BAULKHAM HILLS HIGH SCHOOL
MARKING COVER SHEET

YEAR 12 EXT 1
JUNE 2008

STUDENTS NAME:____________________
TEACHERS NAME:____________________

QUESTION	MARK
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
TOTAL	
PERCENTAGE	

YEAR 12 EXTENSION 1
MATHEMATICS ASSESSMENT
JUNE 2008

TIME : 70 MINUTES

NAME		RESULT	
DIRECTIONS	▪ Full working should be shown in every question. Marks may be deducted for careless or badly arranged work. ▪ Use black or blue pen only (*not pencils*) to write your solutions. ▪ No liquid paper is to be used. If a correction is to be made, one line is to be ruled through the incorrect answer.		
QUESTION 1.	Differentiate (a) $\sin^{-1}\frac{x^2}{2}$ (b) $\cos^{-1}\left(e^x\right)$		2 2
QUESTION 2.	Find the following : (a) $\int\frac{dx}{\sqrt{25-x^2}}$ (b) $\int_{\frac{1}{3}}^{\frac{1}{\sqrt{3}}}\frac{dx}{1+9x^2}$		2 3
QUESTION 3.	State the domain and range for: $y=3\sin^{-1}4x$ and hence draw a neat sketch of this function showing clearly the co-ordinates of the end points.		2 2

QUESTION 4.	The acceleration of a particle moving in a straight line is given by: $a = 2x - 3$ where x is the displacement, in metres, from the origin O and t is the time in seconds. Initially the particle is at rest at x = 4. a) If the velocity of the particle is $v\ ms^{-1}$ show that: $v^2 = 2(x^2 - 3x - 4)$ b) Show that the particle does not pass through the origin. c) Determine the position of the particle when $v = 10\ ms^{-1}$. Justify your answer.	3 2 2
QUESTION 5.	Without the use of a calculator, evaluate the following showing all working: a) $\cos\left(\tan^{-1}\left(\frac{-2}{3}\right)\right)$ b) $\tan\left[\sin^{-1}\frac{3}{5} + \cos^{-1}\frac{12}{13}\right]$	2 3
QUESTION 6.	Evaluate $\int_0^{\ln 2} \frac{e^x}{4 + e^{2x}}\,dx$ using the substitution $u = e^x$.	4
QUESTION 7.	Find $\int \cos^2 2x\,dx$.	2
QUESTION 8.	Consider the function $f(x) = 3x - x^3$ a) Sketch $y = f(x)$ showing the x and y intercepts and the co-ordinates of the stationary points. b) Find the largest domain containing the origin for which $f(x)$ has an inverse function $f^{-1}(x)$. c) State the domain of $f^{-1}(x)$. d) Find the gradient of the function $f^{-1}(x)$ at $x = o$.	3 1 1 1
QUESTION 9.	The velocity $v\ ms^{-1}$ of a particle moving along the x axis is given by $v^2 = 15 - 2x - x^2$, where x m. is its displacement from the origin. a) Show that this motion is SHM. b) Find the amplitude of this motion. c) Find its greatest velocity.	2 2 2

QUESTION 10.		
	A particle is projected with an initial velocity of $60\ ms^{-1}$ at an angle of 45^o to the horizontal. Taking $g = 10\ ms^{-1}$ find:	
	a) Expressions for the horizontal and vertical displacements.	4
	b) The Cartesian equation of the particle.	2
	c) The greatest height reached by the particle.	3
	d) The angle that the path of the particle makes with the horizontal after 6 seconds.	2
	THE END	

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, \ n \neq -1; \ x \neq 0, \ if \ n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}, \ a \neq 0$$

$$\int \cos ax dx = \frac{1}{a}\sin ax, \ a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a}\cos ax, \ a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a}\tan ax, \ a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a}\sec ax, \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a}, \ a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln(x + \sqrt{x^2 - a^2}), \ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln(x + \sqrt{x^2 + a^2})$$

NOTE: $\ln x = \log_e x, \ x > 0$

EXT 1 JUNE 2008

1 a) $\dfrac{\frac{2x}{2}}{\sqrt{1-\frac{x^4}{4}}}$ 1 1 (2)

$= \dfrac{x}{\frac{1}{2}\sqrt{4-x^2}}$ $\boxed{4}$

$= \dfrac{2x}{\sqrt{4-x^2}}$

b) $\dfrac{-e^x}{\sqrt{1-e^{2x}}}$ 1 1 (2)

2) a) $\displaystyle\int \frac{dx}{\sqrt{25-x^2}}$

$= \sin^{-1}\frac{x}{5} + C$ 1 1 (for c) (2)

b) $\displaystyle\int_{\frac{1}{3}}^{\frac{1}{\sqrt3}} \frac{dx}{1+9x^2}$

$= \frac{1}{9}\displaystyle\int_{\frac{1}{3}}^{\frac{1}{\sqrt3}} \frac{dx}{\frac{1}{9}+x^2}$ $\boxed{5}$

$= \frac{1}{9}\cdot\frac{3}{1}\left[\tan^{-1}3x\right]_{\frac13}^{\frac{1}{\sqrt3}}$ 1

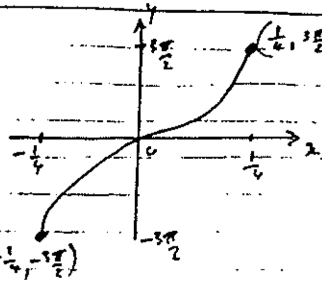
$= \frac{1}{3}\left[\tan^{-1}\sqrt3 - \tan^{-1}1\right]$ (3)

$= \frac{1}{3}\left[\frac{\pi}{3}-\frac{\pi}{4}\right]$ 1

$= \frac{\pi}{36}$ 1

3) D $\rightarrow$ 1 $-\frac14 \le x \le \frac14$ or $|x|\le\frac14$

R $\rightarrow$ 1 $-\frac{3\pi}{2} \le y \le \frac{3\pi}{2}$ or $|y|\le\frac{3\pi}{2}$



1 SHAPE
1 END PTS
$\boxed{4}$

4) a) $a = \dfrac{d^2x}{dt^2} = \dfrac{d\left(\frac{v^2}{2}\right)}{dx} = 2x-3$

$\therefore \frac{v^2}{2} = x^2-3x+C_1$ 1

$v^2 = 2x^2-6x+C_2$

at $t=0$ $v=0$ $x=4$ 1

$\therefore 32-24+C_2=0 \quad \therefore C_2=-8$

$\therefore v^2 = 2x^2-6x-8$

$\therefore v^2 = 2(x^2-3x-4)$ 1 (3)

b) subst $x=0$ 1

$\therefore v^2=-8$

no soln

$\therefore x \ne 0$ 1 (2)

$\therefore$ does not pass thru 0.

c) subst $v=10$

$\therefore 100 = 2(x^2-3x-4)$

$\therefore x^2-3x-4=50$

$x^2-3x-54=0$

$(x-9)(x+6)=0$ 1 (2)

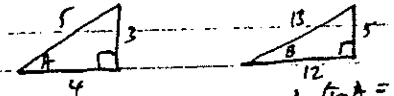
$\therefore x=9$ or -6

but particle starts at $x=4$ and doesn't pass thru 0, 1

$\therefore x \ne -6$

$\therefore x=9$ $\boxed{7}$

5) let $\sin^{-1}\frac{3}{5} = A$ $\cos^{-1}\left(\frac{12}{13}\right)=B$



1 $\tan A = \frac34$

$\tan B = \frac{5}{12}$

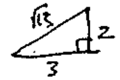
$\therefore \tan(A+B) = \dfrac{\tan A+\tan B}{1-\tan A\tan B}$

$= \dfrac{\frac34+\frac{5}{12}}{1-\frac34\cdot\frac{5}{12}}$ 1

$= \dfrac{\frac{36+20}{48}}{1-\frac{15}{48}}$ (3)

$= \dfrac{56}{33}$ 1 $\boxed{5}$

b) $\cos\left(\tan^{-1}\left(-\frac{2}{3}\right)\right)$



$\frac{\pi}{2}$ $A<0$ as $\tan^{-1}<0$

$\therefore \cos\left(\tan^{-1}\left(\frac{-2}{3}\right)\right) = \dfrac{3}{\sqrt{13}}$ 2 (2)

1 MARK FOR $-\frac{3}{\sqrt{13}}$

6) $\displaystyle\int_0^{\ln 2} \frac{e^x}{4+e^{2x}}\,dx$

let $u=e^x$ $\therefore du = e^x dx$

$x=\ln 2$ $u=2$
$x=0$ $u=1$ } 1

$\therefore = \displaystyle\int_1^2 \frac{du}{4+u^2}$ 1

$= \frac{1}{2}\left[\tan^{-1}\frac{u}{2}\right]_1^2$ 1

$= \frac12\left[\tan^{-1}1 - \tan^{-1}\frac12\right]$ $\boxed{4}$

$= \frac{\pi}{8} - \frac12\tan^{-1}\frac12$ 1

7) $\displaystyle\int \cos^2 2x\,dx$

$\cos 4x = 2\cos^2 2x - 1$

$\therefore \cos^2 2x = \frac12(1+\cos 4x)$

$\therefore I = \frac12\displaystyle\int(1+\cos 4x)\,dx$ 1 $\boxed{2}$

$= \frac12\left(x+\frac14\sin 4x\right)$ 1

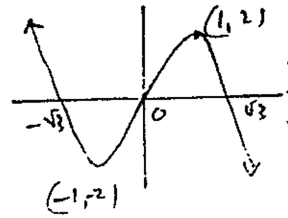
$= \frac{x}{2} + \frac18\sin 4x + C$

8) a) $f(x) = 3x-x^3$

$f'(x) = 3-3x^2 = 0$ $x=\pm$

stat pts at $(1,2)$ $(-1,-2)$

x intercepts $(\sqrt3,0)$, 0, $(-\sqrt3,0$



1 SHAPE
1 INTERCEPTS
1 FOR STAT
(3)

b) Dom of $f^{-1}(x)$ $-1\le x\le 1$
$|x|\le 1$

c) Dom of $f^{-1}(x)$ $-2 \le x \le$
$|x|\le 2$

d) $\dfrac{d\,f(x)}{dx} = 3$ at 0

$\therefore \dfrac{d\,f^{-1}(x)}{dx} = \frac13$ at 0. 1 $\boxed{6}$

(9) a) $v^2 = 15 - 2x - x^2$

$\frac{v^2}{2} = \frac{15}{2} - x - \frac{1}{2}x^2$ ½

$\frac{d\left(\frac{v^2}{2}\right)}{dx} = -1 - x = -1(x+1)$ (2)

½

this is SHM about $x = -1$ $x = 1$

b) when $v = 0$

$x^2 + 2x - 15 = 0$

$(x+5)(x-3) = 0$ 1 (2)

∴ $x = -5$ or $+3$

∴ oscillates between -5 and $+3$

∴ AMP $= \frac{8}{2} = 4$ 1

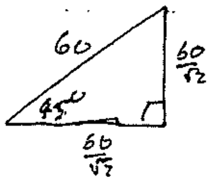
c) max velocity when $\frac{d\left(\frac{v^2}{2}\right)}{dx} = 0$ [6]

ie $x = -1$ 1 (2)

∴ $v^2 = 15 + 2 - 1 = 16$ 1

∴ max velocity $= 4\ m\,s^{-1}$

(10)



at $t=0$ $v_y = \frac{60}{\sqrt{2}}$ $v_x = \frac{60}{\sqrt{2}}$

$\frac{d^2y}{dt^2} = -10$ 1

$\frac{dy}{dt} = c - 10t$ $c = \frac{60}{\sqrt{2}}$

$\frac{dy}{dt} = \frac{60}{\sqrt{2}} - 10t$ 1

$y = \frac{60t}{\sqrt{2}} - 5t^2 + c$

∴ $y = \frac{60t}{\sqrt{2}} - 5t^2$

$\frac{d^2x}{dt^2} = 0$

$\frac{dx}{dt} = \frac{60}{\sqrt{2}}$

$x = \frac{60t}{\sqrt{2}} + c$ (4)

at $x=0$ $t=0$ ∴ $c = 0$

∴ $y = \frac{60t}{\sqrt{2}} - 5t^2$ 1 $x = \frac{60t}{\sqrt{2}}$ 1

b) subst $t = \frac{\sqrt{2}}{60}x$ 1

∴ $y = \frac{60}{\sqrt{2}} \cdot \frac{\sqrt{2}}{60}x - \frac{5 \cdot 2\,x^2}{3600}$ (2)

$y = x - \frac{x^2}{360}$ 1

c) max height when $\frac{dy}{dt} = 0$ 1

∴ $t = \frac{6}{\sqrt{2}}$ 1

(3)

∴ $y = \frac{60}{\sqrt{2}} \cdot \frac{6}{\sqrt{2}} - 5 \cdot \frac{36}{2}$

$= \frac{360 - 180}{2}$ 1

$= 90\ m$

d) when $t = 6$

$\frac{dy}{dt} = \frac{60}{\sqrt{2}}(1 - \sqrt{2})$ $\frac{dx}{dt} = \frac{60}{\sqrt{2}}$ 1

$\tan^{-1}\alpha = \left|\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right| = \sqrt{2} - 1$

$= 22\frac{1}{2}^\circ$ ½