SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS

HSC ASSESSMENT TASK 3

JUNE 2008

Time Allowed: 70 minutes

Instructions:

- Write using blue or black pen
- Approved calculators may be used
- Attempt all questions
- All necessary working must be shown. Mark may not be awarded for careless or badly arranged work
- Marks indicated are a guide only and may be varied if necessary
- Start each question on a new side of a page
- A table of standard integrals is supplied

Name:

Q1	Q2	Q3	Q4	Q5	Total

Question 1 **(11 marks)** **Marks**

a) Write 100° in radians in terms of π 1

b) Evaluate $log_{10}5$ correct to 3 significant figures 1

c) Find $lim \lim_{x \to 0} \frac{sin2x}{x}$ 2

d) Solve $cosx = \frac{\sqrt{3}}{2}$ for $0 \leq x \leq 2\pi$ 2

e) Sketch $y = 2\sin(\pi x)$ over the domain $0 \leq x \leq 2$ 2

f) If $\log_4 Y = 3.22$ evaluate $\log_4 4Y$ 2

g) Find the exact value of $sin\frac{7\pi}{4}$ 1

Question 2 **(11 marks)**

a) Differentiate with respect to x:

(i) $y = e^{3x}$ 1

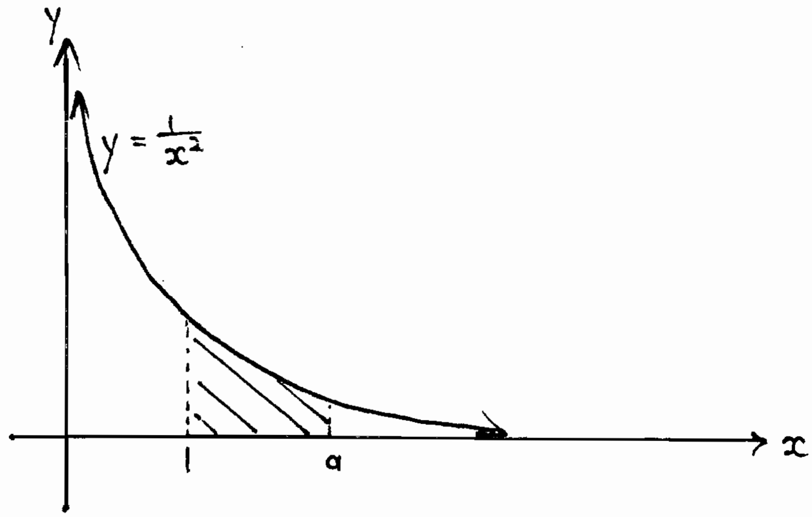
(ii) $y = \cos(1 - x^2)$ 2

(iii) $y = log_e\frac{x^2+1}{x}$ 2

(iv) $y = e^x sinx$ 2

(v) $y = 10^x$ 1

b) 3



The shaded area above is equal to $\frac{2}{3}\,unit^2$. Find a

Question 3 (11 marks)

a) Find

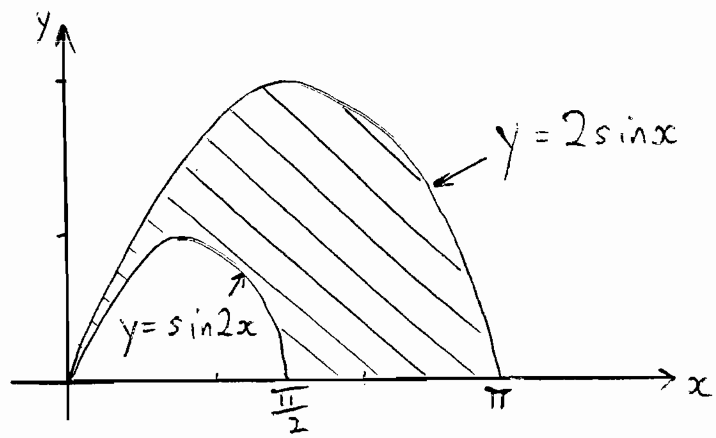
(i) $\int 2 + \frac{3}{x}\ dx$ 1

(ii) $\int sec^2(6x + 1)dx$ 1

(iii) $\int 3e^{2x}\,dx$ 1

(iv) $\int_{\frac{\pi}{2}}^{\pi} cos\,\frac{x}{2}\,dx$ (exact value) 3

b) Calculate the area of the shaded region below. 3



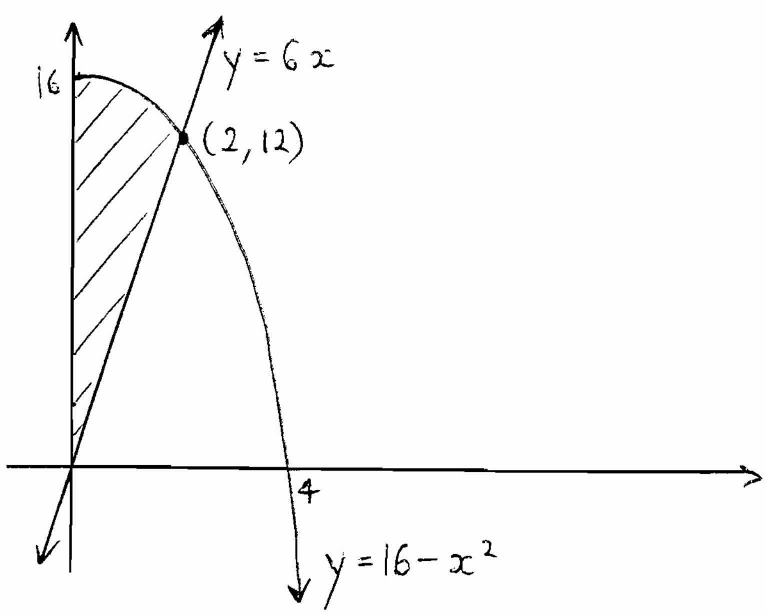
c) By writing $cosecx$ as $(sinx)^{-1}$.

Show that $\frac{d}{dx}(cosecx) = -cosecx\ cotx$ 2

Question 4 (11 marks)

a) Find $\int \sin\left(\frac{\pi}{4} - x\right) dx$ 2

b) 3



The region above is rotated around the y axis. Find the volume of the solid formed to the nearest whole number.

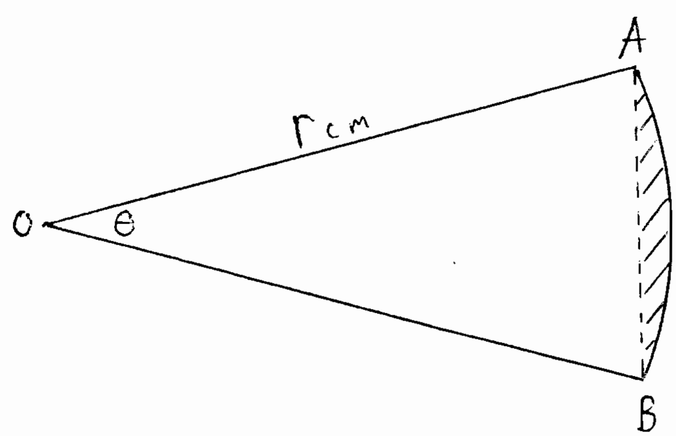
c) Evaluate $\int_0^{\frac{\pi}{3}} \frac{1+cos^3 x}{cos^2 x} dx$ 3

d) (i) Show that $\frac{d}{dx}(x \log_e x) = 1 + \log_e x$ 1

(ii) Hence find $\int \log_e x dx$ 2

Question 5 (11 marks) **Marks**

a) The sector OAB below has an area of $2\pi cm^2$. The arc has length $\frac{\pi}{2} cm$.



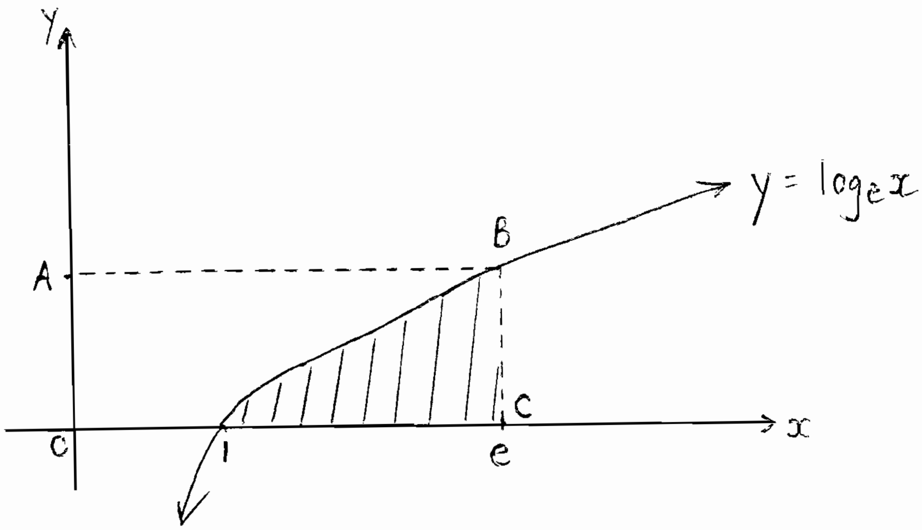
(i) Use this information to form 2 equations. 2

(ii) Hence solve these equations simultaneously to find r and θ 2

(iii) Now find the area of the minor segment shaded above 2

correct to 2 decimal places

b)



(i) Using the graph above find the y value at point B 1

(ii) Hence find the area of rectangle ABCO. 1

(iii) Hence or otherwise find the shaded area. 3

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Teacher's Name: __________ **Student's Name/Nº:** __________

Solutions to 2008 Yr 12 2 Unit Ass. Task 3

Question 1

a) $100° = 100 \times \frac{\pi}{180}$

$= \frac{5\pi}{9}$ radians (1)

b) $\log_{10} 5 = 0.6987$

$= 0.699$ to 3 sig. fig. (1)

c) $\lim_{x \to 0} \frac{\sin 2x}{x}$

$= 2 \lim_{x \to 0} \frac{\sin 2x}{2x}$

$= 2$ since $\lim_{x \to 0} \frac{\sin x}{x} = 1$. (2)

d) $\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}$ working angle

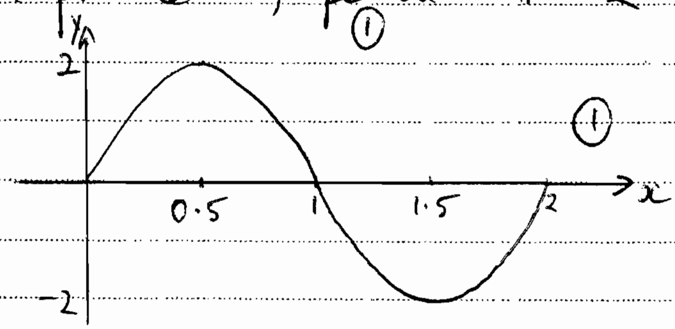
S | A ✓

T | C ✓

$x = \frac{\pi}{6}, \frac{11\pi}{6}$ (2)

e) $y = 2\sin \pi x$

amplitude $= 2$, period $= \frac{2\pi}{\pi} = 2$ (1)



(1)

f) $\log_4 4Y$

$= \log_4 4 + \log_4 Y$ (1)

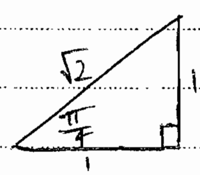
$= 1 + 3.22$

$= 4.22$ (1)

g) $\sin \frac{7\pi}{4}$

$= -\sin \frac{\pi}{4}$ (1)

$= \frac{-1}{\sqrt{2}}$ (1)



Question 2

a) (i) $y = e^{3x}$

$y' = 3e^{3x}$ (1)

(ii) $y = \cos(1 - x^2)$

$y' = -2x\sin(1 - x^2)$ (2)

(iii) $y = \log_e \frac{x^2+1}{x}$

$y = \log_e(x^2+1) - \log_e x$

$y' = \frac{2x}{x^2+1} - \frac{1}{x}$ (2)

(iv) $y = e^x \sin x$

$y' = e^x \cos x + \sin x \, e^x$

$y' = e^x(\sin x + \cos x)$ (2)

(v) $y = 10^x$

$y' = 10^x \log_e 10$ (1)

Teacher's Name: ______ **Student's Name/Nº:** ______

b) $\int_1^a \frac{1}{x^2}\,dx = \frac{2}{3}$ ①

$\left(\frac{-1}{x}\right]_1^a = \frac{2}{3}$ ①

$\frac{-1}{a} - \left(\frac{-1}{1}\right) = \frac{2}{3}$

$\frac{-1}{a} = \frac{-1}{3}$

$a = 3$ ①

(iii) $\int 3e^{2x}\,dx$

$= \frac{3}{2}e^{2x} + c$ ①

Question 3

a) (i) $\int 2 + \frac{3}{x}\,dx$

$= 2x + 3\log_e x + c$ ①

(ii) $\int \sec^2(6x+1)\,dx$

$= \frac{1}{6}\tan(6x+1) + c$ ①

(iv) $\int_{\frac{\pi}{2}}^{\pi} \cos\frac{x}{2}\,dx$

$= \int_{\frac{\pi}{2}}^{\pi} \cos\frac{1}{2}x\,dx$ ①

$\left[2\sin\frac{1}{2}x\right]_{\frac{\pi}{2}}^{\pi}$ ①

$2\left[\sin\frac{\pi}{2} - \sin\frac{\pi}{4}\right]$

$2\left(1 - \frac{1}{\sqrt{2}}\right)$ ①

b) $\text{Area} = \int_0^{\pi} 2\sin x\,dx - \int_0^{\frac{\pi}{2}} \sin 2x\,dx$ ①

$= \left[-2\cos x\right]_0^{\pi} - \left[-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{2}}$ ①

$= \left(-2\cos\pi - -2\cos 0\right) - \left(\frac{-1}{2}\cos\pi - -\frac{1}{2}\cos 0\right)$

$= (2 + 2) - \left(\frac{1}{2} + \frac{1}{2}\right)$

$= 3$ ①

c) $\frac{d}{dx}(\operatorname{cosec} x)$

$= \frac{d}{dx}(\sin x)^{-1}$

$= -(\sin x)^{-2} \times \cos x$ ①

$= \frac{-\cos x}{\sin^2 x}$

$= \frac{-\cos x}{\sin x} \times \frac{1}{\sin x}$

Question 4

a) $\int \sin\left(\frac{\pi}{4} - x\right)dx$

$= +\cos\left(\frac{\pi}{4} - x\right) + c$ ①

Teacher's Name: **Student's Name/N°:**

b) $V = \pi \int_0^{12} x^2\,dy + \pi \int_{12}^{16} x^2\,dy$

$= \pi \int_0^{12} \left(\frac{y}{6}\right)^2 dy + \pi \int_{12}^{16} 16 - y\,dy$ (1)

$= \frac{\pi}{36}\left[\frac{y^3}{3}\right]_0^{12} + \pi\left[16y - \frac{y^2}{2}\right]_{12}^{16}$ (1)

$= \frac{\pi}{36}\left(\frac{1728}{3}\right) + \pi\left[256 - 128 - (192 - 72)\right]$ (1)

$= 75 \text{ units}^3$ (1)

c) $\int_0^{\frac{\pi}{3}} \frac{1+\cos^3 x}{\cos^2 x}\,dx$

$\int_0^{\frac{\pi}{3}} \sec^2 x + \cos x\,dx$ (1)

$\left[\tan x + \sin x\right]_0^{\frac{\pi}{3}}$ (1)

$\tan\frac{\pi}{3} + \sin\frac{\pi}{3}$

$\sqrt{3} + \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$ (1)

or $2 \cdot 6 / 2 \cdot 59$

d) (i) $\frac{d}{dx}(x \log_e x) = x \times \frac{1}{x} + \log_e x$

$= 1 + \log_e x$ (1)

(ii) $\log_e x = \frac{d}{dx}(x\log_e x) - 1$ from part (i)

$\int \log_e x\,dx = \int \frac{d}{dx}(x\log_e x)\,dx - \int dx$ (1)

$= x\log_e x - x + c$ (1)

Question 5

(i) $A = \frac{1}{2}r^2\theta$

$2\pi = \frac{1}{2}r^2\theta$ (1)

$l = r\theta$

$\frac{\pi}{2} = r\theta$ (1)

(ii) $\theta = \frac{4\pi}{r^2}$ sub. into

$\frac{\pi}{2} = r\theta$

$\frac{\pi}{2} = r \times \frac{4\pi}{r^2}$

$r = 8\text{ cm}$ (1) $\therefore \theta = \frac{\pi}{16}$ (1)

(iii) $A = \frac{1}{2}r^2(\theta - \sin\theta)$ (1)

$= \frac{1}{2} \times 8^2\left(\frac{\pi}{16} - \sin\frac{\pi}{16}\right)$

$= 0 \cdot 04\text{ cm}^2$ (1)

Teacher's Name: **Student's Name/N°:**

b) At B, $x = e$

$\therefore y = \log_e e$

$= 1$ ①

(ii) Area $= 1 \times e$

$= e$ ①

(iii) Area = Rectangle $- \int x \, dy$

$= e - \int_0^1 e^y \, dy$ ①

$= e - [e^y]_0^1$ ①

$= e - [e - e^0]$

$= e - e + 1$

$= 1$ unit^2 ①