the point where $x = \pi/6$. 2
- (d) If $y = e^{2x} - 3$, prove that $\frac{dy}{dx} - 2y - 6 = 0$. 2
- (e) $A(2, 3)$ $B(4, -1)$ and $C(-4, -5)$ are the vertices of a triangle.
- (i) Find D and E , the midpoints of AB and AC respectively. 2
- (ii) Show that DE is parallel to BC . 2
- (iii) Find the lengths of DE and BC and hence show that $DE = 1/2BC$. 3

Question 4 (15 marks)

(a) For what values of x is $f(x) = x^3 - 3x^2 - 9x + 2$ an increasing function? 2

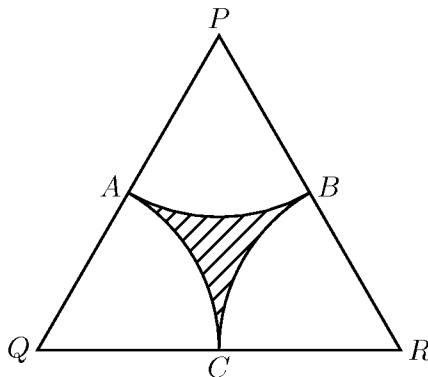
(b) The curve $f(x) = ax^3 - 9x + b$ has a turning point at $(1, 7)$. Find a and b . 3

(c) 2



The diagram shows the graph of $y = f''(x)$. Given that $f(2) = 0$ and $f'(1) = 0$, draw a possible sketch of $y = f(x)$.

(d) 3



PQR is an equilateral triangle with sides 8 cm. A , B and C are the mid-points of its sides. AB , AC and BC are circular arcs, centres P , Q and R . Calculate in cm^2 , correct to two decimal places, the shaded region bounded by the arcs.

(e) Find the values of x for which the curve $y = (x + 1)(x - 2)^2$ is concave down. 3

(f) Find the volume of the solid of revolution when the area bounded by the curve $y = 2 \sec x$ and the x -axis between $x = \pi/6$ and $x = \pi/3$ is rotated about the x -axis. Answer in simplified exact form. 2

Section C

(Use a separate writing booklet.)

Marks

Question 5 (15 marks)

(a) If $y = \ln \left(\frac{1-x}{1+x} \right)$, show that $\frac{dy}{dx} = \frac{-2}{1-x^2}$. 3

Hence evaluate $\int_0^{\frac{1}{2}} \frac{dx}{1-x^2}$.

(b) For the curve $y = x^3 + 6x^2 + 9x + 4$:
(i) Find the coördinates of the stationary points and determine their nature. 3

(ii) Find the coördinates of any points of inflexion. 2

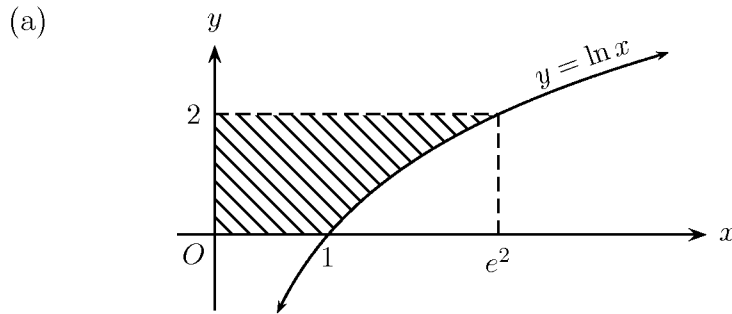
(iii) Sketch the curve, showing essential features, including the y -intercept. 2

(c) Solve $2 \cos x = -1$ for $0 \leq x \leq 2\pi$. 2

(d) If $f(x) = e^{-x^2}$, find
(i) $f'(x)$, 1

(ii) $f''(x)$. 2

Question 6 (15 marks)

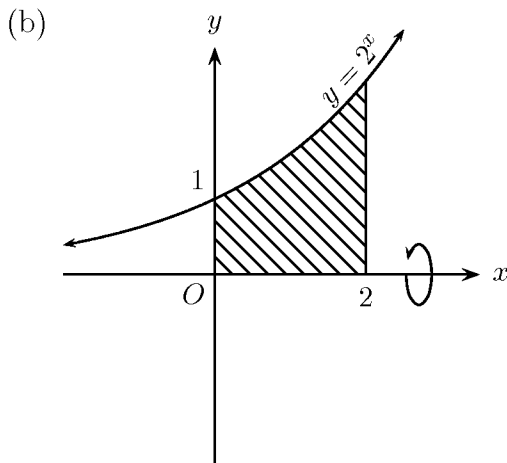


(i) Find the shaded area.

2

(ii) Hence or otherwise find $\int_1^{e^2} \ln x \, dx$.

2



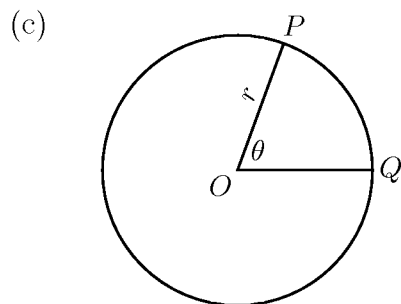
The area under the curve $y = 2^x$, between $x = 0$ and $x = 2$, is rotated about the x -axis.

(i) Show that the volume of the resulting solid is given by $\pi \int_0^2 4^x \, dx$.

2

(ii) Using Simpson's rule with five function values, find the volume of the solid correct to one decimal place.

3



The arc PQ of circle centre O and radius r cm subtends an angle of θ radians at O . The perimeter of the sector POQ is 6 cm.

(i) Show that $r = \frac{6}{\theta + 2}$.

1

(ii) Hence show that the area A cm² is given by $\frac{18\theta}{(\theta + 2)^2}$.

2

(iii) Hence find the maximum area of the sector.

3

End of Paper

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln(x + \sqrt{x^2 - a^2}), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln(x + \sqrt{x^2 + a^2})$$

NOTE: $\ln x = \log_e x$, $x > 0$

Section A

iii) $\frac{\sin x}{x} u$
 v

Question 1

$u = \sin x$ $u' = \cos x$

$v = x$ $v' = 1$

a) $\int 1 - 2x \, dx$

$\frac{d}{dx} = \frac{x \cos x - \sin x}{x^2}$ ①

$= x - \frac{2x^2}{2} + C$

$= x - x^2 + C$ ①

-c can be any
number

iv) $\sqrt{e^x} = (e^x)^{1/2}$
 $= e^{x/2}$

$\frac{d}{dx} = \frac{1}{2} e^{x/2}$
 $= \frac{\sqrt{e^x}}{2}$ ①

b) $\frac{21}{24} = \frac{x}{32}$

d) $e^x = 4$

$\ln e^x = \ln 4$

$x \ln e = \ln 4$

$x = \ln 4$

$= 1.386294361$ ①

$= 1.39$ (3 sig fig)

When 2 (or more)
transversals cut a
series of parallel lines,
the ratios of their
intercepts are equal.

① e) $2 \cos^2 1/4$

ie $x = \frac{21 \times 32}{24}$

$= 2 \times \frac{1}{\sqrt{2}} = \frac{2 \times \sqrt{2}}{\sqrt{2} \sqrt{2}}$

$= 28$ ①

$= \frac{2\sqrt{2}}{2} = \sqrt{2}$ ①

c) i) $\cos 4x$

$\frac{d}{dx} = -4 \sin 4x$ ①

f) $l = r\theta$

$= 40 \times 30 \times \frac{\pi}{180}$

ii) $\ln(4x+5)$

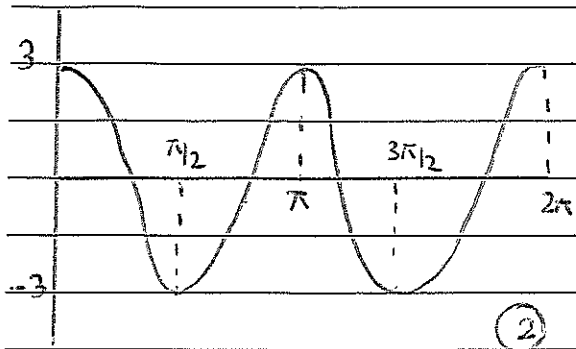
$\frac{d}{dx} = \frac{4}{4x+5}$ ①

$= \frac{20\pi}{3}$ ①

$\frac{d}{dx} = \frac{4}{4x+5}$ ①

$= 20.94395102$

g) $y = 3 \cos 2x$
for $0 \leq x \leq 2\pi$



ii) $\int_0^9 e^{x/3} dx$

$$= \left[\frac{1}{1/3} e^{x/3} \right]_0^9$$

$$= \left[3 e^{x/3} \right]_0^9$$

$$= 3e^3 - 3$$

$$= 3(e^3 - 1)$$

$$= 57.25661077$$

$$= 57.26 \text{ (2dp)} \quad \textcircled{1}$$

h) i) $\log_e 11/4$

$$= 1.011600912$$

$$= 1.01 \text{ (2dp)} \quad \textcircled{1}$$

iii) $\int_{\pi/6}^{\pi/2} \cos \pi x dx$

$$= \left[\frac{1}{\pi} \sin \pi x \right]_{\pi/6}^{\pi/2}$$

$$= \left[\frac{1}{\pi} \sin \frac{\pi}{2} \right] - \left[\frac{1}{\pi} \sin \frac{\pi}{6} \right]$$

$$= \left[\frac{1}{\pi} \right] - \left[\frac{1}{2\pi} \right]$$

$$= \frac{2-1}{2\pi} = \frac{1}{2\pi} \quad \textcircled{2}$$

ii) $\tan 4^\circ$

$$= 1.157821282$$

$$= 1.16 \quad \textcircled{1}$$

Question 2

a) i) $\int_0^{16} \sqrt{x} dx$

$$= \int_0^{16} x^{1/2} dx$$

$$= \left[\frac{x^{3/2}}{3/2} \right]_0^{16}$$

$$= \left[\frac{2x^{3/2}}{3} \right]_0^{16}$$

$$= \frac{2 \cdot 16^{3/2}}{3} - \frac{2 \cdot 0^{3/2}}{3}$$

$$= \frac{4096}{3} \text{ OR } 1365 \frac{1}{3}$$

$$\frac{128}{3} \cdot 42^{3/2}$$

b) $f'(x) = 2x - 5$
point (2, -3)

$$\int (2x - 5) dx$$

$$y = \frac{2x^2}{2} - 5x + C \quad \textcircled{1/2}$$

$$-3 = \frac{2 \cdot 4}{2} - 10 + C$$

$$-3 = 4 - 10 + C$$

$$-3 = -6 + c$$

$$c = 3 \text{ (1)}$$

$$\text{ii) } \int_a^b f(x) dx = -6 \text{ (1)}$$

$$\therefore y = x^2 - 5x + 3 \text{ (12)}$$

$$\text{c) } y = \ln x \quad \text{pt } (1, 0)$$

$$y' = \frac{1}{x} \text{ (12)}$$

$$\text{at } x=1 \quad y' = 1 \text{ (12)}$$

$$\text{grad. of normal} = -1 \text{ (12)}$$

$$y - 0 = -1(x - 1)$$

$$y = -x + 1$$

$$x + y - 1 = 0 \text{ (12)}$$

$$\text{d) } \int_0^1 \frac{3}{1+3x}$$

$$= [\ln(1+3x)]_0^1 \text{ (1)}$$

$$= [\ln(1+3)] - [\ln(1+0)]$$

$$= \ln 4 - \ln 1$$

$$= \ln 4 \text{ (1)}$$

$$= 1.386294361$$

$$= 1.39 \text{ (2 dp)}$$

$$\text{e) } \left| \int_0^4 f(x) \right|$$

$$= |9 - 15| = |-6|$$

$$= 6 \text{ (1)}$$

①

Section B 2 unit Task 3 July 2009

15

(3)(a)(i) $y = x^2$ and $y = 2x^2 - 4$

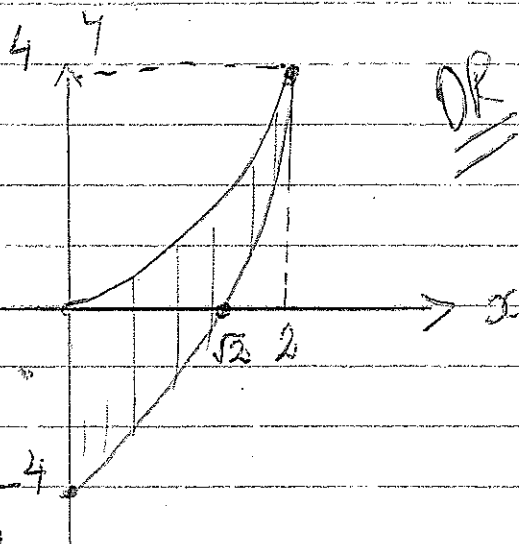
$2x^2 - 4 = x^2$

$x^2 = 4$

$x = \pm 2$

~~$x = 2, y = 4$~~ $P(2, 4)$ (1)

(ii)



OR area = $\int_0^2 x^2 dx - \int_{\sqrt{2}}^2 (2x^2 - 4) dx$

+ $\left| \int_0^{\sqrt{2}} (2x^2 - 4) dx \right|$

= $\left[\frac{x^3}{3} \right]_0^2 - \left[\frac{2x^3}{3} - 4x \right]_{\sqrt{2}}^2$

~~$A = \int_0^2 (x^2 - 2x^2 + 4) dx$~~

~~$= \int_0^2 (-x^2 + 4) dx$~~

~~$= \left[-\frac{x^3}{3} + 4x \right]_0^2$~~

~~$= -\frac{8}{3} + 8$~~

~~$= \frac{16}{3}$~~

= $\frac{8}{3} - \left[\left(\frac{16}{3} - 8 \right) - \left(\frac{4\sqrt{2}}{3} - 4\sqrt{2} \right) \right]$

+ $\left| \left(\frac{4\sqrt{2}}{3} - 4\sqrt{2} \right) \right|$

= $\frac{8}{3} - \left[-\frac{8}{3} - \frac{8\sqrt{2}}{3} \right]$

+ $\left| -\frac{8\sqrt{2}}{3} \right|$

$A = \frac{8}{3} + \frac{8}{3} - \frac{8\sqrt{2}}{3} + \frac{8\sqrt{2}}{3} = \frac{16}{3} \text{ u}^2$ (2)

(2)

$$3 \text{ (b) } \lim_{x \rightarrow 0} \frac{\sin 2x}{3x}$$

$$\frac{2}{3} \lim_{x \rightarrow 0} \frac{\sin 2x}{2x}$$

$$= \frac{2}{3} \times 1 = \frac{2}{3} \quad (1)$$

$$(c) \ y = 4 \tan 2x$$

$$y' = 4 \times \sec^2 2x \times 2$$

$$= 8 \sec^2 2x$$

$$\text{at } x = \frac{\pi}{6}, \quad m = 8 \times \sec^2 \left(2 \times \frac{\pi}{6} \right)$$

$$= 8 \sec^2 \left(\frac{\pi}{3} \right)$$

$$= 8 \times \frac{1}{\cos^2 \left(\frac{\pi}{3} \right)}$$

$$= 8 \times \frac{1}{\left(\frac{1}{2} \right)^2} = 8 \times \frac{1}{\frac{1}{4}} = 32 \quad (2)$$

$$(d) \ y = e^{2x} - 3$$

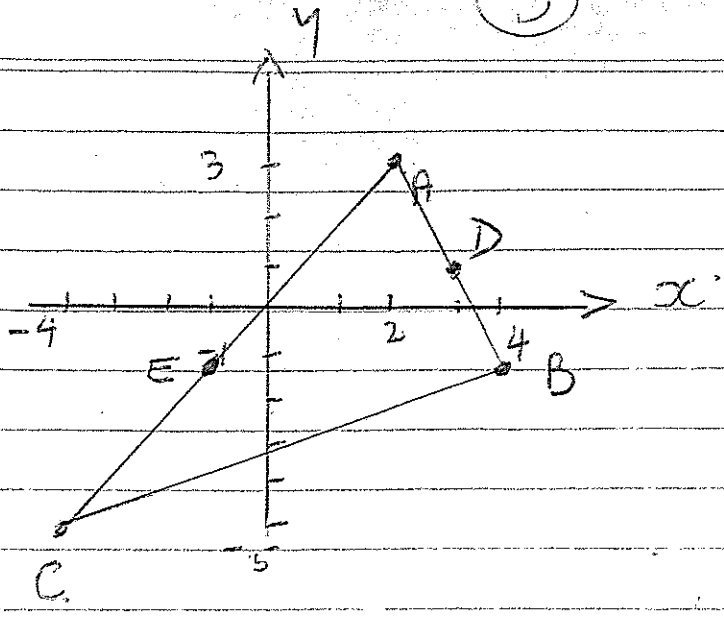
$$\frac{dy}{dx} = 2e^{2x}$$

$$\text{now } \frac{dy}{dx} - 2y - 6 = 2e^{2x} - 2(e^{2x} - 3) - 6$$

$$= 2e^{2x} - 2e^{2x} + 6 - 6 = 0 \quad (2)$$

(3)

(2)



(i) midpt AB $D \left(\frac{2+4}{2}, \frac{3-1}{2} \right) = D(3, 1)$ ①

midpt AC $E \left(\frac{2-4}{2}, \frac{3-5}{2} \right) = E(-1, -1)$ ①

(ii) m of DE $\frac{-1-1}{-1-3} = \frac{-2}{-4} = \frac{1}{2}$

m of BC $\frac{-5-1}{-4-4} = \frac{-6}{-8} = \frac{3}{4}$ $DE \parallel BC$ ②

(iii) dist DE $\sqrt{(-1-3)^2 + (-1-1)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

dist BC $\sqrt{(-4-4)^2 + (-5-1)^2} = \sqrt{64+36} = \sqrt{100} = 10$

SO $DE = \frac{1}{2} BC$ $2\sqrt{5} = \frac{1}{2} \times 10$

③

④

15.

Q4/ (a) $f(x) = x^3 - 3x^2 - 9x + 2$

$f'(x) = 3x^2 - 6x - 9$

let $3x^2 - 6x - 9 > 0$

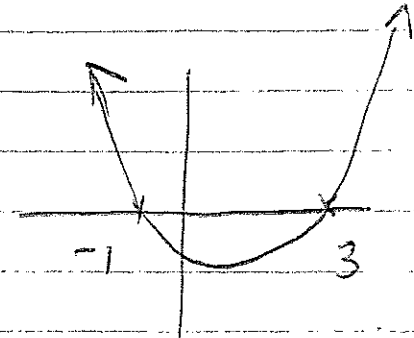
$3(x^2 - 2x - 3) > 0$

$x^2 - 2x - 3 > 0$

$(x-3)(x+1) > 0$

So $x < -1$ and $x > 3$.

②



(b) $f(x) = ax^3 - 9x + b$

$f'(x) = 3ax^2 - 9$

when $x=1$, $f'(x)=0$. So $3a-9=0$

$3a=9$

$a=3 //$

So $f(x) = 3x^3 - 9x + b$

$(1, 7)$ on curve, So $7 = 3 - 9 + b$

$7 = -6 + b$

$b = 13. //$

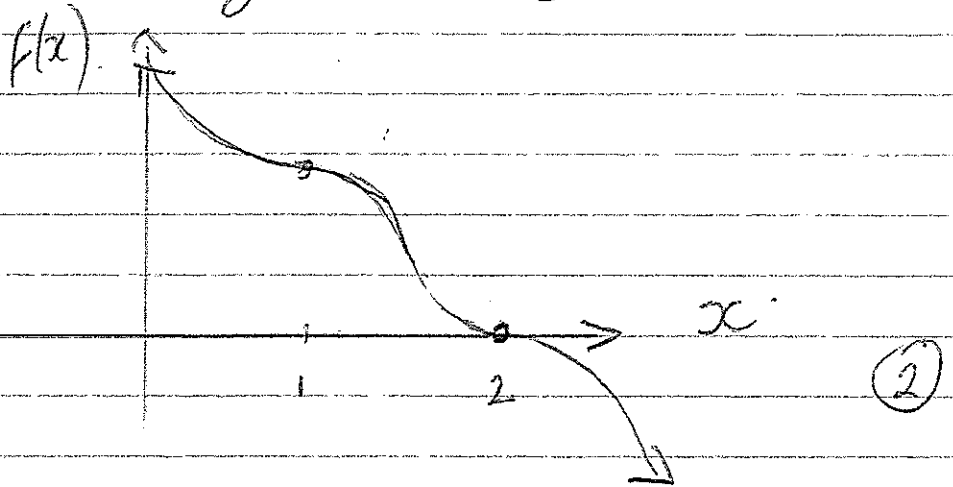
③

(5)

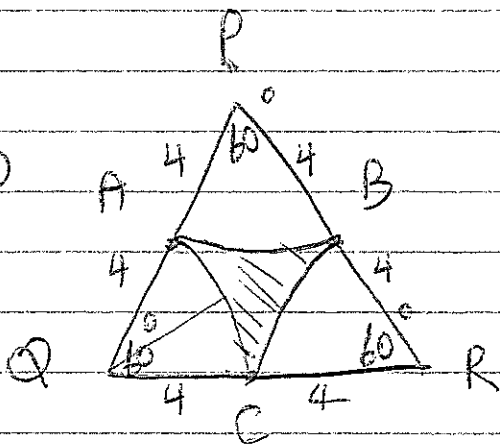
(c) diagram tells us that at $x < 2$, curve is concave up. At $x > 2$ curve is concave down at $x = 2$ point of inflection.

Also $f(2) = 0 \Rightarrow (2, 0)$ on curve.

$f'(1) = 0 \Rightarrow$ tangent is horizontal at $x = 1$.



(d)



$$\begin{aligned} \text{Area } \triangle PQR &= \frac{1}{2} \times 8 \times 8 \times \sin 60 \\ &= 32 \times \frac{\sqrt{3}}{2} = 16\sqrt{3} \end{aligned}$$

$$\text{Area each sector } A = \frac{1}{2} \times \frac{\pi}{3} \times 4^2 = \frac{8\pi}{3}$$

$$3 \times \text{sectors} = 3 \times \frac{8\pi}{3} = 8\pi$$

$$\text{shaded area} = (16\sqrt{3} - 8\pi) \text{ cm}^2$$

$$= 2.58 \text{ cm}^2 \quad \text{--- (3)}$$

$$\begin{aligned}
 (e) \quad y &= (x+1)(x-2)^2 \quad (6) \\
 &= (x+1)(x^2 - 4x + 4) \\
 &= x^3 - 4x^2 + 4x + x^2 - 4x + 4 \\
 &= x^3 - 3x^2 + 4
 \end{aligned}$$

$$y' = 3x^2 - 6x$$

$$y'' = 6x - 6$$

concave down when $y'' < 0$, $6x - 6 < 0$

$$6x < 6$$

$$x < 1$$

(3)

$$(f) \quad V = \pi \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 4 \sec^2 x \, dx$$

$$= 4\pi \left[\tan x \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = 4\pi \left[\tan \frac{\pi}{3} - \tan \frac{\pi}{6} \right]$$

$$= 4\pi \left[\frac{\sqrt{3}}{1} - \frac{1}{\sqrt{3}} \right]$$

$$= 4\pi \left[\frac{3-1}{\sqrt{3}} \right]$$

$$= 4\pi \times \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{8\pi\sqrt{3}}{3} \, \mu^3$$

(2)

$$(14.51 \, \mu^3 \quad 2DP)$$

Question 5

(a) $y = \ln \left(\frac{1-x}{1+x} \right)$

i.e. $y = \ln(1-x) - \ln(1+x)$

$$\frac{dy}{dx} = \frac{-1}{1-x} - \frac{1}{1+x}$$

$$= \frac{-(1+x) - (1-x)}{1-x^2}$$

$$= \frac{-2}{1-x^2}$$

(1/2)

$$\therefore \int_0^{1/2} \frac{dx}{1-x^2} = -\frac{1}{2} \ln \left[\frac{1-x}{1+x} \right]_0^{1/2}$$

$$= -\frac{1}{2} \left[\ln \left(\frac{1}{3} \right) - \ln 1 \right]$$

(1/2)

$$= -\frac{1}{2} \ln \frac{1}{3} \text{ or } \frac{1}{2} \ln 3$$

(b) $y = x^3 + 6x^2 + 9x + 4$

(i) St. pts $y' = 3x^2 + 12x + 9 = 0$

i.e. $(x+3)(x+1) = 0$

at $x = -3$ or $x = -1$ (1)

$$y'' = 6x + 12$$

When $x = -3$, $y'' = -6 < 0 \Rightarrow$ MAX T.P. (1)

at $(-3, 4)$

When $x = -1$, $y'' = 6 > 0 \Rightarrow$ MIN T.P.

(1) at $(-1, 0)$

(ii) $y'' = 6x + 12 = 0$ when $x = -2$

\therefore Possible inflexion

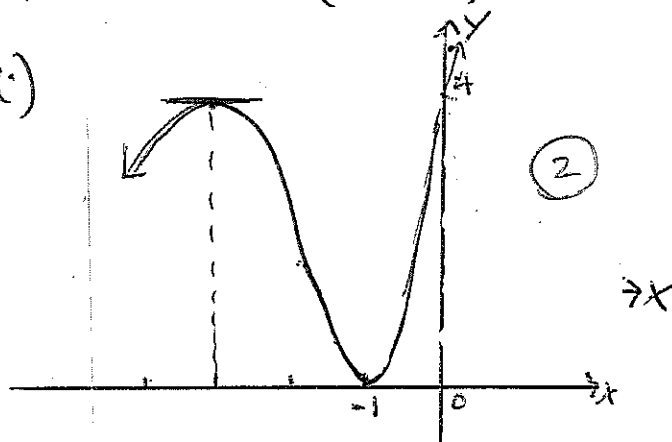
x	$<$	-2	$>$
y''	$+$	0	$-$

(2)

Change of concavity \Rightarrow

P.O.I. at $(-2, 2)$

(iii)



(2)

(c) $2\cos x = -1 \Rightarrow \cos x = -\frac{1}{2}$

(2)

(d) $f(x) = e^{-x^2}$

(i) $f'(x) = -2xe^{-x^2}$ (1)

(ii) $f''(x) = -2x \cdot e^{-x^2} \cdot -2x + -2e^{-x^2}$
 $= +2e^{-x^2}(1 - 2x^2)$ (2)

$$f''(x) = 4x^2 e^{-x^2} - 2e^{-x^2}$$

Question 6

(a) (i) $\text{Area} = \int_0^2 e^y dy$ ✓
 $= [e^y]_0^2$
 $= (e^2 - 1) \text{ units}^2$ ✓

(ii) $\int_1^{e^2} \ln x dx = 2e^2 - (e^2 - 1)$
 $= e^2 + 1$ ✓

(b) (i) $V = \pi \int_0^2 y^2 dx = \pi \int_0^2 (2^x)^2 dx = \pi \int_0^2 4^x dx$ ✓

(ii) $\pi \int_0^2 4^x dx \doteq \pi \left[\frac{1-0}{6} (4^0 + 4^{0.5} + 4^1) + \frac{2-1}{6} (4^1 + 4^{1.5} + 4^2) \right]$
 $\therefore \doteq \pi \left[\frac{1}{6} (13) + \frac{1}{6} (52) \right] \doteq \frac{65\pi}{6} \doteq 34.0$

(c) (i) Perimeter = $r + r + r\theta$

$\therefore 6 = 2r + r\theta \Rightarrow 6 = r(2 + \theta)$

$\therefore r = \frac{6}{2 + \theta}$ ✓

(ii) $A = \frac{1}{2} r^2 \theta = \frac{1}{2} \left(\frac{6}{2 + \theta} \right)^2 \theta$
 $= \frac{1}{2} \cdot \frac{36}{(2 + \theta)^2} \cdot \theta$
 $= \frac{18\theta}{(\theta + 2)^2}$

(iii) $\frac{dA}{d\theta} = \frac{(\theta + 2)^2 \cdot 18 - 18\theta \cdot 2(\theta + 2)}{(\theta + 2)^4} = \frac{18(\theta + 2)(2 - \theta)}{(\theta + 2)^4} = \frac{18(2 - \theta)}{(\theta + 2)^3}$ ✓

Stat. pt when $\theta = 2$ ✓

Using 1st derivative to prove max when $\theta = 2$

$\therefore A_{\text{max}} = \frac{18(2)}{(2 + 2)^2} = \frac{36}{16} = 2\frac{1}{4} \text{ cm}^2$ ✓

①	↓		
θ	1.9	2	2.1
$\frac{dA}{d\theta}$	+	0	-