SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS

HSC ASSESSMENT TASK 3

JUNE 2011

Time Allowed: 70 minutes

Instructions:

- Write using blue or black pen.
- Approved calculators may be used.
- Attempt all questions.
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- Start each question on a new side of a page.
- A table of standard integrals is supplied.

Name: ______________________________

Teachers Name ______________________________

Q1	Q2	Q3	Q4	Q5	Total
/12	/12	/12	/12	/12	/60

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0 \ \text{if} \ n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, a \neq 0$$

$$\int \sec ax \tan ax dx = -\frac{1}{a} \sec ax, a \neq 0$$

$$\int \frac{1}{a \ + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, a > 0, -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln(x + \sqrt{x^2 - a^2}), x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln(x + \sqrt{x^2 + a^2})$$

Note: $\ln x = \log_e x, x > 0$

Question 1 (12 marks)

a) Express 2.37 radians in degrees and minutes (correct to nearest minute) **1 mark**

b) Find the exact value of $\sin \frac{2\pi}{3}$ **2 marks**

c) Evaluate $2\cos\frac{\pi}{5}$ correct to three significant figures **2 marks**

d) Express $135°$ in radians, in terms of π. **1 mark**

e) Evaluate $\int_0^{\frac{\pi}{12}} sec^2\ 3x\ dx$ **2 marks**

f) Differentiate $\cos(3x - 1)$ **1 mark**

g) Differentiate $2x^{-3}$ **1 mark**

h) Find the primitive of $\sqrt[3]{x^4}$? (Answer in index form) **2 marks**

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Question 2 (12 marks):

(a) Differentiate $y = 3\tan x$ **1 mark**

(b) For the curve $y = cos\ 4x$

(i) find the amplitude, **1 mark**

(ii) find the period, **1 mark**

(iii) sketch the curve for $0 \leq x \leq \pi$, showing clearly the positions of any intercepts and turning points. **2 marks**

(c) Find the following integrals:

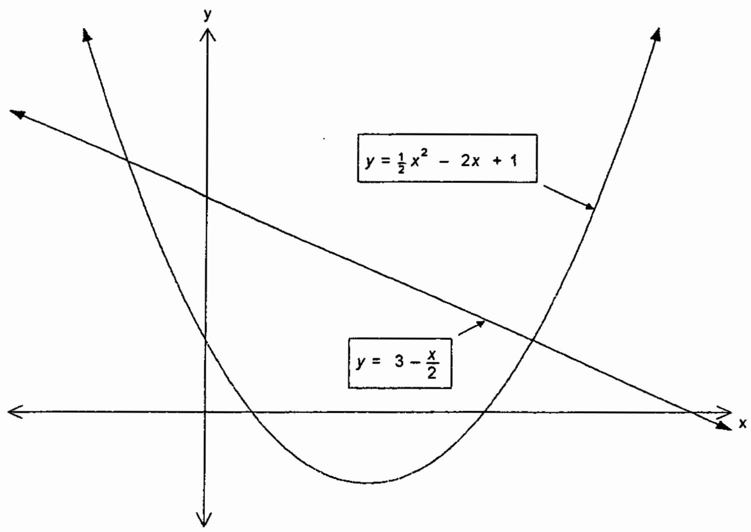
(i) $\int (x^3 - 4x^2 + 3)\,dx$ **1 mark**

(ii) $\int \frac{3x^5 + 4x^3 - 2}{2x^3}\,dx$ **2 marks**

d) Below is the graph of $y = 3 - \frac{x}{2}$ and $y = \frac{1}{2}x^2 - 2x + 1$

(i) Find the points of intersection of the two functions. **2 marks**

(ii) Find the area between the curves. **2 marks**



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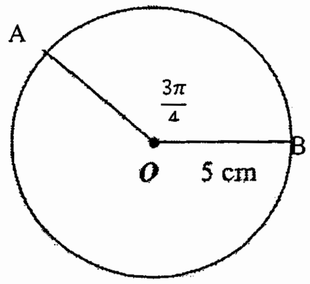
Question 3 (12 marks)

(a) Solve for the domain $0 \leq x \leq 2\pi$, giving exact answers:

i. $\sin x = -\frac{\sqrt{3}}{2}$ **2 marks**

ii. $\tan 2x = 1$ **3 marks**

(b) A circle with a centre O has a radius of 5 cm. A sector subtends an angle of $\frac{3\pi}{4}$ at the centre of the circle



i. What is the length of the arc of this sector? (give the exact answer) **2 marks**

ii. What is the area of the sector? (Answer in exact form) **2 marks**

iii. A chord is drawn from A to B. Show that the area of the minor segment that is formed is

$$\frac{25\,(3\pi - 2\sqrt{2})}{8} \ cm^2$$ **3 marks**

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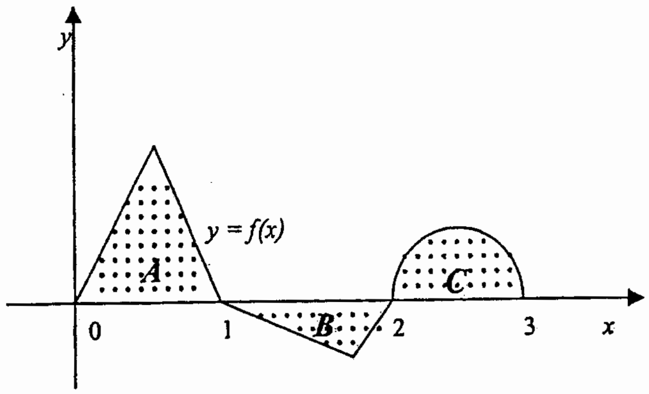
Question 4 (12 marks)

a) Consider the following diagram and question:



Given that the area shaded A is 6 units^2, area B is 2 units^2 and area C is 3 units^2, find $\int_0^3 f(x)dx$.

Maryanne's solution was:

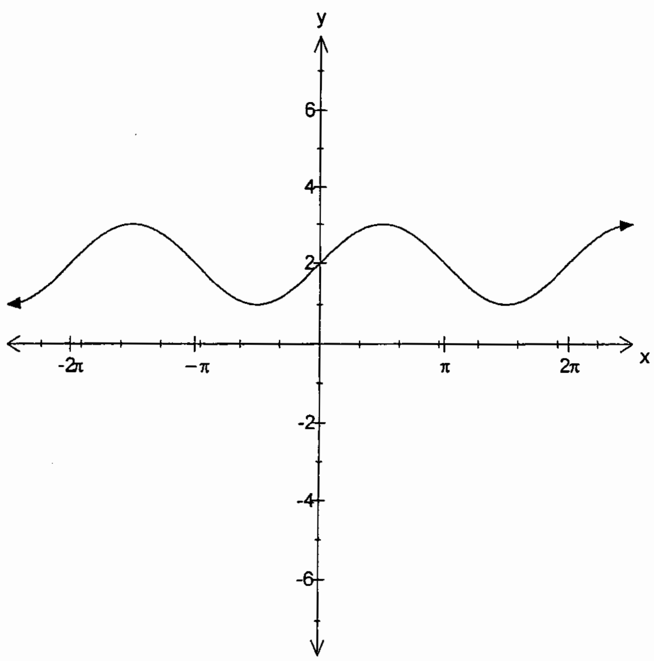
$\int_0^3 f(x)dx = 6 + 2 + 3$ (line 1)

$= 11$ units^2 (line 2)

i. Maryanne's solution has one error in EACH of her lines of working. Clearly *explain* (in words) what were her errors **2 marks**

ii. Write the correct solution **1 mark**

(b) Write an equation for this trigonometric function. **1 mark**



(c) Find the area in the first quadrant enclosed by the curve $y = x^2$, the y-axis and the line $y = 4$ **2 marks**

(d) Five values of the function $f(t)$ are shown in the table.

t	3	4	5	6	7
f(t)	11.2	9.8	12.7	13.4	20.5

3 marks

Use Simpson's rule with the five values given in the table to estimate $\int_3^7 f(t)dt$

(e) The gradient of a curve is given by $\frac{dy}{dx} = 1 - 6\sin 3x$. The curve passes through the point (0,7). What is the equation of the curve? **3 marks**

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Question 5 (12 marks)

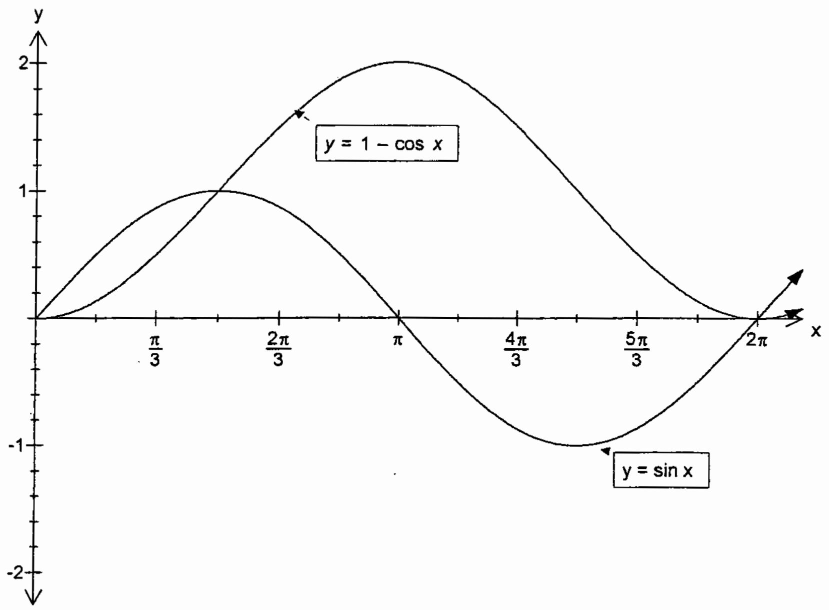
a) Differentiate $y = x^2 tan5x$ **2 marks**

b) Evaluate $\int_0^2 (2x+1)^4 dx$ **2 marks**

c) Find the equation (in exact form) of the normal to the curve **2 marks**

$y = 3\cos x$ at the point $\left(\frac{\pi}{3}, 1\frac{1}{2}\right)$

d) Below is the graph of $y = \sin x$ and $y = 1 - \cos x$ over the domain $0 \leq x \leq 2\pi$



(i) Write the values of x for which $\sin x = 1 - \cos x$ in the domain $0 \leq x \leq \pi$ **2 marks**

(ii) Evaluate the integral $\int_0^{\pi} (1 - \cos x - \sin x)\, dx$ **2 marks**

(iii) Calculate the area between $y = \sin x$ and $y = 1 - \cos x$ over the domain $0 \leq x \leq \pi$ **2 marks**

END OF TEST

SOLUTIONS.

Q1

(a) $2.37 = 135°47'$

(b) $\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

(c) $2\cos \pi/5 = 1.62$

(d) $135° = \frac{3\pi}{4}$

(e) $\int_0^{\pi/12} \sec^2 3x \, dx$

$= \left[\frac{1}{3}\tan 3x\right]_0^{\pi/12}$

$= \frac{1}{3}(\tan \pi/4 - \tan 0)$

$= 1/3$

(f) $\frac{dy}{dx} = -3\sin(3x-1)$

g) $\frac{dy}{dx} \quad -6x^{-4}$

h) $\frac{3}{7}x^{7/3} + K$

Q2 $y = 3\tan x$

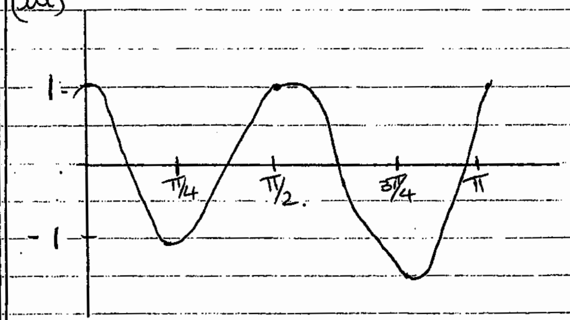
$y' = 3\sec^2 x$

(b) $y = \cos 4x$.

(i) amp $= 1$

(ii) period $= \frac{2\pi}{n} = \frac{2\pi}{4} = \frac{\pi}{2}$.

(iii)



(c) $\int(x^3 - 4x^2 + 3)\,dx = \frac{x^4}{4} - \frac{4x^3}{3} + 3x + K$

(ii) $\int \frac{3x^5}{2x^3} + \frac{4x^3}{2x^3} - \frac{2}{2x^3} \, dx$

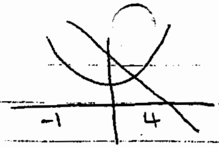
$= \int \frac{3}{2}x^2 + 2 - \frac{1}{x^3} \, dx$

$= \frac{x^3}{2} + 2x + \frac{x^{-2}}{+2} + K$.

$= \frac{x^3}{2} + 2x + \frac{1}{2x^2} + K$.

[redacted]

(a)



(d) $y = 3 - \frac{x}{2} \qquad y = \frac{1}{2}x^2 - 2x + 1$

(i) $3 - \frac{x}{2} = \frac{1}{2}x^2 - 2x + 1$

$6 - x = x^2 - 4x + 2$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4, -1$

(ii) $A = \int_{-1}^{4} 3 - \frac{x}{2} - \left(\frac{1}{2}x^2 - 2x + 1\right)dx$.

$= \int_{-1}^{4} 3 - \frac{x}{2} - \frac{1}{2}x^2 + 2x - 1 \, dx$

$= \int_{-1}^{4} 2 + \frac{3x}{2} - \frac{1}{2}x^2 \, dx$

$= \left[2x + \frac{3x^2}{4} - \frac{x^3}{6}\right]_{-1}^{4}$

$= \left[\frac{28}{3} - \frac{17}{6}\right]$

$= 10\frac{5}{12}$

$= 10.416$

Q3

(i) $\sin x = -\frac{\sqrt{3}}{2}$

$x = 4\pi/3, \ \frac{5\pi}{3}$

(ii) $\tan 2x = 1$

$2x = \pi/4, \ 5\pi/4, \ 9\pi/4, \ \frac{13\pi}{4}$

$x = \pi/8, \ \frac{5\pi}{8}, \ \frac{9\pi}{8}, \ \frac{13\pi}{8}$

(b) (i) $l = r\theta$

$= \frac{3\pi}{4} \times 5$

$= 15/4\,\pi$

(ii) $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 25 \times \frac{3\pi}{4}$

$= \frac{75\pi}{8} \quad u^2$

(iii) $A = \frac{1}{2}r^2(\theta - \sin\theta)$

$= \frac{1}{2} \times 25\left(\frac{3\pi}{4} - \sin\frac{3\pi}{4}\right)$

$= \frac{25}{2}\left(\frac{3\pi}{4} - \frac{2\sqrt{2}}{4}\right)$

$= \frac{25}{2}\left(\frac{3\pi - 2\sqrt{2}}{4}\right)$

$= \frac{25(3\pi - 2\sqrt{2})}{8} \ cm^2$.

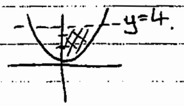
Q4

(a) (i) Maryanne has found the area while the question asked for the evaluation of an integral and she should not have used "units". Also, she should have evaluated that part of the integral from $x=1$ to $x=2$ as a negative as it is below the x axis.

(ii) $\int_0^3 f(x)\,dx = 6 + (-2) + 3$
$= 7$

(b) $y = 2 + \sin x$

(c) $y = x^2$ $\quad x = \sqrt{y}$



$= \int_0^4 y^{1/2}\,dy$

$= \left[\frac{2y^{3/2}}{3}\right]_0^4$

$= \left(\frac{2\times 2^3}{3} - 0\right)$

$= \frac{16}{3}\ u^2$

(d)
$A = \frac{2}{6}\left[f(3) + f(5) + 4f(4) + f(5) + f(7) + 4f(6)\right]$

$= \frac{1}{3}\left[11.2 + 12.7 + (4\times 9.8) + 12.7 + 20.5 + (4\times 13.4)\right]$

$= \frac{1}{3}(149.9)$

$= 49.97$

(e) $\frac{dy}{dx} = 1 - 6\sin 3x \quad (0,7)$

$y = x + 2\cos 3x + K.$

$7 = 0 + 2 + K.$

$K = 5$

$y = x + 2\cos 3x + 5$

Q5

(a) $y = x^2 \tan 5x$

$y' = uv' + vu'$

$= x^2 5\sec^2 5x + 2x\tan 5x$

(b) $\int_0^2 (2x+1)^4\,dx$

$= \int_0^2 \left[\frac{(2x+1)^5}{10}\right]_0^2 dx$

$= \left(\frac{5^5}{10}\right) - \frac{1}{10}$

$= \frac{1562}{5} \quad = 312.4$

(c) $y = 3\cos x$

$y' = -3\sin x \quad$ at $x = \pi/3$

$y' = -3\sin \pi/3$

$= \frac{-3\sqrt{3}}{2} \qquad M_N = \frac{2}{3\sqrt{3}}$

$y - \frac{3}{2} = \frac{2}{3\sqrt{3}}\left(x - \frac{\pi}{3}\right)$

$y = \frac{2x}{3\sqrt{3}} - \frac{2\pi}{9\sqrt{3}} + \frac{3}{2}.$

(d) (i) $\pi/2,\ 0$

(ii) $\int_0^\pi (1 - \cos x - \sin x)\,dx$

$= [x - \sin x + \cos x]_0^\pi$

$= (\pi - \sin\pi + \cos\pi) - (0 - \sin 0 + \cos 0)$

$= (\pi - 0 - 1) - (0 - 0 + 1)$

$= (\pi - 1) - (1)$

$= \pi - 2$

(iii) $\int_0^{\pi/2} \sin x - (1 - \cos x) + \int_{\pi/2}^{\pi} 1 - \cos x - \sin x\ dx$

$= [-\cos x - x + \sin x]_0^{\pi/2} + [x - \sin x + \cos x]_{\pi/2}^{\pi}$

$= (\cos(\pi/2) - \frac{\pi}{2} + \sin \pi/2) + (-\cos 0 - 0 + \sin 0) + (\pi - \sin\pi + \cos\pi) - (\pi/2 - \sin \pi/2 + \cos \pi/2)$

$= (0 - \pi/2 + 1) - (-1 - 0 + 0) + (\pi - 0 + -1) - (\pi/2 - 1 + 0)$

$= (-\pi/2 + 1) - (-1) + (\pi - 1) - (\pi/2 - 1)$

$= -\pi/2 + 1 + 1 + \pi - 1 - \pi/2 + 1$

$= 2\ u^2$