Name: .... **Maths Class:** ....

SYDNEY TECHNICAL HIGH SCHOOL

YEAR 12 HSC COURSE

Mathematics

June 2010

TIME ALLOWED: 70 minutes

Instructions:

- Write your name and class at the top of this page, and on all your answer sheets.
- Hand in your answers attached to the rear of this question sheet.
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Marks indicated are a guide only and may be varied at the time of marking

(FOR MARKERS USE ONLY)

1	2	3	4	5	TOTAL
/12	/10	/11	/10	/11	/54

QUESTION 1: (12 Marks)

Marks

(a) Find $\int \frac{dx}{\sqrt{x}}$ **2**

(b) Give the exact value of $\int_0^{\frac{\pi}{8}} \sec^2 2x\,dx$ **3**

(c) Give the exact value of $\cos\frac{7\pi}{4}$ **1**

(d) Solve the equation $\tan x = -\sqrt{3}$ for all values of x in the Domain $0 \le x \le \pi$ **2**

(e) The minute hand of a clock is 3 cm in length. What area is swept out by the minute hand over a 40 minute period? **2**

(f) Find $\frac{d}{dx}\sin(3x^2 + 5)$ **2**

QUESTION 2: (10 Marks)

Marks

You are given the curve $y = x^3 - 9x$

(i) Where does this curve cut the x-axis? **1**

(ii) Find all stationary points on the curve, and determine their nature. **4**

(iii) Find the point of inflexion. **2**

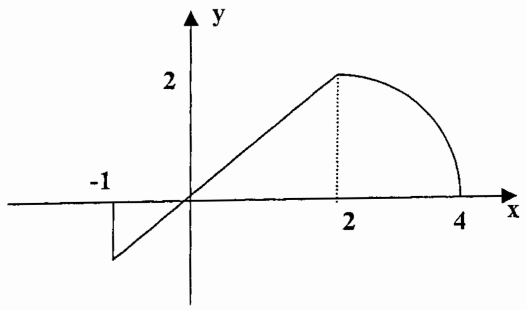
(iv) Sketch the curve showing all features you have just found. **3**

QUESTION 3: (11 Marks)

Marks

(a) The graph of $y = f(x)$ for some function $f(x)$, is drawn below:

Use this graph to find the value of $\int_{-1}^{4} f(x)dx$ **3**



(b) The value of a certain function $g(x)$ for values of x from 1 to 5 is given in the table below: **3**

x	1	2	3	4	5
$g(x)$	1	0.8	1.5	2	1.6

Using Simpson's Rule with 5 function values, find an approximation for the area enclosed by the curve $y=g(x)$, the x-axis and the lines $x=1$ and $x=5$

(c) You are given that $\int_{1}^{a} \frac{dx}{x^2} = \frac{1}{2}$ **2**

Find the value of a.

(d) Find the area between the curve $y = 4 - x^2$ and the x-axis. **3**

QUESTION 4: (10 Marks):

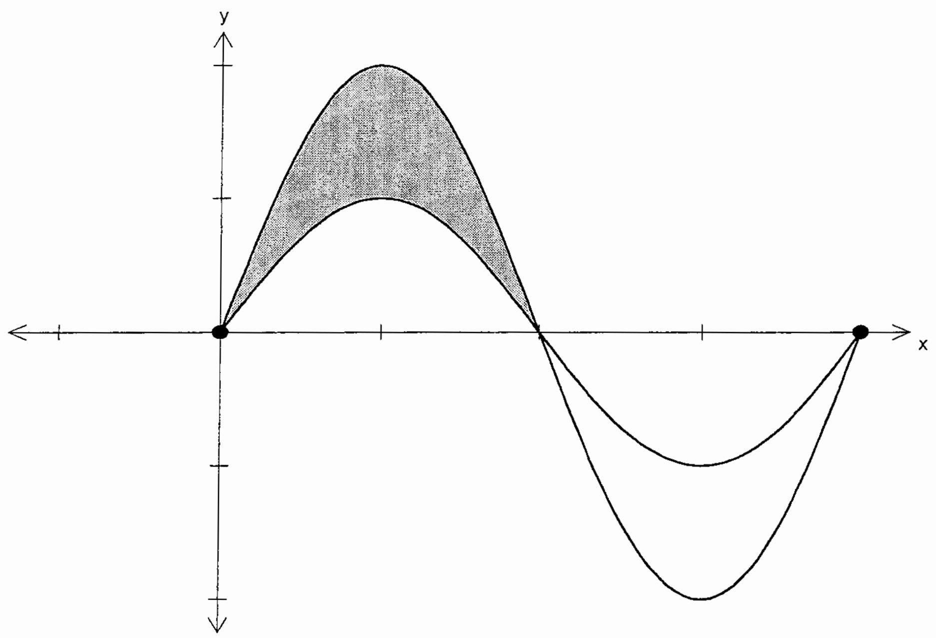
Marks

(a) An open water tank has a square base of side x m and vertical sides. It is to be built out of $75m^2$ sheet metal (the base is also metal)

(i) If V is the volume of the water tank, show that $V = \frac{x}{4}(75 - x^2)$ **2**

(ii) Find the dimensions of the tank if it is to hold as much water as possible. **4**

(b) (i) Rough sketches of the curves $y = 2\sin 2x$ and $y = \sin 2x$ are drawn below. ***Neatly*** *copy this diagram onto your answer sheet* and complete the labelling of both axes **2**



(ii) Find the value of the shaded area. **2**

QUESTION 5: (11 Marks)

Marks

(a)

(i) Draw the curve $y = \tan x$ for the Domain $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ **2**

(ii) Find the equation of the tangent to the curve $y = \tan x$ at the point where $x = \frac{\pi}{4}$ **3**

(iii) By considering your diagram, how many solutions are there of the equation **1**

$\tan x = 2x + 1 - \frac{\pi}{2}$ in the domain $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

(b)

(i) Show that $\int_0^3 (x+1)^{\frac{3}{2}} dx = \frac{62}{5}$ **2**

(ii) Explain, algebraically, why $(x+1)^{\frac{3}{2}} = x\sqrt{x+1} + \sqrt{x+1}$ **1**

(iii) Using your answers to parts (i) and (ii) above, find the value of **2**

$$\int_0^3 x\sqrt{x+1}\,dx$$

END OF EXAMINATION PAPER

2 UNIT SOLUTIONS (and MARKING)

QUESTION 1: (12 MARKS)

(a) $\int \frac{dx}{\sqrt{x}} = 2x^{1/2} + k$

2 MARKS This is the only place in the paper where mark is deducted for no "

(b) $\int_0^{\pi/8} \sec^2 2x\,dx = \frac{1}{2}\tan 2x\Big]_0^{\pi/8}$ ← 1 MARK

$= \frac{1}{2}\left(\tan\frac{\pi}{4} - \tan 0\right)$ ← 1 MARK

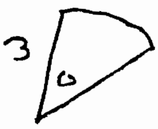
$= \frac{1}{2}.$ ← 1 MARK

(c) $\cos\frac{7\pi}{4} = \cos\frac{\pi}{4}$ 1 MARK

$= \frac{1}{\sqrt{2}}$

(d) $x = \cancel{\frac{\pi}{3}}, \frac{2\pi}{3}, \frac{5\pi}{3}$ 1 MARK each part.

(e)



40 mins $= \frac{2}{3}$ hour.

$\therefore \theta = \frac{4\pi}{3}$

$\therefore A = \frac{1}{2}(3)^2 \cdot \frac{4\pi}{3}$

$= 6\pi \text{ cm}^2$

1 MARK for $\frac{4\pi}{3}$

1 MARK for answer

(f) $\frac{d}{dx}\sin(3x^2+5) = 6x\cos(3x^2+5)$

1 for cos

1 for $6x$.

QUESTION 2: (10 MARKS)

$$y = x^3 - 9x$$

(i) Cuts at $(0,0)$ and $(3,0)$ and $(-3,0)$ — 1 ONLY

(ii) $\dfrac{dy}{dx} = 3x^2 - 9$

$\dfrac{d^2y}{dx^2} = 6x$

At T.P.'s $\dfrac{dy}{dx} = 0$

$\Rightarrow x = \sqrt{3}$ or $x = -\sqrt{3}$

$y = +6\sqrt{3}$ $\quad$ $y = -6\sqrt{3}$

$y'' > 0$ $\quad$ $y'' < 0$

$\Rightarrow$ MIN T.P. at $(\sqrt{3}, -6\sqrt{3})$ $\quad$ $\Rightarrow$ MAX T.P. at $(-\sqrt{3}, 6\sqrt{3})$

1 for each point
1 for each max/min
(ie. 4)

(iii) at I.P.'s $\dfrac{d^2y}{dx^2} = 0$

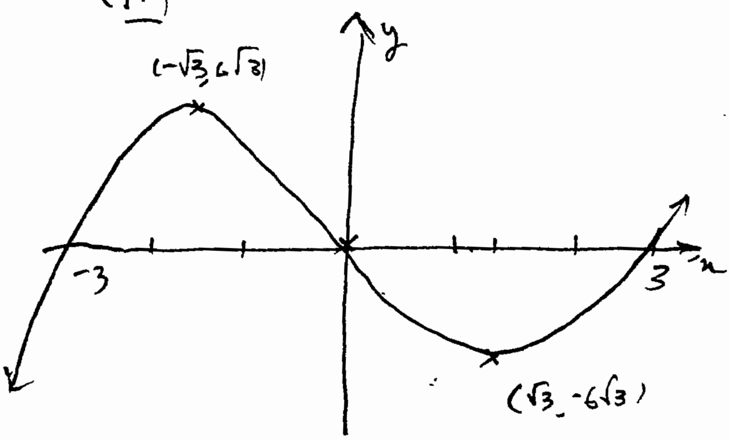
$\Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases}$ $\quad$ $-$ve | | +ve at -1, 0, 1

2nd derivative changes sign.

$\therefore$ I.P. at $(0,0)$

1 for $(0,0)$
1 for derivative changing sign

(iv)



1 for x-axis intercepts
1 for T.P.'s
1 for I.P. at $(0,0)$

QUESTION 3:

(a) $A_1 = \frac{1}{2}(1\times 1) = \frac{1}{2}$

$A_2 = \frac{1}{2} \times 2 \times 2 = 1$

$A_3 = \frac{1}{4} \times \pi \times (2)^2$

$= \pi$.

$\therefore$ ~~Area~~

$\therefore$ Integral $= -\frac{1}{2} + 1 + \pi$

$= \pi + \frac{1}{2}$

3 MARKS

2 MARKS for the areas plus 1 for adding two and subtracting one area

(ie only 2 for $\pi + \frac{3}{2}$)

(b) $A_1 = \frac{1}{3}\left[1 + 1.5 + 4 \times 0.8\right] = 1.9$

$A_2 = \frac{1}{3}\left[1.5 + 1.6 + 4 \times 2\right] = 3.7$

$\therefore A \approx 5.6\ u^2$

1 for each part

OR

There are other S.R. formulae

(c) $\displaystyle\int_1^a \frac{dx}{x^2} = \left[-x^{-1}\right]_1^a$

$= -\frac{1}{a} + 1 = \frac{1}{2}$

$\therefore \frac{1}{a} = \frac{1}{2}$

$\therefore a = 2$

1 for correct integral

1 for $a = 2$

(d) Cuts x-axis at $(2,0)$ and $(-2,0)$

$\therefore A = \displaystyle\int_{-2}^{2} 4 - x^2\, dx$

$= \left[4x - \frac{1}{3}x^3\right]_{-2}^{2}$

$= 8 - \frac{8}{3} - \left(-2 + \frac{8}{3}\right)$

$= \frac{32}{3}\ u^2$

1 for limits of $x = 2$, $x = -2$

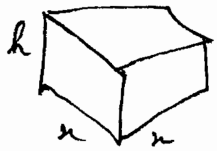
1 for correct integration

1 for answer

CAN ALSO BE DONE by $2\displaystyle\int_0^2 4 - x^2\, dx$.

QUESTION 4:

(a)



(i) $SA = x^2 + 4xh = 75$

$VOL = x^2 h$

$= x^2\left(\dfrac{75 - x^2}{4x}\right)$

$= \dfrac{x}{4}\left(75 - x^2\right)$

(1) for getting $h = \dfrac{75 - x^2}{4x}$.

(1) for substituting into volume formula

(ii) $\dfrac{dV}{dx} = \dfrac{75}{4} - \dfrac{3x^2}{4}$

$\dfrac{d^2V}{dx^2} = -\dfrac{6x}{4}$

$= -3x/2$

(1) for both $\dfrac{dV}{dx}$ and $\dfrac{d^2V}{dx^2}$

At max vol, $\dfrac{dV}{dx} = 0$

(1) for recognising this

$\Rightarrow \quad 3x^2 = 75$

$x = 5$ ~~or~~ -5 ← NOT A SOL^N

$h = 5/2$

$V = \dfrac{125}{2}$

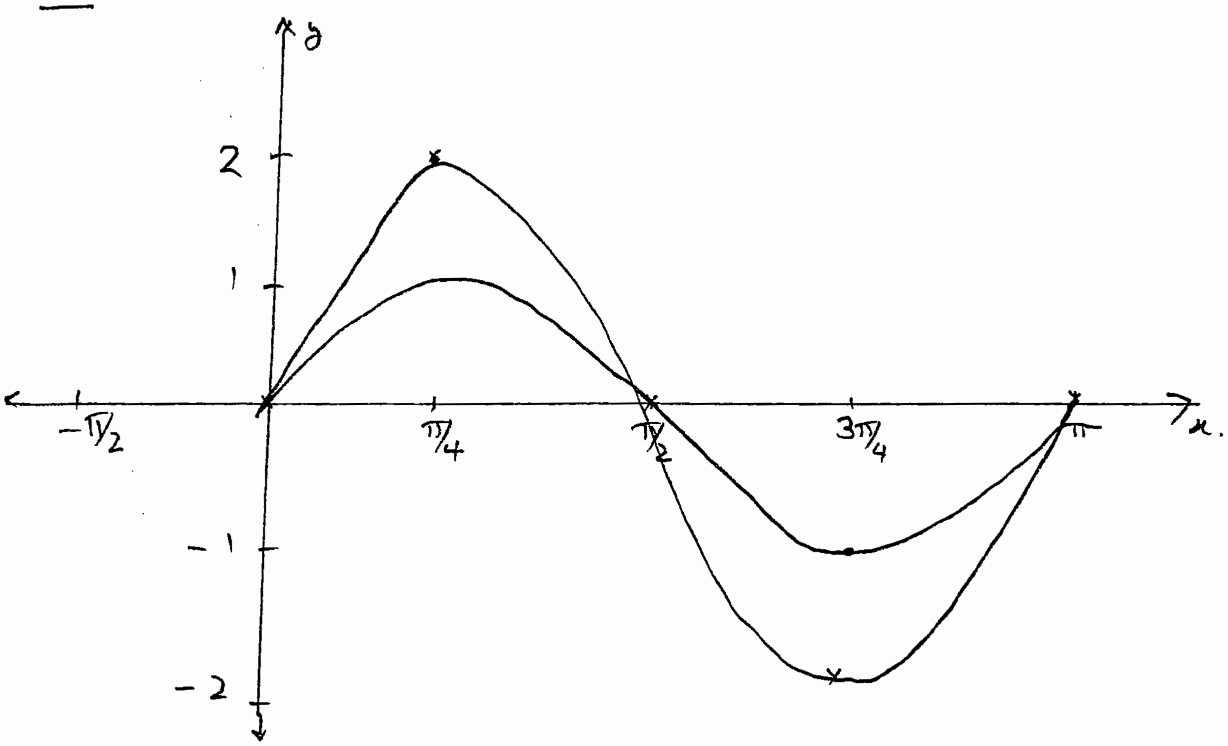
$V'' < 0 \Rightarrow$ MAX.

(1) for testing MAX

$\therefore$ Tank is $5 \times 5 \times 5/2$ m.

(1) for dimensions

4 (b)



2 MARKS

1 for vertical axis labelling

1 for horizontal axis labelling

[DON'T BE TOO TOUGH – as long as they give the idea they know what the dimensions are!]

(ii)

$$A = \int_0^{\pi/2} 2\sin 2x - \sin 2x \, dx$$ ← 1 for this line

$$= \left[-\frac{1}{2}\cos 2x\right]_0^{\pi/2}$$

$$= -\tfrac{1}{2}\cos\pi + \tfrac{1}{2}\cos 0$$

$$= 1 - \tfrac{1}{2}$$

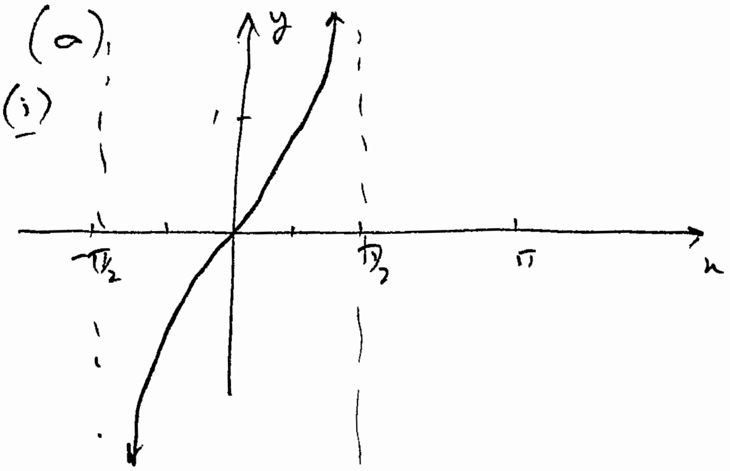
$$= \tfrac{1}{2}\,u^2$$

1 for answer

QUESTION 5:

(a)

(i)



(1) for assymptotes

(1) for shape

(ii) $\frac{dy}{dx} = \sec^2 x.$ ← (1)

At $x=\pi/4$ $\quad m_T = \sec^2 \pi/4$

$= 2$ ← (1)

∴ Equation is

$y - 1 = 2(x - \pi/4)$

∴ $y = 2x - \pi/2 + 1$ ← (1)

(iii) TWO ← (1) RIGHT OR WRONG.

(b) (i) $\int_0^3 (x+1)^{3/2}\,dx = \frac{2}{5}(x+1)^{5/2}\Big]_0^3$ ← (1) for integral

$= \frac{2}{5}(4)^{5/2} - \frac{2}{5}$

$= 62/5$

(1) for process

(ii) $(x+1)^{3/2} = (x+1)\sqrt{x+1}$

$= x\sqrt{x+1} + \sqrt{x+1}$

anything acceptable = (1)

(iii) $\int_0^3 x\sqrt{x+1}\,dx = \int_0^3 (x+1)^{3/2}\,dx - \int_0^3 \sqrt{x+1}\,dx$ ← (1) for reasoning thi

$= 62/5 - \left[\frac{2}{3}(x+1)^{3/2}\right]_0^3$

$= 62/5 - 16/3 + 2/3 = \underline{26/15}$ (1)