SYDNEY TECHNICAL HIGH SCHOOL

Mathematics

HSC ASSESSMENT TASK 3
JUNE 2007

General Instructions

- Working time allowed – 70 minutes
- Write using black or blue pen
- Approved calculators may be used
- A table of standard integrals is provided
- All necessary working should be shown
- Start each question on a new page
- Attempt all questions

NAME :______________________

QUESTION 1	QUESTION 2	QUESTION 3	QUESTION 4	QUESTION 5	TOTAL

Question 1 (11 marks) **Marks**

a) Evaluate e^7 giving your answer correct to 3 significant figures. 2

b) Find the exact value of $\cos\dfrac{5\pi}{6}$. 1

c) Find a primitive of $3e^x + \cos x$ 2

d) Convert 0.56 radians to degrees giving your answer to the nearest degree. 1

e) Find the equation of the tangent to $y = \cos\dfrac{x}{2}$ at the point $(\pi, 0)$. 3

f) Sketch the curve $y = 4\cos 2x$ for $0 \le x \le 2\pi$. 2

Question 2 (11 marks) (Start a new page) **Marks**

a) Differentiate with respect to x :

i) $\tan x$ 1

ii) $\sin(x^2 + 1)$ 2

iii) $\dfrac{e^{2x}}{x}$ 2

b) The area bounded by $y = e^{2x}$ and the x axis from $x = 1$ to $x = 3$ 3

is rotated about the x axis. Find the volume of the solid of revolution formed.

c) Find the value of k for which $y = e^{-2x}$ satisfies the equation 3

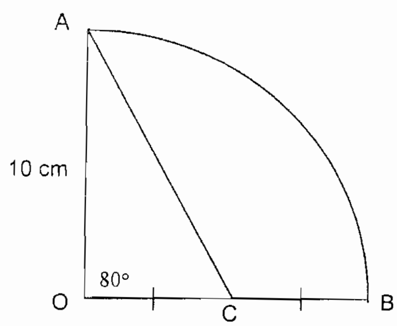
$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + ky = 0$$

Question 3 (11 marks) (Start a new page) **Marks**

a) Solve $\tan x - 1 = 0$ for $0 \le x \le 2\pi$. (answer must be in radians) 2

b) Sketch the curve $y = e^x - 1$ 1

c)



not to scale

OAB is a sector with radius 10 centimetres.

$\angle AOB = 80°$. C is the midpoint of OB.

i) Find the exact length of the arc AB. 2

ii) Find the area enclosed by the arc AB and the lines AC and CB. 3

d) i) Two values of the function $y = \dfrac{x^2}{x+9}$ are shown in the table.

x	0	3	6	9
y	0	0.75		

Copy the table onto your answer sheet and fill in the missing values. 1

ii) Use the trapezoidal rule with 3 intervals (4 function values) to estimate 2

$$\int_0^9 \frac{x^2}{x+9}\,dx$$

Question 4 (11 marks) (Start a new page) **Marks**

a) i) State the period of the function $y = \sin(2x - \pi)$. 1

ii) State the amplitude of the function $y = \sin(2x - \pi)$ 1

b) i) Use the standard integral table to find $\int \sec 2x \tan 2x \, dx$ 1

ii) Find $\int e^{4x} + 1 \, dx$ 1

c) i) Evaluate $\int_0^{\frac{\pi}{8}} \sec^2 2x \, dx$ 2

ii) Evaluate $\int_0^2 \sin \frac{\pi x}{4} \, dx$ 2

d) Find the area enclosed by the curve $y = \sin x$ for $0 \le x \le 2\pi$, 3

the line $y = 1$ and the y axis.

Question 5 (11 marks) (Start a new page) **Marks**

a) i) On the same set of axes draw neat sketches of the functions 2

$y = \sin x$ for $-2\pi \le x \le 2\pi$ and $y = 1 - \dfrac{x}{4}$

ii) How many solutions does the equation $\sin x = 1 - \dfrac{x}{4}$ have ? 1

b) The curve $y = x + \cos x$ has one stationary point for x between 0 and 2π. 4

Find this stationary point and determine its nature.

c) i) Find $\dfrac{d}{dx}(x e^{2x})$ 2

ii) Use the above result to find $\int x e^{2x}\, dx$ 2

End of Paper.

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \, \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

SOLUTIONS - ASS TASK 3 2007

QUESTION 1

a. 1100

b. $-\frac{\sqrt{3}}{2}$

c. $3e^{x} + \sin x$

d. 32°

e. $y' = -\frac{1}{2}\sin\frac{x}{2}$

when $x = \pi$

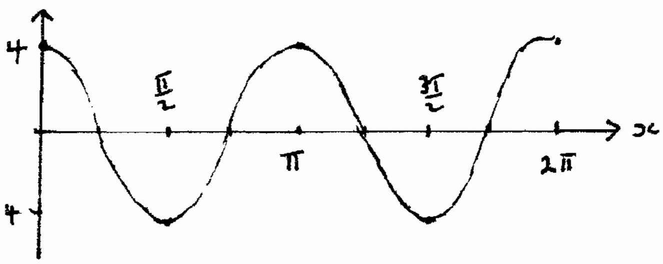
$y' = -\frac{1}{2}$

$\therefore m_T = -\frac{1}{2} \quad (\pi, 0)$

$y - 0 = -\frac{1}{2}(x - \pi)$

$x + 2y = \pi$

f.



QUESTION 2

a. i) $\sec^2 x$

ii) $2x\cos(x^2+1)$

iii) $\frac{e^{2x}(2x-1)}{x^2}$

b. $V = \pi\int_1^3 e^{4x}\,dx$

$= \frac{\pi}{4}\left[e^{4x}\right]_1^3$

$= \frac{\pi}{4}\left[e^{12} - e^{4}\right]$

c. $y' = -2e^{-2x} \quad y'' = 4e^{-2x}$

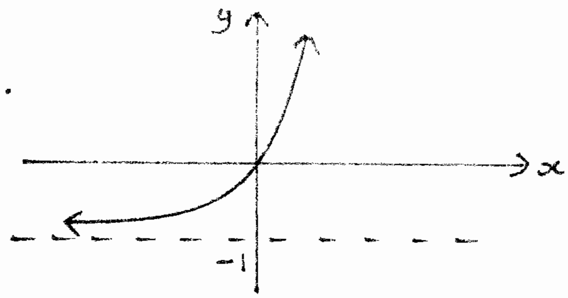
$4e^{-2x} + 3(-2e^{-2x}) + ke^{-2x} = 0$

$e^{-2x}(4 - 6 + k) = 0$

QUESTION 3

a. $x = \frac{\pi}{4}, \frac{5\pi}{4}$

b.



c. i) $80^{\circ} = \frac{4\pi}{9}$

$l = 10 \times \frac{4\pi}{9}$

$= \frac{40\pi}{9}$ cm

ii)

$A = \frac{1}{2}\cdot 10^2 \cdot \frac{4\pi}{9} - \frac{1}{2}\cdot 10 \cdot 5 \cdot \sin 80^{\circ}$

$= 45.2$ cm^2

d. i)

x	0	3	6	9
y	0	0.75	2.4	4.5

ii) $\int_0^9 \frac{x^2}{x+9}\,dx$

$\approx \frac{3}{2}\left[0 + 4.5 + 2\times(0.75 + 2.4)\right]$

$= 16.2$

Question 4

a. i) Period = π

ii) Amplitude = 1

b. i) $\frac{1}{2}$ Sec $2x + c$

ii) $\frac{1}{4} e^{4x} + x + c$

c. i) $\int_0^{\frac{\pi}{8}} \text{Sec}^2 2x \, dx$

$= \frac{1}{2} \tan 2x \Big]_0^{\frac{\pi}{8}}$

$= \frac{1}{2} \tan \frac{\pi}{4} - \frac{1}{2} \tan 0$

$= \frac{1}{2}$

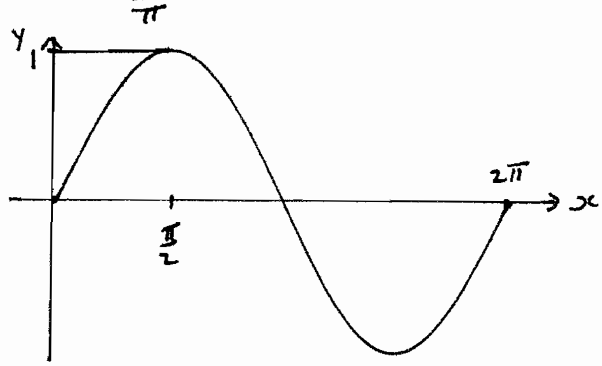
ii) $\int_0^2 \text{Sin} \frac{\pi x}{4} \, dx$

$= \left[-\frac{4}{\pi} \text{Cos} \frac{\pi x}{4} \right]_0^2$

$= -\frac{4}{\pi} \left[\text{Cos} \frac{\pi}{2} - \text{Cos } 0 \right]$

$= \frac{4}{\pi}$

d.



$A = \frac{\pi}{2} - \int_0^{\frac{\pi}{2}} \text{Sin } x \, dx$

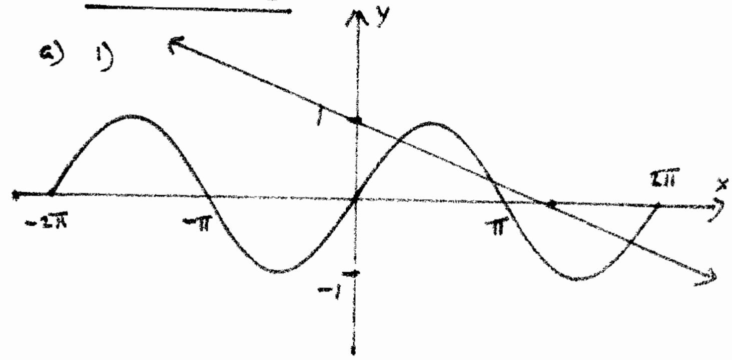
$= \frac{\pi}{2} - \left[-\text{Cos } x \right]_0^{\frac{\pi}{2}}$

$= \frac{\pi}{2} - \left[-\text{Cos} \frac{\pi}{2} + \text{Cos } 0 \right]$

$= \frac{\pi}{2} - 1$ sq units

Question 5

a) i)



ii) 3 solutions

b) $y = x + \text{Cos } x$

$y' = 1 - \text{Sin } x$

st. pts when $y' = 0$

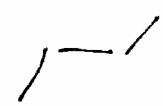
$\text{Sin } x = 1$

$x = \frac{\pi}{2}$

$y = \frac{\pi}{2}$

test

x	$\frac{\pi}{2}^{-ve}$	$\frac{\pi}{2}$	$\frac{\pi}{2}^{+ve}$
y'	+ve	0	+ve



∴ horizontal point of inflexion at $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

c) i) $\frac{d}{dx}\left(xe^{2x}\right) = e^{2x} + 2xe^{2x}$

ii) $\int x e^{2x} \, dx$

$= \frac{1}{2}\left(x e^{2x} - \int e^{2x} dx \right)$

$= \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} + c$