GIRRAWEEN HIGH SCHOOL

EXTENSION 1
TIME: 85 Mins

TASK 3
JUNE, 2005

Instructions
1. Each question is to be answered on a separate sheet of paper
2. Show all necessary working
3. Sketches must be large ,have ruled axes and be neatly drawn
4. Marks will be deducted for careless and badly arranged work.

QUESTION 1 (25 Marks)

(a) Find the exact value of

(i) $\sin^{-1}(1)$ (ii) $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ (iii) $\sin\left(\tan^{-1}\left(\frac{5}{12}\right)\right)$ 5

(b) Find the derivatives of

(i) $y=\sin^{-1}\frac{x}{2}$ (ii) $y=\sin^{-1}2x$ 2

(iii) $y=x\tan^{-1}x$ (iv) $y=\tan^{-1}(\sin x)$ 4

(c) For $f(x)=3\cos^{-1}2x$

(i) Evaluate $f\left(\frac{1}{2}\right)$ 1

(ii) Find $f'(x)$ 2

(iii) State the domain and range of f(x) 2

(d) Sketch $y=\tan^{-1}x$ clearly stating domain and range 4

(e) (i) Write down the formula for sin(x+y) 1

~~(ii) If $x=\sin^{-1}(\frac{3}{5})$ and $y=\sin^{-1}(\frac{5}{13})$. Find sin(x+y) without using a calculator. 4~~

QUESTION 2 (23 Marks)

Evaluate

(a) (i) $\int\frac{dx}{x^2+3}$ (ii) $\int\frac{dx}{\sqrt{1-4x^2}}$ 4

(iii) $\int_0^1\frac{1}{\sqrt{4-x^2}}dx$ (iv) $\int_0^{\frac{1}{\sqrt{2}}}\frac{1}{2t^2+1}dt$ 6

(b) Use the substitution $u=e^x$ to evaluate

$$\int \frac{e^x}{1+e^{2x}}dx$$ 4

(c) Use the substitution u=1-x to find exact value

$$\int_0^1 x\sqrt{1-x}\,dx$$ 5

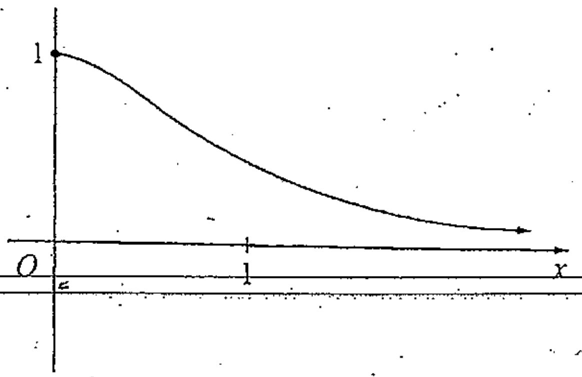
(d) Write down the general solution to

$$\cos 2x=\frac{1}{2}$$ 2

(e) Without evaluating the integral explain why $\int_{-1}^{1} \tan^{-1} x\, dx = 0$ 2

QUESTION 3 (20 Marks)

(a) The diagram below shows a sketch of the graph of y=f(x)



(i) Explain why the function y=f(x) has an inverse function $y=f^{-1}(x)$ 1

(ii) Copy this diagram onto your answer sheet.
On the same set of axes sketch the graph of $y=f^{-1}(x)$ 2

(iii) State the domain and range of $y=f^{-1}(x)$. 2

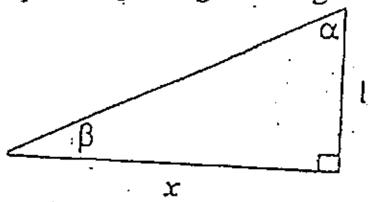
(b) At what points on the curve $y=\cos^{-1} x$ is the gradient $\frac{-2}{\sqrt{3}}$. 4

QUESTION 3 (cont)

(c) (i) Differentiate $f(x) = \tan^{-1} x + \tan^{-1}\frac{1}{x}$ for $x \neq 0$ and show that $f'(x) = 0$. 2

(ii) What does the result in (i) imply about f(x). 1

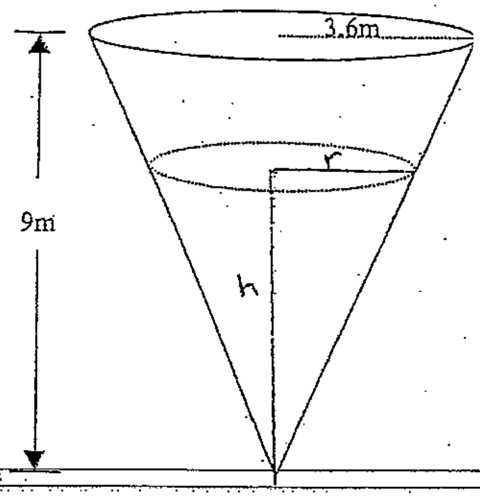
(iii) By considering the given right angled triangle or otherwise find the value of f(x) 2



(d) The diagram shows a milk storage unit in the shape of an inverted cone.

When the gate at the bottom is open, milk pours out at a rate equal to $\frac{\sqrt{h}}{10} m^3/s$

Given that volume of cone is $\frac{1}{3}\pi r^2 h$



(i) Show that $r = \frac{2h}{5}$. 3

(ii) At what rate is the height changing when h=7.2. 3

QUESTION 4 (22 Marks)

(a)The acceleration of a particle x metres from origin O at the time t seconds is given by $\ddot{x} = -\frac{1}{2}e^{-x}$. If its velocity v is 1m/s when x=0 find its velocity when x=4. 4

(b) A projectile is fired from the top of a cliff at an angle of 60° to the horizontal. The cliff was 40 metres high and the speed of projection was 20m/s.

Neglecting air resistance and assuming acceleration due to gravity is 10 m/s^2

(i) Using the origin as the point of projection.
Show that the equations for the horizontal and vertical components of the particles displacement are given by 5

$x = 10t, \; y = -5t^2 + 10\sqrt{3}t$

(ii) Find the maximum height the projectile reaches. 2

(iii)How long does the projectile take to land in the sea. 3

(iv) Find the distance from the foot of the cliff to the point where the projectile hits the sea. 2

(c) A softball player hits the ball at a velocity of 30m/s and the ball just clears a 1.5m fence 80m away. Given the horizontal and vertical components are

$x = 30t\cos\theta$ and $y = -5t^2 + 30t\sin\theta$

(i) Show the Cartesian equation is

$$y = -\frac{x^2}{180}\sec^2\theta + x\tan\theta$$ 3

(ii) Hence find the angle at which the ball was hit? 3