Name:__________________________

Class:__________________________

SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS

HSC ASSESSMENT TASK 3

JUNE 2006

Time Allowed: 70 minutes

Instructions

- Write your name and class at the top of this page, and at the top of each answer sheet.
- At the end of the examination, this examination paper must be attached to the front of your answers.
- Attempt all questions.
- All necessary working must be shown. Marks will be deducted for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- A table of standard integrals is supplied.

Q1	Q2	Q3	Q4	Q5	Total
11	12	12	12	10	57

Question 1 (11 Marks)

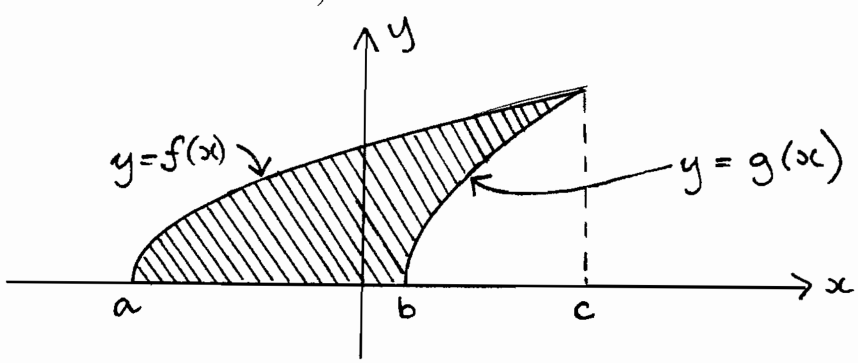
a) Find the exact value of $\sin\dfrac{2\pi}{3}$ 1

b) Find $\cos 1.5^{c}$ correct to 3 decimal places. 1

c) Express 2.25π radians in degrees. 1

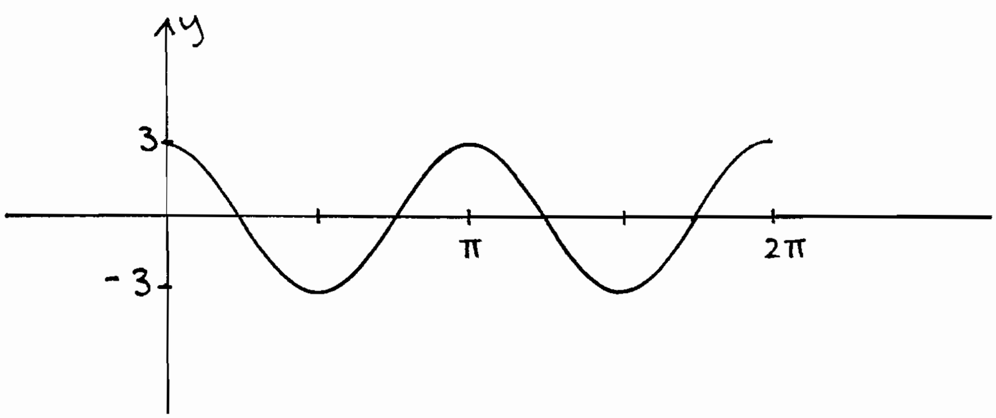
d) Find $\lim\limits_{x \to 0} \dfrac{\sin x}{2x}$ 1

e) Express the shaded area below as either the <u>sum</u> or <u>difference</u> of two integrals (correct notation must be used) 2



f) The curve below has been drawn from $x = 0$ to $x = 2\pi$. The curve has equation in the form

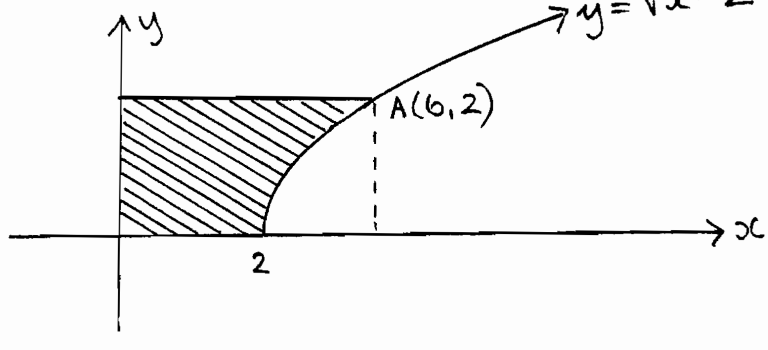
$y = a\cos bx$. Find a and b. 2



g) Draw a neat sketch of $y = f(x)$ in the domain $a \le x \le b$ given that $f'(x) > 0$ and $f''(x) > 0$ in the domain and $f(a) = 0$ 3

Question 2 **(start a new page)** **(12 marks)**

a) The curve $y = \sqrt{x-2}$ is shown below. 3

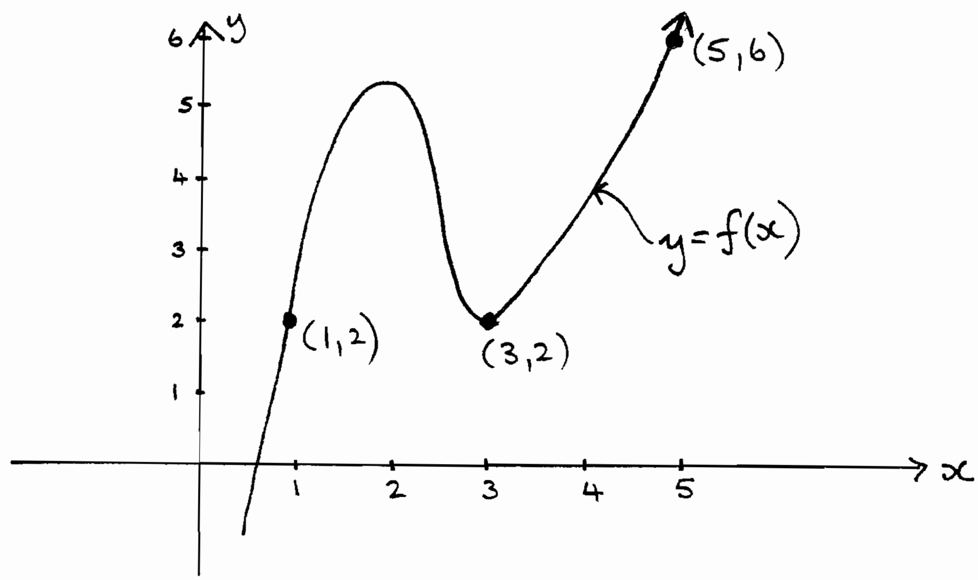


A(6,2) lies on the curve.

Find the shaded area.

b) i) Find the approximate area enclosed by the curve $y = f(x)$, the x axis and the lines $x = 1$ and $x = 5$, by using 3 function values and the Trapezoidal Rule.

The curve $y = f(x)$ is shown below. 2



ii) Is your answer in part i) an under or over estimate of the exact area.

Explain your answer. 1

c) The curve $y = \sqrt{\cos \pi x}$ from $x = 0$ to $x = \frac{1}{2}$ is rotated around the x axis. What is the volume of the solid of revolution generated? 3

d) Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x \, dx$ 3

Question 3 (start a new page) (12 marks)

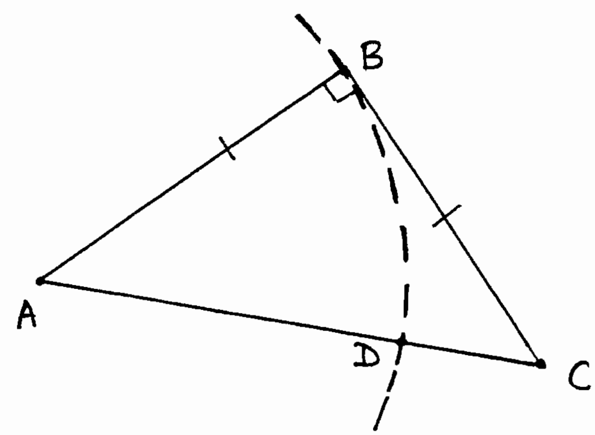
a) Solve $2\cos^2 x + 3\cos x - 2 = 0$ for x, if $0 \le x \le 2\pi$ 3

b) i) Find $\frac{d}{dx}(\sin^2 x)$ 2

ii) Find $\frac{d}{dx}(\sin x . \cos 2x)$ 2

iii) Find $\int \sin(2x+1)dx$ 2

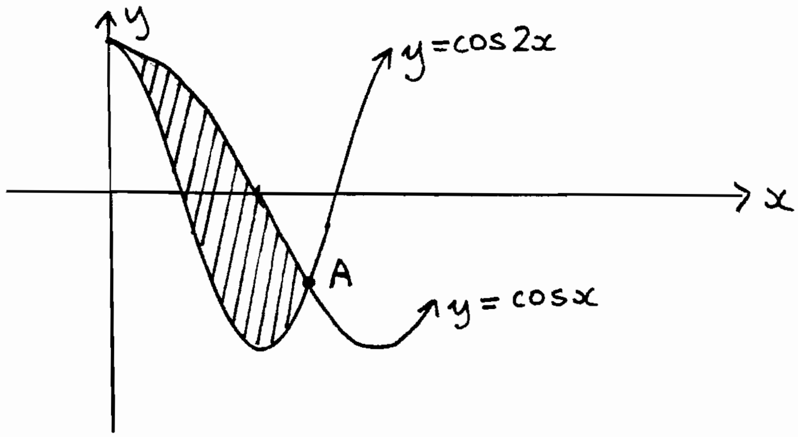
c) ABC is an isosceles right angled triangle. AB=BC=4cm. An arc, centre A and radius 4cm is drawn to cut the side AC at D.



Show the area of the portion BDC is $2(4-\pi)cm^2$ 3

Question 4 (start a new page) (12 marks)

a) The diagram shows parts of the curves $y = \cos x$ and $y = \cos 2x$ 4



The coordinates of A are $(\frac{2\pi}{3}, -\frac{1}{2})$

Show that the shaded area is $\frac{3\sqrt{3}}{4} unit^2$.

b) Prove that the curve $y = x + 2\cos x$ has a maximum turning point at x=$\frac{\pi}{6}$ in the domain $0 \le x \le \frac{\pi}{2}$ (do not sketch the curve). 4

c) i) Sketch the parabola $y = x^2 - x$ indicating where it cuts the x axis. 1

ii) The area enclosed by the parabola $y = x^2 - x$ and the x axis is rotated around the x axis. Find the volume of the solid generated. 3

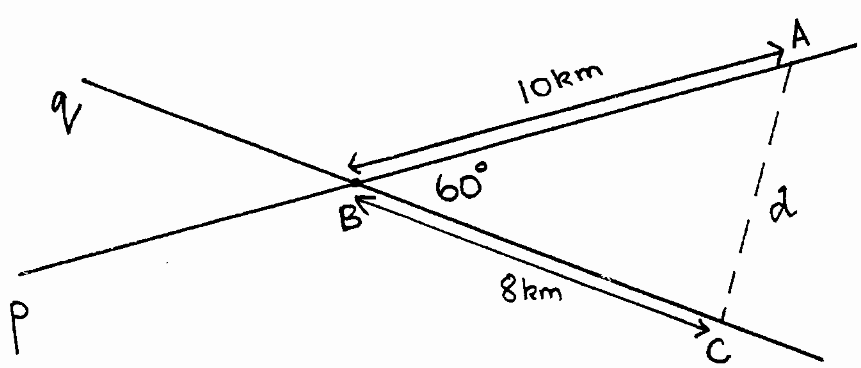
Question 5 (start a new page) 10 marks

a) A couple borrow \$320,000 at 6% p.a. The interest on the loan is compounded monthly on the balance owing. The loan is to be repaid in equal monthly instalments over 30 years. Let the monthly instalment be \$M and the amount owing after n months be $\$A_n$.

i) Find an expression for $\$A_1$, the amount owing after one month. 1

ii) Find the monthly instalment if the loan is to be fully repaid in 30 years. 4

b) Two streets p and q intersect at B at an angle pf $60°$. Andrew is at A, 10 km from B and walks towards B at 5km/h. Con is at C, 8km from B and walks towards B at 6km/h.



After t hours Andrew has walked 5t km towards B and Con has walked 6t km towards B.

i) Use the cosine rule to show the distance d between Andrew and Con can be given by $d^2 = 31t^2 - 96t + 84$ 2

ii) Hence find how many hours (correct to 2 decimal places) until Andrew and Con are the least distance apart. 3

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

TEACHERS COPY - showing suggested marks

STHS 2UNIT HSC ASSESSMENT TASK 3 JUNE 2006

Question 1

a) $\sin \frac{2\pi}{3} = \sin(\pi - \frac{\pi}{3})$

$= \sin \pi/3$

$= \sqrt{3}/2$ (1)

b) $\cos 1.5 = 0.071$ (1)

c) $2.25\pi = 405°$ (1) d) $1/2$ (1)

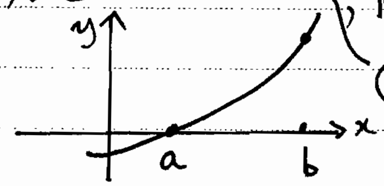
e) $A = \int_a^c f(x)\,dx - \int_b^c g(x)\,dx$ (1) (1)

f) $a = 3$ $\frac{2\pi}{b} = \pi$ $\therefore b = 2$

$\therefore y = 3\cos 2x$ (2)

g) $f'(x) > 0$ $\therefore$ +ve gradient (1)

$f''(x) > 0$ $\therefore$ conc. ↑, pt $(a,0)$ (1) (1)



Question 2

a) $A_y = \int_0^2 (y^2+2)\,dy$ (1)

$y = \sqrt{x-2}$

$\therefore y^2 = x - 2$

$y^2 + 2 = x$

$\therefore A_y = \left[\frac{y^3}{3} + 2y\right]_0^2$ (1)

$= \frac{8}{3} + 4$ (1)

$= 6\frac{2}{3}$ unit^2

bi) $A = \frac{2}{2}\left[2 + 6 + 2(2)\right]$ — (1)

$= 12$ unit^2 — (1)

ii) under estimate as area

(1) { from $x=1$ to $x=3$ becomes approximated to 4 and area above line $y=2$ is

c) $V = \pi \int_0^{1/2} \left(\sqrt{\cos \pi x}\right)^2 dx$ (1)

$= \pi \int_0^{1/2} \cos \pi x \, dx$

$= \pi \left[\frac{1}{\pi} \sin \pi x\right]_0^{1/2}$

$= \sin \frac{\pi}{2} - \sin 0$ (1)

$= 1$ unit^3 (1)

d) $\int_{\pi/6}^{\pi/3} \sec^2 x \, dx$

$= \left[\tan x\right]_{\pi/6}^{\pi/3}$ (1)

$= \tan \frac{\pi}{3} - \tan \frac{\pi}{6}$ (1)

$= \sqrt{3} - \frac{1}{\sqrt{3}}$ (1)

Question 3

a) $2\cos^2 x + 3\cos x - 2 = 0$

(1) $(2\cos x - 1)(\cos x + 2) = 0$

$\cos x = \frac{1}{2}$ $\cos x = -2$

(1) $x = \frac{\pi}{3}, \frac{5\pi}{3}$ (1) no solution

b) i) $\frac{d}{dx}(\sin x)^2 = 2\cos x . \sin x$ (2)

ii) $u = \sin x$ $v = \cos 2x$

$u' = \cos x$ $v' = -2\sin 2x$

(2) $\frac{dy}{dx} = \cos x . \cos 2x - 2\sin x . \sin 2x$

iii) $\int \sin(2x+1)\,dx$ (2)

$= \frac{1}{2}\cos(2x+1) + c$

do not take off mk if no "c"

c) $\hat{ABC} = \frac{\pi}{4}$

Area BDC = area ΔABC − sector ABD — (1)

Question 4

a) $A = \int_0^{2\pi/3} (\cos x - \cos 2x)\,dx$ — (1)

$= \left[\sin x - \frac{1}{2}\sin 2x\right]_0^{2\pi/3}$ — (1)

$= \sin\frac{2\pi}{3} - \frac{1}{2}\sin\frac{\pi}{3}$

$= \frac{\sqrt{3}}{2} + \frac{1}{2}\cdot\frac{\sqrt{3}}{2}$

(1) $= \frac{2\sqrt{3} + \sqrt{3}}{4}$

$= \frac{3\sqrt{3}}{4}$ units^2 — (1)

b) $y = x + 2\cos x$

$\frac{dy}{dx} = 1 - 2\sin x$

$\frac{d^2y}{dx^2} = -2\cos x$ — (1)

st pt $\frac{dy}{dx} = 0$ $\quad 1 - 2\sin x = 0$ — (1)

$\therefore \sin x = 1/2$

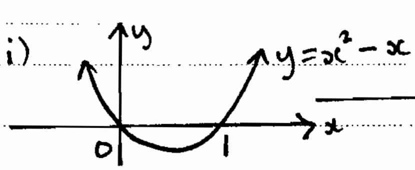
(1) — $x = \pi/6$ if $0 \le x \le \pi/2$

test max/min for $x = \pi/6$

$\frac{d^2y}{dx^2} = -2\cos\frac{\pi}{6} < 0$ — (1)

$\therefore$ max

$\therefore$ max turning pt at $x = \frac{\pi}{6}$

c) i) 

— (1)

ii) $V = \pi\int_0^1 (x^2 - x)^2\,dx$ — (1)

$= \pi\int_0^1 x^4 - 2x^3 + x^2\,dx$

$= \pi\left[x^5 \ldots - x^4 + x^3\right]^1$ — (1)

Question 5

a) \$320,000

6% pa $\Rightarrow$ .5% p.m

30 yrs $\Rightarrow$ 360 months

i) $\$A_1 = 320{,}000\left(1 + \frac{.5}{100}\right)^1 - M$

$= 320{,}000(1.005)^1 - M$ — (1)

ii) $\$A_2 = (320{,}000(1.005) - M)1.005 - M$

(1) $= 320{,}000(1.005)^2 - 1.005M - 1$

$\therefore$

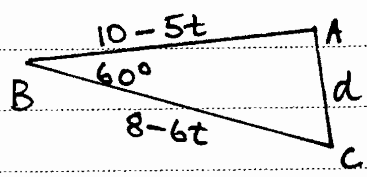
$\$A_{360} = 320{,}000(1.005)^{360} - M(1 + 1.005 + \ldots + 1.005^{359})$

(1) G.P. $a = 1$ $r = 1.005$ $n = 360$

$\$A_{360} = 0$ as loan repaid

$M\left[\frac{1(1.005^{360} - 1)}{1.005 - 1}\right] = 320{,}000(1.005)^{360}$ — (1)

$M = \$1918.56$ — (1)

b) 

(1)

i) $d^2 = (10-5t)^2 + (8-6t)^2 - 2(10-5t)(8-6t)\cos 60°$

$= 100 - 100t + 25t^2 + 64 - 96t + 36t^2$

$- (80 - 100t + 30t^2)$

$\therefore d^2 = 84 - 96t + 31t^2$ — (1)

ii) $\frac{d(d^2)}{dt} = -96 + 62t$ — (1)

$\frac{d^2(d^2)}{dt^2} = 62 > 0$ $\therefore$ minimum

st pt $-96 + 62t = 0$ $\therefore t = 1.548$ hrs

(1) $\therefore t = 1.55$ hr (2 dec pl)

STHS 2UNIT HSC ASSESSMENT TASK 3 JUNE 2006

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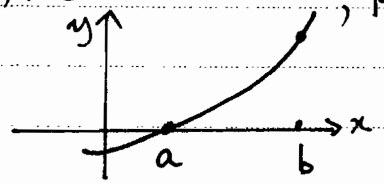

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ii) under estimate as area from $x=1$ to $x=3$ becomes approximated to 4 and area above line $y=2$ is

c) $V = \pi \int_0^{1/2} (\sqrt{\cos \pi x})^2\,dx$

$= \pi \int_0^{1/2} \cos \pi x\,dx$

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$= \sin \frac{\pi}{2} - \sin 0$

$= 1$ unit^3

d) $\int_{\pi/6}^{\pi/3} \sec^2 x\,dx$

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$(2\cos x - 1)(\cos x + 2) = 0$

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$x = \frac{\pi}{3}, \frac{5\pi}{3}$

b) i) $\frac{d}{dx}(\sin x)^2 = 2\cos x . \sin x$

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$= \frac{\sqrt{3}}{2} + \frac{1}{2}\cdot\frac{\sqrt{3}}{2}$

$= \frac{2\sqrt{3} + \sqrt{3}}{4}$

$= \frac{3\sqrt{3}}{4}$ units^2

b) $y = x + 2\cos x$

$\frac{dy}{dx} = 1 - 2\sin x$

$\frac{d^2y}{dx^2} = -2\cos x$

st pt $\frac{dy}{dx} = 0$ $\quad 1 - 2\sin x = 0$

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$x = \pi/6$ if $0 \le x \le \pi/2$

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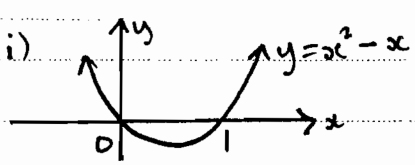
c) i)



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$= 320{,}000\,(1.005)^1 - M$

ii) $\$A_2 = \left(320{,}000\,(1.005) - M\right)1.005 - M$

$= 320{,}000\,(1.005)^2 - 1.005M - 1$

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$\$A_{360} = 320{,}000\,(1.005)^{360} - M(1 + 1.00 + \ldots + 1.005^{359})$

G.P. $\quad a = 1 \quad r = 1.005 \quad n = 360$

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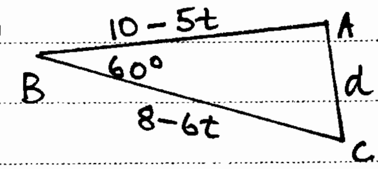
b)



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$\frac{d^2(d^2)}{dt^2} = 62 > 0 \quad \therefore$ minimum

st pt $-96 + 62t = 0 \quad \therefore t = 1.548$ hrs

$\therefore t = 1.55$ hr (2 dec pl)