SYDNEY TECHNICAL HIGH SCHOOL

YEAR 12 ASSESSMENT TASK 3

JUNE 2004

MATHEMATICS: 2 UNIT

Time Allowed: 70 minutes

Instructions:

- Write your name and class at the top of this page, and at the top of each answer sheet
- At the end of the examination this examination paper must be attached to the front of your answers.
- All questions are of equal value. Attempt all questions.
- All necessary working must be shown. Marks will be deducted for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- Standard integrals are attached at the back of this paper.

Name: ....

Teacher: ....

Q1	Q2	Q3	Q4	Q5	Total
/12	/12	/12	/12	/12	/60

Question 1 (12 Marks)

a) For the parabola $x^2 - 4x - 8y - 36 = 0$

find : i) the co-ordinates of the vertex (1)

ii) the focal length (1)

iii) the co-ordinates of the focus (1)

iv) the equation of the directrix (1)

b) If $x^{1.0986} = 3$ find:

i) $\log_x 3$ (1)

ii) $\log_x 81$ (1)

c) Evaluate $\int_0^{\pi/3} \sin 2x \, dx$ (2)

d) Differentiate $y = x \tan x$ (2)

e) Differentiate $y = \log_e(4 - x^2)$ (2)

Question 2

a) Sketch the graph of $y = \cos\frac{x}{4}$ *for* $-4\pi \le x \le 4\pi$ (2)

b) (i) Evaluate $\int_0^{2\pi} \cos\frac{x}{4}\,dx$ (2)

(ii) For the function in (i) $y = \cos\frac{x}{4}$, complete the table below

x	0	$\pi/2$	π	$\frac{3\pi}{2}$	2π
y					

Hence approximate $\int_0^{2\pi} \cos\frac{x}{4}\,dx$ using Simpson's rule with 5 function values and answering correct to 2 decimal places. (3)

Question 2 (cont)

(iii) Find the percentage error given by using Simpson's rule to approximate the integral (answer correct to 1 decimal place). (1)

c) (i) Find the co-ordinates of the point on the curve $y = x \ln x$ for which the gradient of the tangent is equal to 2. (2)

(ii) Hence find the equation of the normal at this point. (2)

Question 3

a) Solve the equation $5e^{2x} = 0.4213$. Give your answer to 4 decimal places. (2)

b) By using the substitution $u = e^x$ or otherwise, solve the equation $e^{2x} - 6e^x + 5 = 0$
Answer to 3 decimal places where necessary (3)

c) Solve for θ in the domain $0 \le \theta \le 2\pi$: (3)

$$4\sin\theta - 3\cos\theta = 0$$

d) Differentiate

$$y = \sqrt{\sin 2x}$$ (2)

e) Find the derivative of $y = e^{\cos x}$ (1)

Question 4

a) Find

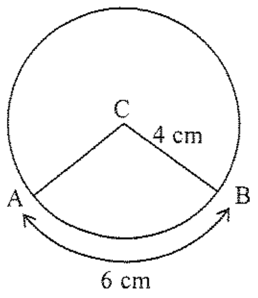
i) $\int e^{5x} dx$ (1)

ii) $\int \sqrt{e^x} dx$ (2)

b) Express $\frac{2\pi^c}{5}$ in degrees (1)

Question 4 (cont.)

c)



In the diagram, AB is an arc of length 6cm in a circle, centre C, radius 4cm. Find the

i) size of angle ACB in radians (1)

ii) area of sector ACB (1)

iii) area of the minor segment cut off by the chord AB (2)

d) (i) Find the area bounded by the x $axis$, the curve $y = e^x$ and the lines $x = 0$ & $x = \log_e 5$. (2)

(ii) This area is rotated about the x $axis$.

Find the volume of the solid so formed. (Leave answer in terms of π). (2)

Question 5

a) Solve for x, $\sin^2 2x = \frac{1}{4}(0 \le x \le 180°)$ (2)

b) (i) Find the value of k for which the equation $x^2 - (k-5)x + (k-7) = 0$ has roots which are reciprocals of one another. (1)

(ii) If α and β are the roots of this equation find

A) $\alpha + \beta$ (1)

B) $\frac{1}{\alpha} + \frac{1}{\beta}$ (1)

c) Find the values of P, Q and R if $3x^2 + 5x - 1 \equiv P(x+1)^2 + Q(x+1) + R$. (3)

d) Find all the values of k for which $(k+2)x^2 + x - 3 = 0$ has real roots. (2)

e) Evaluate $\int_1^2 \frac{x^3 - 3x + 2}{x^2} dx$ (2)

End of Exam.

Question 1

a) $x^2 - 4x + 4 = 8y + 36 + 4$

$(x-2)^2 = 8y + 40$

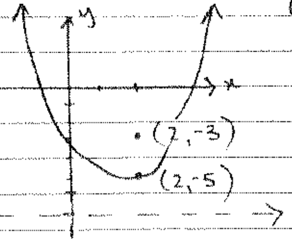
$(x-2)^2 = 8(y+5)$

i) Vertex $(2,-5)$ ①

ii) $4a = 8 \quad \therefore a = 2$ ①

iii) Focus $(2,-3)$ ①

iv) $y = -7$ ①



1.0986

b) $x = 3$

i) $\log_x 3 = 1.0986$ ①

ii) $\log_x 81 = \log_x 3^4$

$= 4\log_x 3$

$= 4.3944$ ①

c) $\int_0^{\pi/3} \sin 2x \, dx = \left[-\frac{1}{2}\cos 2x\right]_0^{\pi/3}$ ①

$= -\frac{1}{2}\left[\cos\frac{2\pi}{3} - \cos 0\right]$

$= -\frac{1}{2}\left[-\frac{1}{2} - 1\right]$

$= 3/4$ ①

d) $\frac{d}{dx}(x\tan x)$

$u = x \qquad v = \tan x$

$u' = 1 \qquad v' = \sec^2 x$ ②

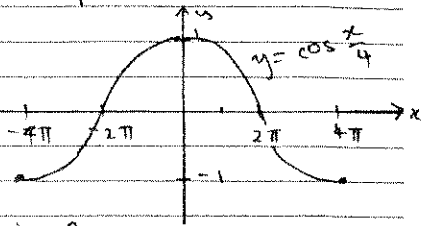
$\frac{dy}{dx} = \tan x + x\sec^2 x$

e) $\frac{d}{dx}\ln(4-x^2) = \frac{-2x}{4-x^2}$ ②

Question 2

a) period $= \frac{2\pi}{1/4} = 8\pi$

amplitude $= 1$



b) i) $\int_0^{2\pi} \cos\frac{x}{4}\, dx$

$= \left[4\sin\frac{x}{4}\right]_0^{2\pi}$

$= 4\left(\sin\frac{\pi}{2} - \sin 0\right)$

$= 4$

ii)

	F				L
x	0	π/2	π	3π/2	2π
y	1	.924	.707	.383	0
		y_1	y_2	y_3	

$S = \frac{\pi/2}{3}\left[1 + 0 + 4(.924 + .383) + 2 \times .707\right]$

$= 4.00$

c) i) $y = x\ln x \quad \therefore \frac{dy}{dx} = 2$

$u = x \qquad v = \ln x$

$u' = 1 \qquad v' = \frac{1}{x}$

$\frac{dy}{dx} = \ln x + 1$

$\ln x + 1 = 2$

$\ln x = 1$

$\log_e x = 1 \quad \therefore e = x$

$\therefore$ pt (e, e)

ii) $m_N = -\frac{1}{2}$ at (e,e)

eqn $y - e = -\frac{1}{2}(x - e)$

$2y - 2e = -$[illegible]$+ e$

$x + 2y - 3e = 0$

Question 3

a) $5e^{2x} = .4213$

$e^{2x} = \frac{.4213}{5}$

$2x\ln e = \ln(.08426)$

$= -1.2369$ (4d.p)

b) $e^{2x} - 6e^x + 5 = 0$

$u^2 - 6u + 5 = 0$

$(u - 5)(u - 1) = 0$

$u = 5 \qquad u = 1$

$\therefore e^x = 5 \qquad e^x = 1$

$x = 1.609 \qquad x = 0$

c) $4\sin\theta - 3\cos\theta = 0$

$4\sin\theta = 3\cos\theta$

$\tan\theta = \frac{3}{4}$ (S A / T C)

$\theta = .64^c, \ 3.79^c$

d) $y = (\sin 2x)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(2\cos 2x)(\sin 2x)^{-1/2}$

$= \frac{\cos 2x}{\sqrt{\sin 2x}}$

e) $\frac{d}{dx}\left(e^{\cos x}\right) = -\sin x . e^{\cos x}$

Question 4

a) i) $\int e^{5x}\, dx = \frac{1}{5}e^{5x} + C$

ii) $\int e^{x/2}\, dx = 2e^{x/2} + c$

OR $2\sqrt{e^x} + c$

b) $\frac{2\pi}{5} = 72^\circ$

c) i) $6 = 4 \times \theta$

$\theta = 1.5^c$

$\therefore A\hat{C}B = 1.5^c$

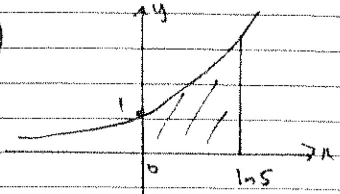
ii) $A = \frac{1}{2} \times 4^2 \times 1.5$

$= 12\text{ cm}^2$

iii) $A = \frac{1}{2} \times 4^2(1.5 - \sin 1.5)$

$= 4.02\text{ cm}^2$

d)



i) $A = \int_0^{\ln 5} e^x\, dx$

$= \left[e^x\right]_0^{\ln 5}$

$= e^{\ln 5} - e^0$

$= 4\ u^2$

ii) $V = \pi\int_0^{\ln 5} e^{2x}\, dx$

$= \pi\left[\frac{e^{2x}}{2}\right]_0^{\ln 5}$

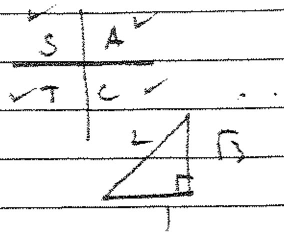
$= \frac{\pi}{2}\left[e^{2\ln 5} - e^0\right]$

$= \frac{\pi}{2}[25 - 1]$

$= 12\pi$ unit^3

Question 5

a) $\sin^2 2x = \frac{1}{4}$



$\sin 2x = \pm\frac{1}{2}$

acute $2x = 30°$

$\therefore 2x = 30°, 150°, 210°, 330°$

$x = 15°, 75°, 105°, 165°$

b) i) Reciprocal roots $\alpha, \frac{1}{\alpha}$

$\therefore$ product is 1 $\therefore \frac{c}{a} = 1$

$\frac{k-7}{1} = 1$

$k = 8$

ii) A) $\alpha + \beta = \frac{k-5}{1} = 3$

B) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{3}{1}$

c) $3x^2 + 5x - 1 = P(x+1)^2 + Q(x+1) + R$

$= P(x^2+2x+1) + Qx + Q + R$

$= Px^2 + 2Px + P + Qx + Q + R$

$= P(x^2) + x(2P+Q) + P+Q+R$

$\therefore P = 3$ $\quad 2P + Q = 5$ $\quad 3 - 1 + R = -1$

$6 + Q = 5$ $\quad R = -3$

$Q = -1$

d) Real Roots $\Delta \geq 0$ $\quad 1 - 4(k+2)\times -3 \geq 0$

$1 + 12(k+2) \geq 0$

$1 + 12k + 24 \geq 0$

$12k \geq -25$

$k \geq \frac{-25}{12}$

e) $\int_1^2 \frac{x^3 - 3x + 2}{x^2}\,dx = \int_1^2 x - \frac{3}{x} + 2x^{-2}\,dx$

$= \left[\frac{x^2}{2} - 3\ln x - \frac{2}{x}\right]_1^2$

$= \left[\left(2 - 3\ln 2 - 1\right) - \left(\frac{1}{2} - 0 - 2\right)\right] = 2\frac{1}{2} - 3\ln 2$