BAULKHAM HILLS HIGH SCHOOL
MARKING COVER SHEET

2011
YEAR 12 JUNE ASSESSMENT
EXTENSION 1

STUDENT NAME:____________________

TEACHERS NAME:____________________

QUESTION	MARK
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	/
PERCENTAGE	%

Topics Tested: Further Integration, Newton's Method & Halving the Interval Method,
Applications of Calculus (Related Rates, Velocity as a function of x, Projectile Motion, Simple Harmonic Motion)

Year 12 Extension 1 Mathematics Assessment
JUNE 2011

Time : 60 minutes

NAME		RESULT

DIRECTIONS

- Full working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Use black or blue pen only (*not pencil*) to write your solutions.
- No liquid paper is to be used. If a correction is to be made, one line is to be ruled through the incorrect answer.

QUESTION 1

Evaluate

$$\int_0^1 6x\sqrt{9-x^2}\,dx \text{ using the substitution } u = 9 - x^2$$

Answer to 2 significant figures. **4**

QUESTION 2

Taking $x = \frac{\pi}{3}$ as the first estimate for the root of $f(x) = 3\cos x - 4\tan x$ use one application of Newton's method to find a closer approximation for the root to 2 decimal places. **3**

QUESTION 3

A particle is moving along the x axis and its velocity (v) in metres/second at position x metres from the origin is given by $v^2 = 16 + 4x - 2x^2$

(a) Show that the particle undergoes Simple Harmonic Motion. **2**

(b) Find the amplitude of the motion. **2**

(c) What is the frequency of the motion? **1**

(d) What is the maximum acceleration of the particle? **1**

QUESTION 4

Find

$$\int \frac{e^{3x}}{1+e^{6x}}\,dx \text{ using the substitution } u = e^{3x}$$

3

QUESTION 5

The displacement of an object as it moves in Simple Harmonic Motion about the origin is given by,

$$x = 4\sin(3t + \alpha)$$

Initially the particle is 2 cm to the right of the origin.

Find (i) α **1**

(ii) the maximum speed and when it first reaches this speed . **4**

QUESTION 6

The acceleration of a particle is given by $a = x - 1$, where x is the displacement from the origin. Initially it's at the origin moving with a velocity(v) of 1 m/s

(a) Show that $v^2 = (x-1)^2$ **2**

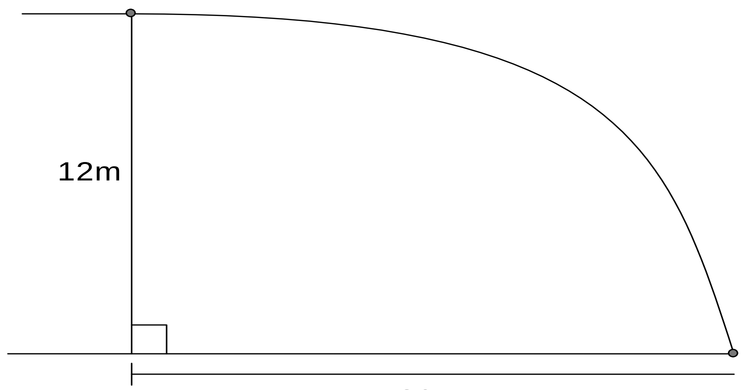
(b) Find the speed of the particle when $x = \frac{1}{2}$ **1**

(c) Show that $x = 1 - e^{-t}$ **3**

(d) Will the particle reach $x = 2$? Justify your answer. **1**

QUESTION 7

A ball is projected horizontally from the top of a 12 metre building. It hits the ground at a point 20 metres from the base of the building



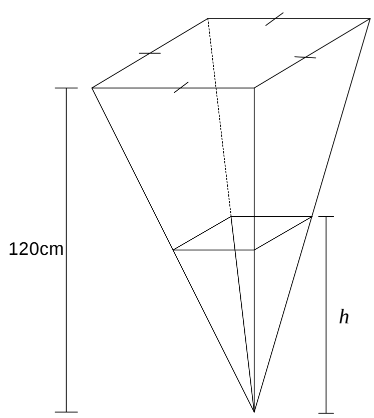
(a) Taking the base of the building as the origin derive the equations of motion Take $g = 10\ m/s^2$ **2**

(b) Find the time that it takes for the ball to hit the ground. (Answer to 1 decimal place). **1**

(c) Find the initial speed of the ball. **1**

(d) Find the velocity and angle with which the ball hits the ground. **3**

QUESTION 8



Water is poured into a container at a rate of $400 cm^3/min$ .The container is an inverted square pyramid of height $120cm$ and base edge $60cm$

(a) Show that the volume of the water in the container when the depth of the water is $h\ cm$ is given by $V = \frac{h^3}{12}$. **2**

(b) Find the rate at which the water level is rising when the depth of the water is $40cm$. **2**

(c) Find the time taken for the water level to reach a depth of $100cm$. **3**

QUESTION 9

Find

$$\int (2sinx + cosx)^2\, dx$$ **4**

~ END OF EXAM ~

1. [4] $\int_0^1 6x\sqrt{9-x^2}\,dx$ $\quad u = 9-x^2$

$\frac{du}{dx} = -2x \rightarrow dx = \frac{du}{-2x}$

when $x=1$ $u=8$; $x=0$ $u=9$ } (1)

$6x\sqrt{9-x^2}\,dx = {}^{-3}\,6x\sqrt{u}\cdot\frac{du}{-2x}$

$\therefore \int_0^1 6x\sqrt{9-x^2}\,dx = -3\int_9^8 u^{\frac{1}{2}}\,du$ (1)

$= -3\cdot\frac{2}{3}\left[u^{\frac{3}{2}}\right]_9^8$ (1)

$= -2\left[\sqrt[2]{8^3} - \sqrt[2]{9^3}\right]$

$= 8\cdot7$ (1)

2. $f(x) = 3\cos x - 4\tan x$ [3]

$f'(x) = -3\sin x - 4\sec^2 x$

$f(\frac{\pi}{3}) = 3\cos(\frac{\pi}{3}) - 4(\tan\frac{\pi}{3})$

$= -5\cdot428$ (1)

$f'(\frac{\pi}{3}) = -3\sin(\frac{\pi}{3}) - 4\sec^2(\frac{\pi}{3})$

$= -18\cdot598$ (1)

$\therefore a_1 = \frac{\pi}{3} - \left(\frac{-5\cdot428}{-18\cdot598}\right)$

$= 0\cdot76$ (1)

3. a) [6] $v^2 = 16 + 4x - 2x^2$

$\frac{1}{2}v^2 = 8 + 2x - x^2$

$a = 2 - 2x$ (1)

$= -2(x-1)$

which is in the form (1)

$\ddot{x} = -n^2(x-b)$ ∴ in SHM.

b) $v=0$ $\quad -2(x-2x-8)=0$

$-2(x-4)(x+2)=0$

$x = 4, -2.$ (1)

c) $n=\sqrt{2}$ ∴ Freq'y $= \frac{n}{2\pi} = \frac{\sqrt{2}}{2\pi}$ (1) $\left(\frac{1}{\pi\sqrt{2}}\right)$

d) Max acc'n when $x=-2$

$\ddot{x} = -2(-2-1)$

$= 6$ unit²/sec. (1)

4/ [3] $\int \frac{e^{3x}}{1+e^{6x}}\,dx$ $\quad u = e^{3x}$ $(u^2 = e^{6x})$

$\frac{du}{dx} = 3e^{3x} \rightarrow dx = \frac{du}{3e^{3x}}$ (1)

$\frac{e^{3x}}{1+e^{6x}}\,dx = \frac{e^{3x}}{1+u^2}\cdot\frac{du}{3e^{3x}}$

$\therefore \int \frac{e^{3x}}{1+e^{6x}}\,dx = \frac{1}{3}\int\frac{1}{1+u^2}\,du$ (1)

$= \frac{1}{3}\tan^{-1}(u) + c$

$= \frac{1}{3}\tan^{-1}(e^{3x}) + c$ (1)

5/ (i) $x = 4\sin(3t+\alpha)$ [5]

when $t=0$ $x=2$

sub in $\frac{2}{4} = \sin(0+\alpha)$

$\sin\alpha = \frac{1}{2} \rightarrow \alpha = \frac{\pi}{6}$ (1)

$\therefore x = 4\sin(3t + \frac{\pi}{6})$

(ii) Centre of the motion is zero

max speed at $x=0$.

now $a=4$ $n=3$ $x=0$ (1)

now $v^2 = 3^2(4^2 - 0^2)$

$v = \pm 12$ (1)

∴ Max speed is 12 cm/s

Max Speed when $x=0$ $4\sin(3t+\frac{\pi}{6}) = 0$

$3t + \frac{\pi}{6} = 0, \pi, 2\pi, \ldots$

$3t = \frac{5\pi}{6}$ (1)

$t = \frac{5\pi}{18}$ (1)

$t = -\frac{\pi}{18}$ (1) with working

[7] a) ... $v=1$

$\frac{1}{2}v^2 = \int x - 1\,dx$

$= \frac{x^2}{2} - x + c$ (1)

when $x=0$ $v=1$

$\therefore \frac{1}{2} = c.$

$\therefore \frac{1}{2}v^2 = \frac{x^2}{2} - x + \frac{1}{2}$

$v^2 = x^2 - 2x + 1$

$= (x-1)^2$ } (1)

b) when $x = \frac{1}{2}$ $\quad v^2 = (\frac{1}{2}-1)^2$

$v^2 = \frac{1}{4}$

$v = \pm\frac{1}{2}$

∴ speed $= \frac{1}{2}$ m/s. (1)

c) $v^2 = (x-1)^2$

$v = \pm(x-1)$ but when $x=0$ $v=1$

$\therefore v = -x+1$

$\frac{dx}{dt} = 1 - x$ (1)

$\frac{dt}{dx} = \int\frac{1}{1-x}\,dx$

$t = -\ln(1-x) + C$

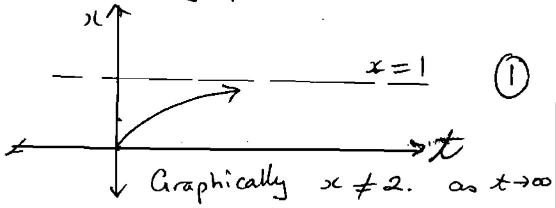
when $t=0$ $x=0$ $\Rightarrow$ $C=0$

$\therefore -t = \ln(1-x)$ (1)

$e^{-t} = 1-x$

$x = 1 - e^{-t}$ (1)

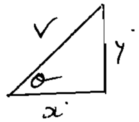
d) the graph of $x = 1-e^{-t}$



(1)

Graphically $x \neq 2$. as $t\to\infty$ $x\to1$.

[7] $x = c_1$ $\quad \dot{y} = -10t + c_2$



$\dot{x} = V\cos\theta$, $\dot{y} = V\sin\theta$ } when $\theta=0$

$\dot{x} = V$

$\dot{y} = 0$

$\therefore \dot{x} = V$ $\quad \dot{y} = -10t$

$x = Vt + c_3$ $\quad y = -5t^2 + c_4$

when $t=0$ $x=0$ $y=12$

$\therefore x = Vt$ $\quad y = -5t^2 + 12$

∴ Eq'ns. $\ddot{x}=0$, $\dot{x}=V$, $x=Vt$ } (1) $\quad \ddot{y}=-10$, $\dot{y}=-10t$, $y=-5t^2+12$ } (1)

b) when $y=0$ $\quad -5t^2 = 12$

$t = \sqrt{\frac{12}{5}}$ $(t>0)$

$= 1\cdot59\ldots$

(1) $= 1\cdot6$ secs

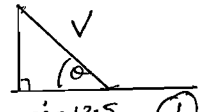
c) $x=20$ when $y=0$, $t=1\cdot6$

$\therefore 20 = V(1\cdot6)$ (1)

$V = \frac{20}{1\cdot6} \rightarrow V \doteqdot 12\cdot5$ m/s.

($V = 12\cdot9$ m/s)

d) At impact



(1)

at $t=1\cdot6$ $\dot{y} = -16$ m/s.

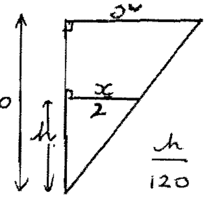
$V = \sqrt{(-16)^2 + (12\cdot5)^2}$

$= 20\cdot3$ m/s. (1)

$\tan\theta = \frac{-16}{12\cdot5}$

Angle $= 180 - 52°$. $= 118°$ (Accept acute angle 52°) (1)

8. a) [7]



x = square edge

$\frac{h}{120} = \frac{\frac{x}{2}}{30}$ (1)

$\therefore x = \frac{h}{2}$

$V = \frac{1}{3} \times x^2 \times h$

$= \frac{1}{3} \cdot \left(\frac{h}{2}\right)^2 \cdot h$ (1)

$V = \frac{h^3}{12}$

b) Find $\frac{dh}{dt}$ when $h = 40$cm. given $\frac{dV}{dt} = 400$

$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$

$\frac{dV}{dh} = \frac{3h^2}{12} = \frac{h^2}{4}$

when $h = 40$ $\frac{dV}{dh} = 400$ (1)

$\therefore 400 = 400 \times \frac{dh}{dt}$

$\therefore \frac{dh}{dt} = 1$ cm/sec. (1)

c)

$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$

$400 = \frac{h^2}{4} \cdot \frac{dh}{dt}$ (1)

$\frac{dh}{dt} = \frac{1600}{h^2}$

$\frac{dt}{dh} = \frac{h^2}{1600}$

$t = \int \frac{h^2}{1600}\, dh$ (1)

$t = \frac{h^3}{4800} + c$

when $t=0$ $h=0 \rightarrow c=0$

$\therefore t = \frac{h^3}{4800}$ when $h = 100$

(1) $t = \frac{(100)^3}{4800} \rightarrow t = 208.3$ min (= 3hrs 28min)

9. ✓ [4] $\int (2\sin x + \cos x)^2\, dx$

$= \int 4\sin^2 x + 4\sin x\cos x + \cos^2 x$

$= \int (4\sin^2 x + 2\sin 2x + 1 - \sin^2 x)\, dx$

$= \int 3\sin^2 x + 2\sin 2x + 1$

$= \int 3\left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) + 2\sin 2x + 1$

(1) $= \int \frac{5}{2} - \frac{3\cos 2x}{2} + 2\sin 2x$ (1) (1)

$= \frac{5x}{2} - \frac{3\sin 2x}{4} - \frac{2\cos 2x}{2} + c$

$= \frac{5x}{2} - \frac{3\sin 2x}{4} - \cos 2x + c$ (1)