Name: File

Class:

SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS

HSC ASSESSMENT TASK 3

JUNE 2005

Time Allowed: 70 minutes

Instructions

- Write your name and class at the top of this page, and at the top of each answer sheet.
- At the end of the examination, this examination paper must be attached to the front of your answers.
- Attempt all questions.
- All necessary working must be shown. Marks will be deducted for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.

Q1	Q2	Q3	Q4	Q5	**Total**
12	12	10	11	11	56

Question 1 (12 marks)

a) Express $315°$ in radians, in terms of π (1)

b) Find the exact value of $\cos \dfrac{5\pi}{4}$ (1)

c) Differentiate

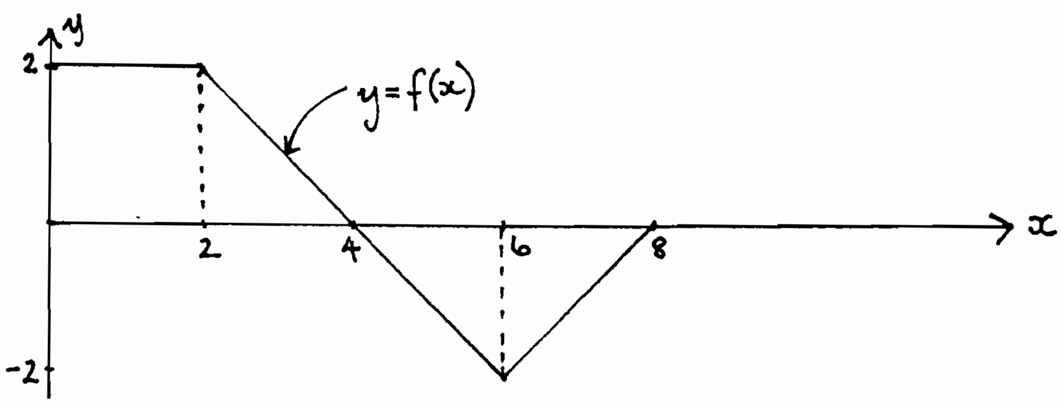
i) $y = 2\sqrt{x}$ (1)

ii) $y = \dfrac{1}{x-1}$ (1)

iii) $y = \dfrac{x+1}{x-1}$ (2)

d) Find the primitive of $(2x+1)^4$ (2)

e) The function $y = f(x)$ is sketched below



i) Evaluate $\int_0^8 f(x)\ dx$ (1)

ii) Find the area enclosed by $y = f(x)$ the x axis, $x = 0$ and $x = 8$ (1)

For parts f) and g) write the correct letter on your answer sheets.

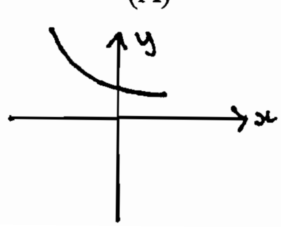
f) Which one of the following is equal to $\int k\ dx$, where k is a constant.

(A) $kx + C$ (B) $\dfrac{k^2}{2} + C$ (C) $\dfrac{k^2 x}{2} + C$ (D) $\dfrac{kx^2}{2} + C$ (1)

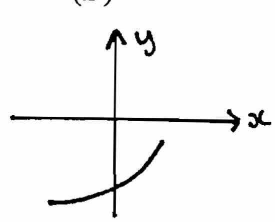
g) For a given function $y = f(x), f(0) < 0, f'(x)(0) > 0$ *and* $f''(x)(0) > 0$.

The graph of $y = f(x)$ in this region would look like: (1)

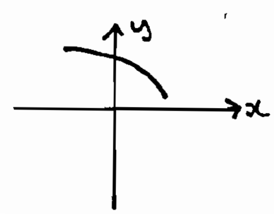
(A)



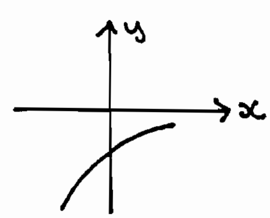
(B)



(C)



(D)



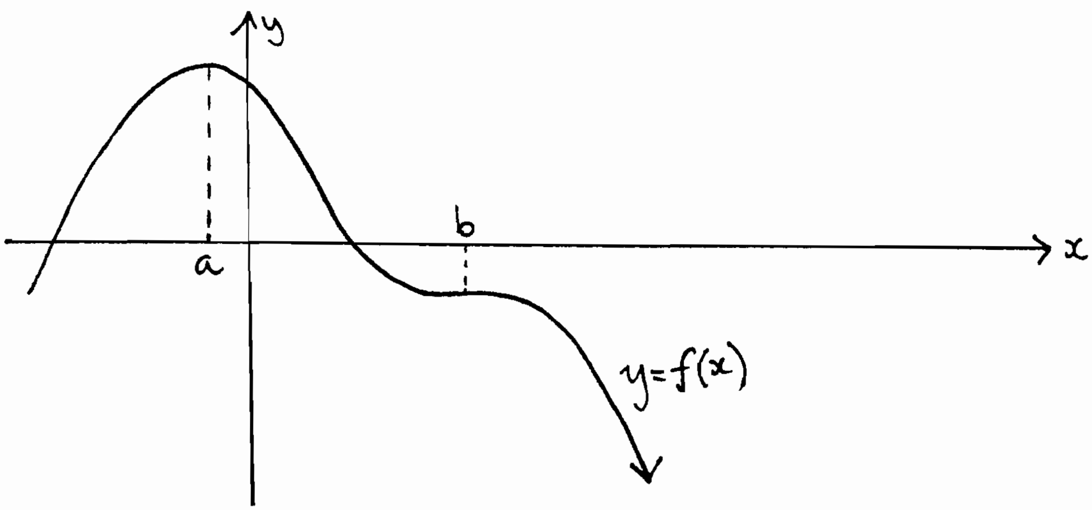
Question 2 (12 marks) (Start a new page)

a) Sketch $y = 2\sin\frac{x}{2}$ *for* $-2\pi \le x \le 2\pi$ (2)

b) Solve $\sin x = -\frac{\sqrt{3}}{2}$ *for* $0 \le x \le 2\pi$ (2)

(Solution(s) <u>must</u> be in radians)

c) Given the sketch of $y = f(x)$ (2)



Make a neat sketch of $y = f'(x)$. Indicate a and b on your sketch.

d) Consider the curve $y = x^3(4 - x)$ in the domain $-2 \le x \le 4$

i) Find the stationary points and determine their nature (3)

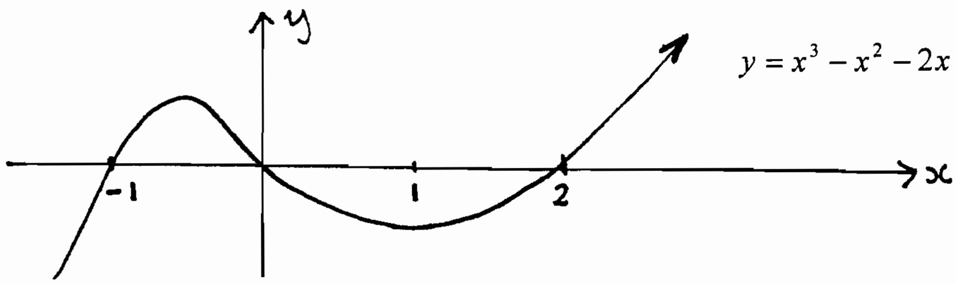
ii) Find any points of inflexion (1)

iii) Sketch the curve in the domain $-2 \le x \le 4$.

Show the co-ordinates of the end points (2)

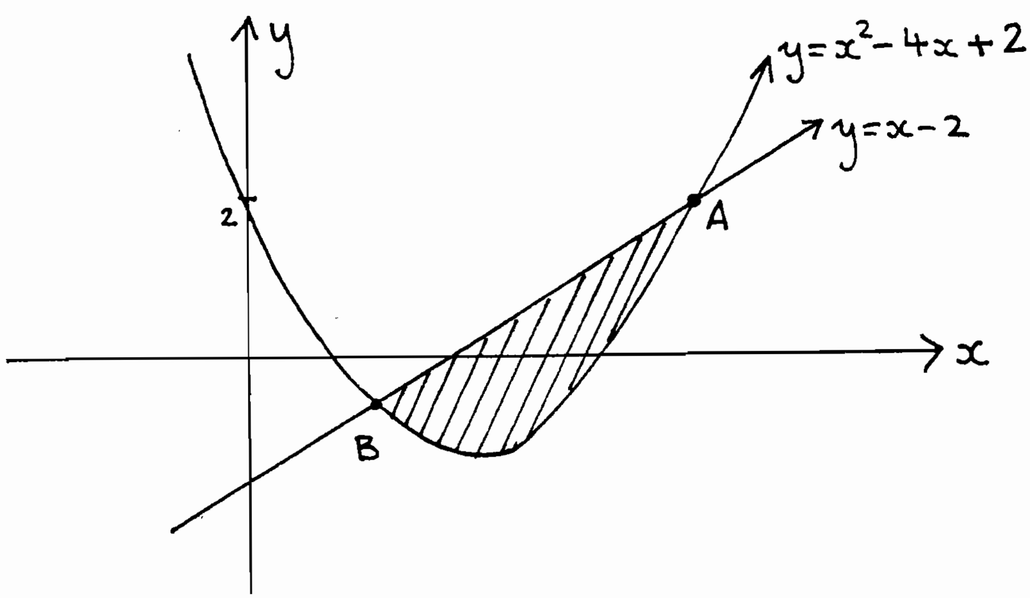
Question 3 (10 marks) (Start a new page)

a) The curve $y = x^3 - x^2 - 2x$ is sketched below (4)



Find the area enclosed by the curve and x axis.

b) The curves $y = x^2 - 4x + 2$ and $y = x - 2$ are sketched below.



i) Find the x co ordinate for A and B. (2)

ii) Find the area of the enclosed shaded region. (4)

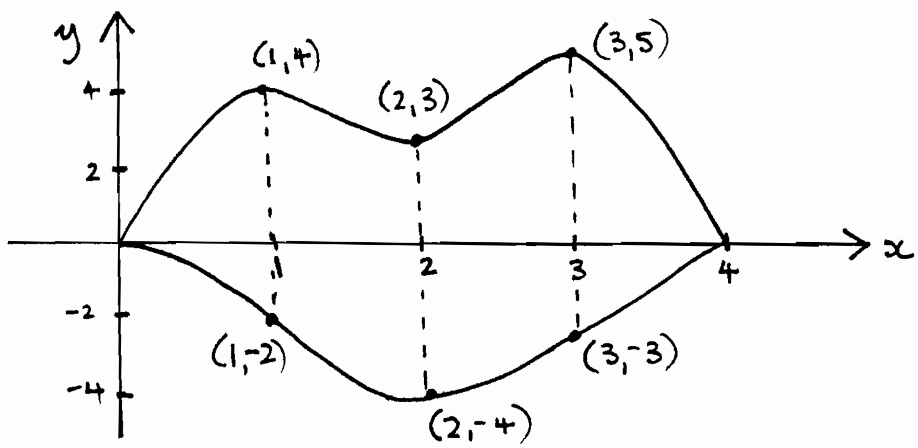
Question 4 (11 marks) (Start a new page)

a) The area below the graph $y = \frac{1}{x}$ in the first quadrant between $x = 1$ and $x = 4$ is rotated around the x axis. What is the volume of the generated solid? (3)

(answer in terms of π).

b) For a certain curve $\frac{d^2y}{dx^2} = 6x - 10$. Find the equation of the curve if it passes through the point (1, 1) with gradient -1. (3)

c)

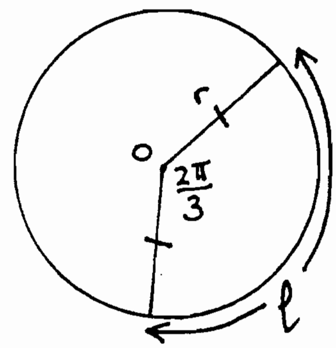


Use the trapezoidal rule and 5 function values to find the area of the enclosed region. (2)

d) If $\int_1^b (2x-1)dx = 6$ and $b > 0$. Find b. (3)

Question 5 (11 marks) (Start a new page)

a) A circular pond has an area of $615.75m^2$. It is divided into sectors one of which contains a central angle of $\frac{2\pi}{3}$ radians.



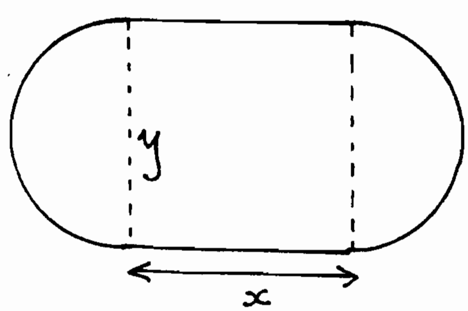
i) Find the radius r of the circle (to 1 dec pl) (1)

ii) Use r to find the length ℓ of the sector (to 1 dec pl) (1)

iii) Find the area of the sector (to 1 dec pl) (1)

b) Solve $\sin x + \cos x = 0$ for $-\pi \le x \le \pi$ (solution must be in radians). (2)

c) A running track of length 400 metres is designed using two sides of a rectangle and two semicircles, as shown. The rectangle has length x metres and the semicircles each have diameter y metres.



(i) Show that $x = \frac{1}{2}(400 - \pi y)$. (1)

(ii) The region inside the track will be used for field events.

Show that its areas is $A = 200y - \frac{\pi y^2}{4}$ (2)

(iii) Hence find the maximum area that may be enclosed to the nearest whole

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Question 1

a) $315° = 315 \times \frac{\pi}{180°} = \frac{7\pi}{4}$

b) $\cos\frac{5\pi}{4} = \cos\left(\pi + \frac{\pi}{4}\right)$

$= -\cos\pi/4$

$= -1/\sqrt{2}$

c) i) $\frac{d}{dx}(2\sqrt{x}) = 2 \times \frac{1}{2}x^{-1/2}$

$= \frac{1}{\sqrt{x}}$

ii) $\frac{d}{dx}(x-1)^{-1} = -(x-1)^{-2}$

$= \frac{-1}{(x-1)^2}$

iii) $u = x+1 \quad v = x-1$

$u' = 1 \quad v' = 1$

$\frac{dy}{dx} = \frac{(x-1)-(x+1)}{(x-1)^2}$

$= \frac{-2}{(x-1)^2}$

d) $\int (2x+1)^4 dx = \frac{(2x+1)^5}{10} + c$

e) i) $\int_0^8 f(x)\,dx = 4 - \frac{2\times 2}{2}$

$= 2$

ii) area $= 4 + 3(2)$

$= 10$ unit^2

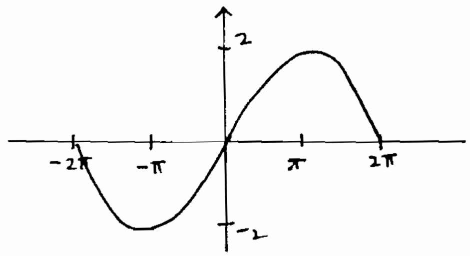
f) A

g) B

Question 2

a) amplitude $= 2$

period $= \frac{2\pi}{1/2} = 4\pi$



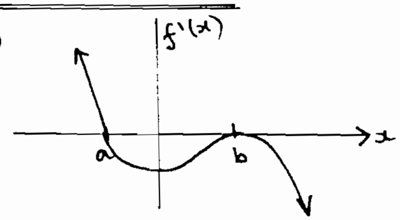
b) $\sin x = -\frac{\sqrt{3}}{2}$

acute $x = \frac{\pi}{3}$

$\therefore x = \frac{4\pi}{3}, \frac{5\pi}{3}$

S	A
T ✓	C ✓

c)



d) $y = x^3(4-x) \quad -2 \le x \le 4$

end pts $(-2, -48) \quad (4, 0)$

i) $y = 4x^3 - x^4$

$\frac{dy}{dx} = 12x^2 - 4x^3$

$\frac{d^2y}{dx^2} = 24x - 12x^2$

st pt $12x^2 - 4x^3 = 0$

$4x^2(3-x) = 0 \quad x = 0, 3$

$\therefore (0,0) \quad y'' = 0$

$(3, 27) \quad y'' < 0$ max

test concavity at $(0,0)$

x	-1	0	1
y''	$-ve$	0	$+ve$

concavity change $\therefore$ horizontal point of inflexion

ii) $\frac{d^2y}{dx^2} = 0 \qquad 24x - 12x^2 = 0$

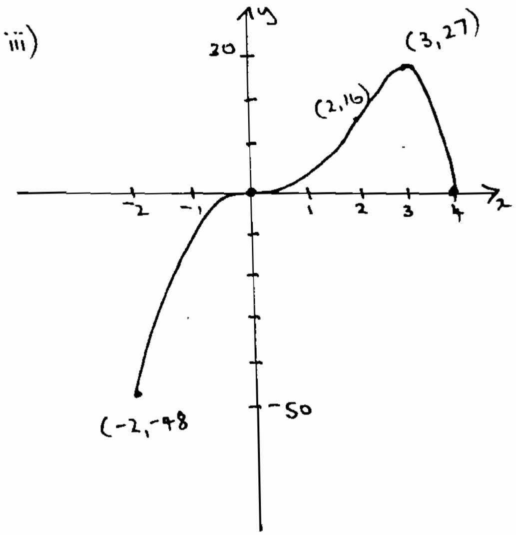
$12x(2-x) = 0$

$x = 0 \quad (0,0)$ horz pt inf

$x = 2 \quad (2, 16)$ pt inf

on continuous curve

iii)



Question 3

a) $A = \int_{-1}^{0} x^3 - x^2 - 2x\,dx + \left|\int_0^2 x^3 - x^2 - 2x\,dx\right|$

$= \left[\frac{x^4}{4} - \frac{x^3}{3} - x^2\right]_{-1}^{0} + \left|\left[\frac{x^4}{4} - \frac{x^3}{3} - x^2\right]_0^2\right|$

$= \left[0 - \left(\frac{1}{4} + \frac{1}{3} - 1\right)\right] + \left|\left(\frac{16}{4} - \frac{8}{3} - 4\right) - 0\right|$

$= \frac{5}{12} + 2\frac{2}{3}$

$= 3\frac{1}{12}$

b) i) $x^2 - 4x + 2 = x - 2$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x = 4, 1$

A $(4, 2)$ B $(1, -1)$

ii) $A = \int_1^4 (x-2) - (x^2 - 4x$

$= \int_1^4 (x - 2 - x^2 + 4x$

$= \int_1^4 (-x^2 + 5x - 4)$

$= \left[-\frac{x^3}{3} + \frac{5x^2}{2} - 4x\right.$

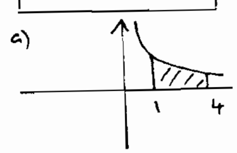
$= \left[\left(-\frac{64}{3} + 40 - 16\right) -\right.$

$= 2\frac{2}{3} - \left(-1\frac{5}{6}\right)$

$= 4\frac{1}{2}$ unit^2

Question 4

a)



$V_x = \pi\int_1^4 \left(\frac{1}{x}\right)$

$= \pi\int_1^4$

$= \pi\left[\frac{x^{-1}}{-1}\right.$

$= \pi\left[-\frac{1}{x}\right]_1^4$

$= \pi\left[-\frac{1}{4} - -1\right]$

$= \frac{3\pi}{4}$ unit^3

b) $\frac{d^2y}{dx^2} = 6x - 10$

$\frac{dy}{dx} = 3x^2 - 10x + c$ sub $x = 1$ $\frac{dy}{dx} = -1$

$-1 = 3 - 10 + c$

$-1 = -7 + c$

$\therefore c = 6$

$\frac{dy}{dx} = 3x^2 - 10x + 6$

$y = x^3 - 5x^2 + 6x + k$

sub $(1,1)$

$1 = 1 - 5 + 6 + k$

$1 = 2 + k$

$k = -1$

$\therefore y = x^3 - 5x^2 + 6x - 1$

c) $A_T = \frac{1}{2}\left[0 + 0 + 2(6 + 7 + 8)\right]$

$= 21 \text{ unit}^2$

use table to get above

x	0	1	2	3	4
y	0	6	7	8	0

d) $\int_1^b (2x - 1)\,dx = \left[x^2 - x\right]_1^b$

$= (b^2 - b) - (1 - 1)$

$\therefore b^2 - b = 6$

$b^2 - b - 6 = 0$

$(x - 3)(x + 2) = 0$

$b = 3$ $b = -2$ but $b > 0$

$\therefore b = 3$ only

Question 5

a) i) $A = \pi r^2$ $\therefore 615.75 = \pi r^2$

$r = 14.0$ cm (1 dec pl)

ii) $l = r\theta$

$= 14 \times 2\pi/3$

$= 29.3$ cm

iii) $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 14^2 \times \frac{2\pi}{3}$

$= 205.3 \text{ cm}^2$

b) $\sin x = -\cos x$

$\tan x = -1$

acute $x = \pi/4$

S ✓	A
T	C ✓

$x = \frac{3\pi}{4}, -\frac{\pi}{4}$

c)

i) $400 = 2\pi \frac{y}{2} + 2x$

$\frac{400 - \pi y}{2} = x$

$\therefore x = \frac{1}{2}(400 - \pi y)$

ii) $A = \pi \times \left(\frac{y}{2}\right)^2 + xy$

$= \frac{\pi y^2}{4} + y\,\frac{1}{2}(400 - \pi y)$

$= \frac{\pi y^2}{4} + 200y - \frac{\pi y^2}{2}$

$= 200y - \frac{\pi y^2}{4}$

iii) $\frac{dA}{dy} = 200 - \frac{2\pi y}{4}$

$= 200 - \frac{\pi y}{2}$

$\frac{d^2A}{dy^2} = -\frac{\pi}{2} < 0 \therefore$ max

st pt $\frac{dA}{dy} = 0$

$200 = \frac{\pi y}{2}$

$y = \frac{400}{\pi}$

$\therefore$ max area $= 200 \times \frac{400}{\pi} - \frac{\pi}{4}\left(\frac{400}{\pi}\right)^2$

$= \frac{80000}{\pi} - \frac{40000}{\pi}$

$= \frac{40000}{\pi}$

Max Area $= 12732 \text{ m}^2$