Name: .... Maths Class: ....

SYDNEY TECHNICAL HIGH SCHOOL

Year 12

Mathematics

HSC Course

Assessment 3

June, 2015

Time allowed: 90 minutes

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- **Begin each question on a new page**
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- A set of Standard Integrals is provided at the rear of this Question Booklet, and may be removed at any time.

Section 1 Multiple Choice
Questions 1-10
10 Marks

Section II Questions 11-15
48 Marks

Section 1 **Objective response Questions**
Total Marks – 5
Attempt 1-5

Answer each question on the multiple choice sheet provided

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1. Which term represents the distance that $y = a\sin(bx)$ extends out from the centre of its graph on the y-axis?

   (A) Amplitude

   (B) Domain

   (C) Period

   (D) Range

2. What are the solutions of $2\cos x = -\sqrt{3}$ for $0 \leq x \leq 2\pi$?

   (A) $\frac{\pi}{6}$ and $\frac{\pi}{6}$

   (B) $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$

   (C) $\frac{\pi}{3}$ and $\frac{2\pi}{3}$

   (D) $\frac{\pi}{6}$ and $\frac{7\pi}{6}$

3. What is the derivative of $e^x\left(x^2 + 2x\right)$?

   (A) $(2x + 2)$

   (B) $e^x(2x + 2)$

   (C) $e^x\left(x^2 - 2\right)$

   (D) $e^x\left(x^2 + 4x + 2\right)$

4. A chord of length 5 cm is drawn in a circle of radius 6 cm. The area of the smaller region inside the circle cut off by the chord, correct to one decimal place, is:

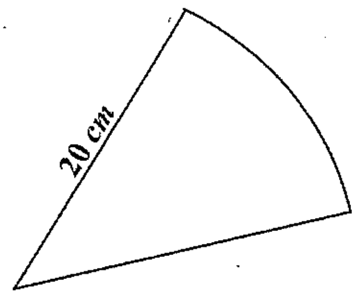
(A) 1.8 cm^2

(B) 2.3 cm^2

(C) 13.6 cm^2

(D) 15.5 cm^2

5. What is the perimeter, P, of the sector below with an angle 36° and radius 20cm?



(A) $P = 0.5 \times 400 \times \left(\frac{\pi}{5} - \sin\frac{\pi}{5}\right)$ *cm*

(B) $P = \left(0.5 \times 400 \times \frac{\pi}{5}\right)$ *cm*

(C) $P = (40 + 36^\circ)$ *cm*

(D) $P = (40 + 4\pi)$ *cm*

6. What is the value of $\int_0^1 \left(e^{3x} - 1\right) dx$?

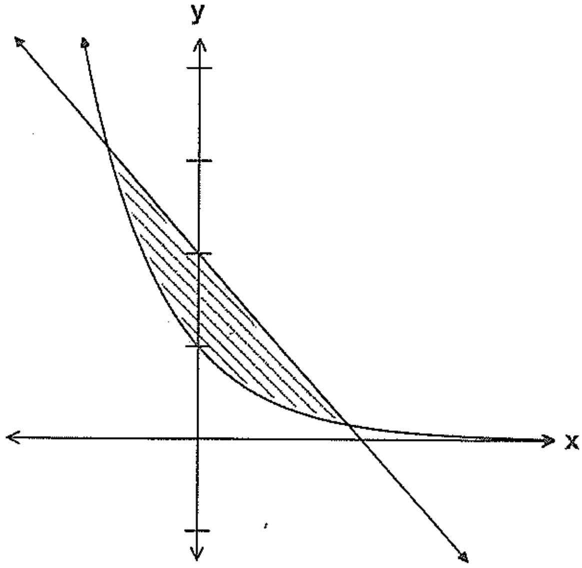
(A) $\frac{e^3}{3}$

(B) $\frac{e^3}{3} - 1$

(C) $e^3 - 1$

(D) $\frac{1}{3}\left(e^3 - 4\right)$

7. The diagram shows the region enclosed by $x + y = 2$ and $y = e^{-x}$



Which of the following pair of inequalities describes the shaded region in the diagram?

(A) $x + y \leq 2$ and $y \leq e^{-x}$

(B) $x + y \leq 2$ and $y \geq e^{-x}$

(C) $x + y \geq 2$ and $y \leq e^{-x}$

(D) $x + y \geq 2$ and $y \geq e^{-x}$

8. What is the greatest value taken by the function $f(x) = 4 - 2\cos x$ for $x \geq 0$?

(A) 2

(B) 4

(C) 6

(D) 8

9. The values for a continuous function are given in the table below.

x	0	1	2	3	4	5	6	7	8
$f(x)$	15	12.5	6	-3	-5	2	3.5	7.5	10

The trapezoidal rule approximation for $\int_0^8 f(x)\,dx$ is:

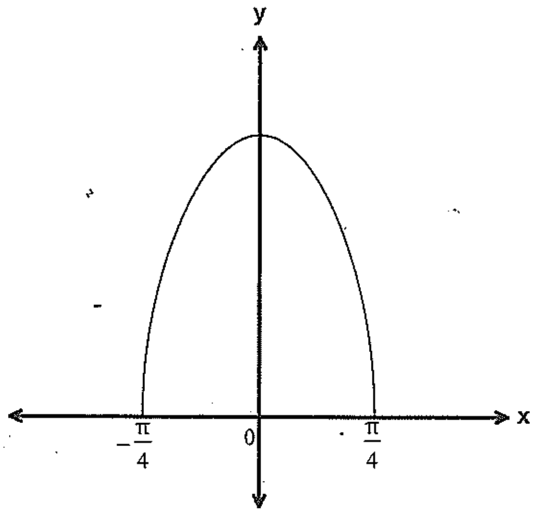
(A) 36

(B) 35.5

(C) 48.5

(D) 49

10. The diagram below shows the region bounded by the curve $y = \sqrt{5\cos^2 x}$ and the x-axis for $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$. The region is rotated about the x-axis to form a solid. Which of the following gives the volume of the solid?



(A) $V = 5\pi \int_0^{\frac{\pi}{4}} \cos^2 x\,dx$

(B) $V = 10\pi \int_0^{\frac{\pi}{4}} \cos^2 x\,dx$

(C) $V = 10\pi \int_0^{\frac{\pi}{4}} \cos^4 x\,dx$

(D) $V = 25\pi \int_0^{\frac{\pi}{4}} \cos^2 x\,dx$

Section II
Total Marks 50
Attempt Questions 11-15

Answer the questions in the booklet provide. Start each question on a NEW sheet of paper.

Question 11 **10 marks**

a) Find the exact value of $\tan\frac{2\pi}{3}$ **2**

b) (i) Find the derivative of $y = \sin^2 x$ **2**

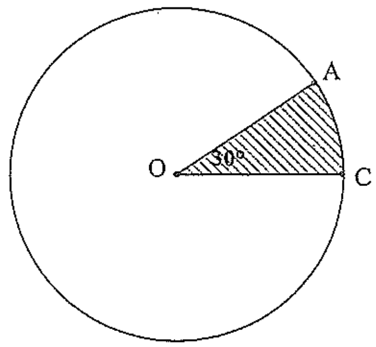
(ii) Find the equation of the tangent to $y = \sin^2 x$ at $x = \frac{\pi}{4}$ **2**

(iii) Find the equation of the normal to $y = \sin^2 x$ at $x = \frac{\pi}{4}$ **2**

(iv) If the tangent meets the x-axis at P and the normal meets the y-axis at Q, find the area of ΔOPQ where O is the origin in exact form. **2**

Question 12 **(Start a New Page)** **10 marks**

a) Differentiate 4 cos $(5x - 3)$ with respect to x: **2**



b) (i) Find the radius of the circle if the area of the shaded sector is $12\pi\ cm^2$ **3**

(ii) Hence find the exact length of the major arc AC **2**

c) Copy the table of values into your writing booklet and supply the missing numbers, for $f(x) = x \sin x$, writing each correct to 3 decimal places.

x	1	1.5	2	2.5	3
$f(x) = x \sin x$	0.841				

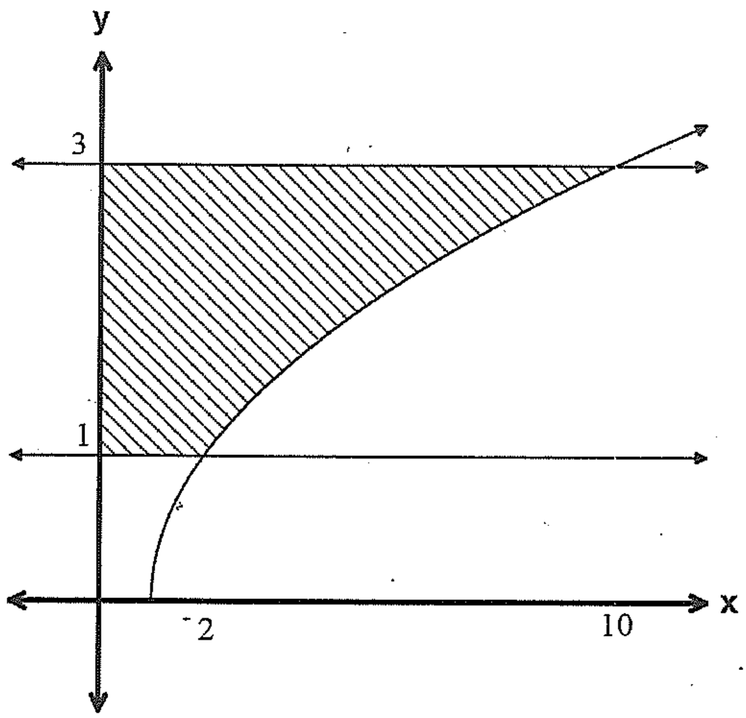
Use Simpson's Rule with 5 function values to find an approximation for $\int_1^3 x \sin x\, dx$ **3**

Question 13 **(Start a New Page)** **8 marks**

a) Differentiate $\dfrac{x}{\cos x}$ **2**

b) Find the equation of the tangent to the curve $y = 3e^x - 1$ at the point where $x = 1$ **3**

c) The diagram shows the shaded region enclosed by the curve $y = \sqrt{x-1}$, the y-axis and the lines y=1 and y=3

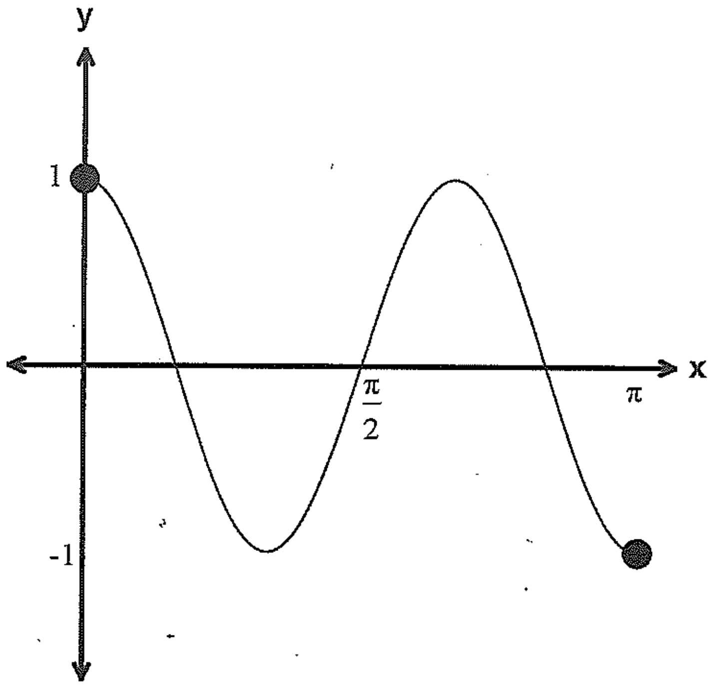


Find the volume of the solid of revolution when the shaded region is rotated about the y-axis. **3**

Question 14 **(Start a New Page)** **10 marks**

a) Find $\displaystyle\int_0^{\frac{\pi}{12}} \sec^2 3x \, dx$ **2**

b) The graph of $y = \cos 3x$ is shown below



(i) Solve $\cos 3x = 0$ for $0 \le x \le \pi$ **2**

(ii) State the amplitude and the period of $y = \cos 3x$ **2**

(iii) Copy this diagram into your booklet showing the x-intercepts
Hence sketch the graph of $y = \sec 3x$ in the domain $0 \le x \le \pi$
showing any asymptotes.
(Hint: The diagram should be one third of your page, use a ruler) **2**

(iv) Using (iii), find the number of solutions to $\sec 3x = x$ in the domain $0 \le x \le \pi$ **2**

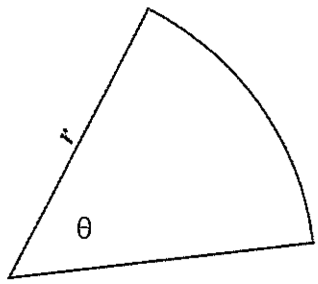
Question 15 **(Start a New Page)** **10 marks**

a) Consider the function $f(x) = \cos^2 x - \sin x$ in the domain $\pi \le x \le \frac{3\pi}{2}$

(i) Find $f'(x)$. **1**

(ii) Find the x-coordinates of the stationary points of $y = f(x)$ and determine their nature **3**

b) The diagram shows a sector of a circle with radius r cm. The angle at the centre is θ radians and the area is $18\ cm^2$



i) Find an expression for r in terms of θ. **1**

ii) Show that P, the perimeter of the sector in cm, is given by **2**

$$P = \frac{6(2+\theta)}{\sqrt{\theta}}$$

iii) Find the minimum perimeter and the value of θ for which this occurs. **3**

End of Exam

☺☺☺

STHS YEAR 12 MATHEMATICS TASK 3 2015

Multiple Choice

1	A	6	D
2	B	7	B
3	D	8	C
4	A	9	A
5	D	10	B

Question 11

a) $\tan\frac{2\pi}{3} = -\tan\frac{\pi}{3}$

$= -\sqrt{3}$

b) $y = \sin^2 x$

Let $u = \sin x$

$du = \cos x$

$y = u^2$

$\frac{dy}{du} = 2u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= 2u \times \cos x$

$= 2\sin x \cos x$

c) When $x = \frac{\pi}{4}$ $y = \frac{1}{2}$

Equation of the tangent

$\frac{dy}{dx} = 2\times\sin\frac{\pi}{4}\times\cos\frac{\pi}{4}$

$= 2\times\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}$

$= 1$

$y - y_1 = m(x - x_1)$

$y - \frac{1}{2} = 1\left(x - \frac{\pi}{4}\right)$

$y = x + \frac{1}{2} - \frac{\pi}{4}$

d) Equation of the normal

$m_2 = -1$

$y - y_1 = m(x - x_1)$

$y - \frac{1}{2} = -1\left(x - \frac{\pi}{4}\right)$

$y = -x + \frac{\pi}{4} + \frac{1}{2}$

e) $Q\left(0, \frac{\pi}{4} + \frac{1}{2}\right)$

$P\left(\frac{1}{2} - \frac{\pi}{4}, 0\right)$

Area of $\Delta OPQ =$

$= \frac{1}{2}\left(\frac{\pi}{4} - \frac{1}{2}\right)\left(\frac{\pi}{4} + \frac{1}{2}\right)$

$= \frac{1}{2}\left(\frac{\pi^2}{16} - \frac{1}{4}\right)$

$= \left(\frac{\pi^2}{36} - \frac{1}{8}\right)$ units^2

Question 12

a) $y = 4\cos(5x - 3)$

$\frac{dy}{dx} = -20\sin(5x - 3)$

b) i) Area of sector $= \frac{1}{2}r^2\theta$

$12\pi = \frac{1}{2}\times r^2\times\frac{\pi}{6}$

$144\pi = r^2\pi$

$r = 12$

ii) Major arc $= l = r\theta$

$\theta = \frac{11\pi}{6}$

$l = 12\times\frac{11\pi}{6}$

$l = 22\pi$ units

c)

x	1	1.5	2	2.5	3
$f(x)$	0.841	1.496	1.819	1.496	0.423

$\int_1^3 x\sin x\,dx =$

$= \frac{0.5}{3}\left[0.841 + 0.423 + 4(1.496 + 1.496) + 2(1.819)\right]$

$= 2.812$ (3 d p)

Question 13

a) $u = x$ $v = \cos x$

$du = 1$ $dv = -\sin x$

$\frac{dy}{dx} = \frac{\cos x \times 1 + x\sin x}{\cos^2 x}$

b) $y = 3e^x - 1$

$\frac{dy}{dx} = 3e^x$

When $x = 1$ $y = 3e - 1$

$m = 3e$

Equation of tangent

$y - (3e - 1) = 3e(x - 1)$

$y - 3e + 1 = 3ex - 3e$

$y + 1 = 3ex$

$y = 3xe - 1$

c) $y = \sqrt{x-1}$

$y^2 = x - 1$

$x = y^2 + 1$

$V = \pi\int_1^3 (y^2 + 1)^2\,dy$

$= \pi\int_1^3 y^4 + 2y^2 + 1\,dy$

$= \pi\left[\frac{y^5}{5} + \frac{2y^3}{3} + y\right]_1^3$

$= \pi\left[\frac{243}{5} + \frac{54}{3} + 3 - \left(\frac{1}{5} + \frac{2}{3} + 1\right)\right]$

$= \pi\left[\frac{348}{5} - \frac{28}{15}\right]$

$= \frac{1016}{15}\pi$ unit^3

Question 14

a) $\int_0^{\pi/12}\sec^2 3x\,dx = \left[\frac{\tan 3x}{3}\right]$

$= \left[\frac{1}{3} - 0\right]$

$= \frac{1}{3}$

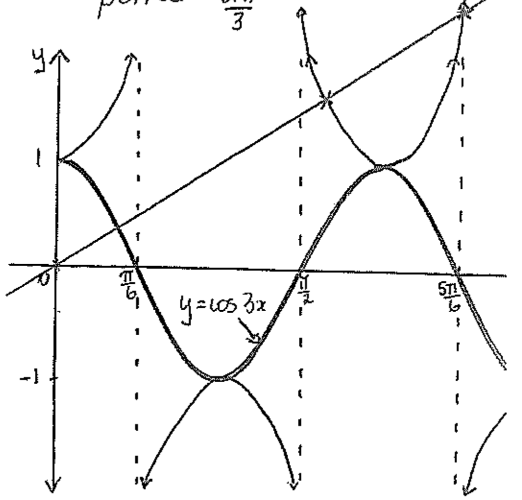
b) i) $\cos 3x = 0$

$3x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

ii) amp $= 1$

period $= \frac{2\pi}{3}$



iv) $\sec 3x = x$

2 solutions from the gr

uestion 15

) $f(x) = \cos^2 - \sin x$

$f'(x) = 2\cos x(-\sin x) - \cos x$

$= -2\sin x \cos x - \cos x$

$=$

) Stationary points occur when $f'(x) = 0$

$-2\sin x \cos x - \cos x = 0$

$-\cos x(2\sin x + 1) = 0$

$-\cos x = 0$ $\quad$ $2\sin x + 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ $\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}$

within the domain

3	$\frac{7\pi}{6}$	4	$\frac{3\pi}{2}$	5
1.26	0	-0.34	0	0.26
/	—	\	—	/

$f(x)$ has a maximum at $x = \frac{7\pi}{6}$

$f(x)$ has a minimum at $x = \frac{3\pi}{2}$

b) i) $A = \frac{1}{2}r^2\theta$

$18 = \frac{1}{2}\theta r^2$

$36 = r^2\theta$

$r = \sqrt{\frac{36}{\theta}}$

$r = \frac{6}{\sqrt{\theta}}$

ii) $P = 2r + r\theta$

$= 2 \times \frac{6}{\sqrt{\theta}} + \frac{6}{\sqrt{\theta}} \times \theta$

$= \frac{6(2+\theta)}{\sqrt{\theta}}$

iii) $P = 12\theta^{-\frac{1}{2}} + 6\theta^{-\frac{1}{2}}$

$P' = -6\theta^{-3/2} + 3\theta^{-\frac{1}{2}}$

Stationary points occur when $P' =$

$0 = -6\theta^{-3/2} + 3\theta^{-1/2}$

$6\theta^{-3/2} = 3\theta^{-1/2}$

$\frac{6}{\theta^{3/2}} = \frac{3}{\theta^{\frac{1}{2}}}$

$6\theta^{\frac{1}{2}} = 3\theta^{3/2}$

$0 = \theta^{\frac{1}{2}}(\theta - 2)$ $\quad$ $\theta \neq 0$

$\theta = 2$

$P'' = 9\theta^{-5/2} - 3/2\,\theta^{-3/2}$

At $\theta = 2$

$P'' = 9(2)^{-5/2} - 3/2\,(2)^{-3/2}$

$= 1.06 > 0$ $\quad$ concave up

$\therefore$ minimum at $\theta = 2$

$P = \frac{6(4)}{\sqrt{2}}$

$P = 12\sqrt{2}$ cm