Name_______________________ Teacher___________________

GOSFORD HIGH SCHOOL

2014

HIGHER SCHOOL CERTIFICATE

ASSESSMENT TASK 3

MATHEMATICS – EXTENSION 1

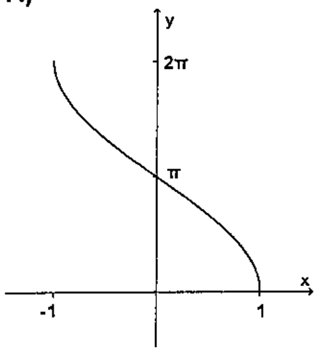
Duration- 60 minutes plus 5 minutes reading time

Questions 1-5	Multiple choice. Answer on the **multiple choice response sheet provided)**	/5
Question 6	Inequalities, Division of a line, Iterative methods.	/7
Question 7	Inverse Functions, Inverse Trigonometric functions	/11
Question 8	Inverse Trigonometric Functions	/11
Question 9	Binomial Theorem	/11
TOTAL		/45

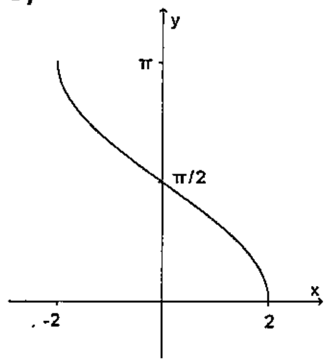
Questions 1-5. 1 mark each. Answer on the multiple choice answer sheet provided.

Question 1. The best representation of the graph of $y = cos^{-1}2x$ is given by:

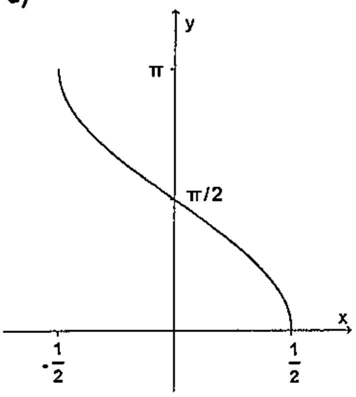
A)



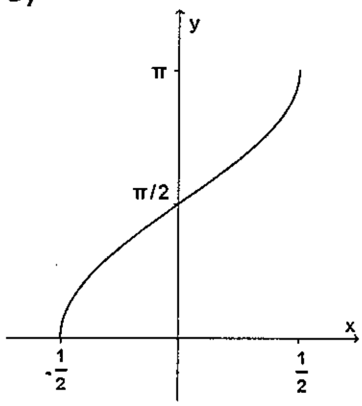
B)



C)



D)



Question 2. The coefficient of the third term in the expansion of $(3 - 2y)^8$ is:

A) -81648 **B)** 81648

C) -108864 **D)** 16128

Question 3. $\int \frac{2}{16+x^2} dx =$

A) $\frac{1}{4} tan^{-1}\left(\frac{x}{4}\right) + c$ **B)** $\frac{1}{2} tan^{-1}\left(\frac{x}{4}\right) + c$

C) $2tan^{-1}\left(\frac{x}{4}\right) + c$ **D)** $\frac{1}{8} tan^{-1}\left(\frac{x}{16}\right) + c$

Question 4. Given that a is an approximation to a root of the equation $y = sin^{-1}x$, then a better approximation using Newton's method is given by:

A) $a - \frac{sin^{-1}a}{\sqrt{1-a^2}}$

B) $a + \sin^{-1} a\sqrt{1-a^2}$

C) $a - \sin^{-1} a\sqrt{1-a^2}$

D) $a - \frac{1}{\sin^{-1} a\sqrt{1-a^2}}$

Question 5. Which of the following relationships is true?

A) ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$

B) ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_r$

C) ${}^nC_r - {}^nC_{r+1} = {}^{n+1}C_{r+1}$

D) ${}^nC_r \times {}^nC_{r+1} = {}^{n+1}C_{r+1}$

Question 6. **START A NEW PAGE.**

a) Solve $\frac{x^2-16}{x} \geq 0$ **(3)**

b) The point Q divides the interval joining $R(6, -3)$ and $P(-2, -5)$ externally in the ratio $3:2$. Find the coordinates of Q. **(2)**

c) The equation $e^x = 4x + 8$ has a root close to $x = 3$. Using 3 as the first approximation and Newton's method once, find an improved value of this root. Give answer correct to 3 decimal places. **(2)**

Question 7. **START A NEW PAGE.**

a) **i)** Show that $\frac{x+2}{x+1} = 1 + \frac{1}{x+1}$ **(1)**

ii) Carefully sketch the function defined by

$$g(x) = \frac{x+2}{x+1} \quad \text{for} \quad -1 < x$$ **(3)**

iii) Find $g^{-1}(x)$ and sketch it on the same graph as $g(x)$. **(3)**

iv) Find any value(s) of x for which $g(x) = g^{-1}(x)$ **(2)**

b) Find the exact value of $sin\left[2tan^{-1}\left(\frac{3}{4}\right)\right]$ **(2)**

Question 8. **START A NEW PAGE.**

a) Differentiate $\sin^{-1} 4x$ **(1)**

b) Find $\int \frac{x+1}{x^2+4} dx$ **(2)**

c) Evaluate $\int_0^2 \frac{1}{\sqrt{16-x^2}} dx$ **(2)**

d) The curve $y = \frac{1}{\sqrt{1+x^2}}$ is rotated about the x axis. Find the volume of the solid enclosed between $x = \frac{1}{\sqrt{3}}$ and $x = \sqrt{3}$. **(3)**

e) Differentiate $x \sin^{-1} x$ and hence find $\int_0^1 \sin^{-1} x \, dx$ **(3)**

Question 9. **START A NEW PAGE.**

a) Using the fact that $(1+x)^4(1+x)^9 = (1+x)^{13}$, show that:

$$^4C_0\ ^9C_4 + \ ^4C_1\ ^9C_3 + \ ^4C_2\ ^9C_2 + \ ^4C_3\ ^9C_1 + \ ^4C_4\ ^9C_0 = \ ^{13}C_4$$ **(3)**

b) By assigning an appropriate value to x in the expansion of $(1+x)^n$, prove

$$^nC_1 + \ ^nC_2 + \ ^nC_3 + \ldots\ldots + \ ^nC_{n-1} = 2^n - 2$$ **(2)**

c) Consider the binomial expansion

$$1 + \binom{n}{1}x + \binom{n}{2}x^2 + \ldots\ldots + \binom{n}{n}x^n = (1+x)^n$$

Show that:

i) $$1 - \binom{n}{1} + \binom{n}{2} - \ldots\ldots + (-1)^n\binom{n}{n} = 0$$ **(1)**

ii) $$\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \ldots\ldots + n\binom{n}{n} = n(2)^{n-1}$$ **(2)**

iii) $$1 - \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} - \ldots\ldots + (-1)^n\frac{1}{n+1}\binom{n}{n} = \frac{1}{n+1}$$ **(3)**

Solutions

1) C

2) ${}^8C_2 . 3^6 (-2)^2$

$= 81648$

C
B
B
C
A

3) B

4) $a_1 = a - \dfrac{\sin^{-1} a}{\frac{1}{\sqrt{1-a^2}}}$

$= a - \sqrt{1-a^2} . \sin^{-1} a$

$\therefore$ C.

5) A

Q 6

a) $\dfrac{x^2-16}{x} \geq 0$

critical point $x \neq 0$

Solve $\dfrac{x^2-16}{x} = 0$

$x^2 = 16$

$x = \pm 4$

× (−4) ✓ (0) × (4) ✓

Test: $x=-5$ F

$x=-1$ T

$x=1$ F

$x=5$ T

$-4 \leq x < 0$ or $x \geq 4$

b) $K(6,-3)$ $P(-2,-5)$

$3:-2$

$\left(\dfrac{-12-6}{1}, \dfrac{6-15}{1}\right)$

$= (-18, -9)$

c) $e^x = 4x + 8$

$e^x - 4x - 8 = 0$

$f(3) = e^3 - 12 - 8$

$= 0.0855$

$f'(x) = e^x - 4$

$f'(3) = e^3 - 4$

$= 16.0855$

$a_1 = a - \dfrac{f(a)}{f'(a)}$

$= 3 - \dfrac{0.0855}{16.0855}$

$= 2.995$

~~Question~~

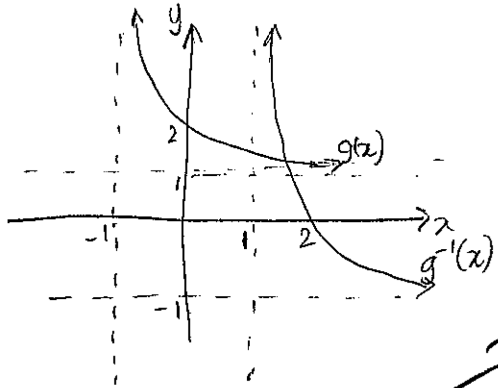
Question 7

a) i) $\dfrac{x+2}{x+1} = \dfrac{x+1+1}{x+1}$

$= \dfrac{x+1}{x+1} + \dfrac{1}{x+1}$

$= 1 + \dfrac{1}{x+1}$

ii) $g(x) = \dfrac{x+2}{x+1}$



iii) $x = \dfrac{y+2}{y+1}$

$xy + x = y + 2$

$xy - y = 2 - x$

$y(x-1) = 2 - x$

$y = \dfrac{2-x}{x-1}$ $= \dfrac{1}{x-1} - \dfrac{(x-1)}{x-1} = \dfrac{1}{x-1} - 1$.

$g: y = 1 + \dfrac{1}{x+1}$

$g^{-1}: x = 1 + \dfrac{1}{y+1}$

$x - 1 = \dfrac{1}{y+1}$

$y + 1 = \dfrac{1}{x-1}$

$y = -1 + \dfrac{1}{x-1}$

iv) $g(x) = g^{-1}(x)$ when $y = x$.

$\therefore \dfrac{2-x}{x-1} = x$

$2 - x = x^2 - x$

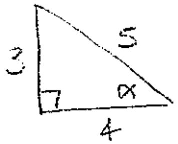
$x^2 = 2$

$x = \sqrt{2}$ ($-\sqrt{2}$ outside domain)

b) $\sin\left[2\tan^{-1}\left(\frac{3}{4}\right)\right]$

$\tan^{-1}\left(\frac{3}{4}\right) = \alpha$

$\tan\alpha = \frac{3}{4}$



$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$= 2 \cdot \frac{3}{5} \cdot \frac{4}{5}$

$= \frac{24}{25}$

Q8

a) $\frac{d}{dx} \sin^{-1} 4x$

$= \frac{4}{\sqrt{1-16x^2}}$

b) $\int \frac{x+1}{x^2+4}\, dx$

$= \int \frac{x}{x^2+4}\, dx + \int \frac{1}{x^2+4}\, dx$

$= \frac{1}{2}\ln(x^2+4) + \frac{1}{2}\tan^{-1}\frac{x}{2} + C$

c) $\int_0^2 \frac{1}{\sqrt{16-x^2}}\, dx$

$= \left[\sin^{-1}\frac{x}{4}\right]_0^2$

$= \sin^{-1}\frac{1}{2} - \sin^{-1} 0$

$= \frac{\pi}{6}$

d) $y = \frac{1}{\sqrt{1+x^2}}$

$V = \pi \int_{1/\sqrt{3}}^{\sqrt{3}} \frac{1}{1+x^2}\, dx$

$= \pi\left[\tan^{-1} x\right]_{1/\sqrt{3}}^{\sqrt{3}}$

$= \pi\left[\tan^{-1}\sqrt{3} - \tan^{-1}\frac{1}{\sqrt{3}}\right]$

$= \pi\left[\frac{\pi}{3} - \frac{\pi}{6}\right]$

$= \frac{\pi^2}{6}$ cubic units

e) $\frac{d}{dx} x\sin^{-1}x = \frac{x}{\sqrt{1-x^2}} + \sin^{-1}x$

$\therefore \sin^{-1}x = \frac{d}{dx} x\sin^{-1}x - \frac{x}{\sqrt{1-x^2}}$

$$\int_0^1 \sin^{-1}x\,dx = \int_0^1 \frac{d}{dx}(x\sin^{-1}x)\,dx - \int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx$$

$$= \left[x\sin^{-1}x\right]_0^1 - \frac{1}{2}\int_0^1 2x(1-x^2)^{-1/2}\,dx$$

$$= \sin^{-1}1 + \left[(1-x^2)^{1/2}\right]_0^1$$

$$= \frac{\pi}{2} + [0-1]$$

$$= \frac{\pi}{2} - 1$$

Question 9

a) ${}^{13}C_4$ is the coefficient of x^4 in $(1+x)^{13}$

in $(1+x)^4(1+x)^9$, the term in x^4 is:

$${}^4C_0 . {}^9C_4 x^4 + {}^4C_1 x\, {}^9C_3 x^3 + {}^4C_2 x^2\, {}^9C_2 x^2 + {}^4C_3 x^3\, {}^9C_1 x + {}^4C_4 x^4\, {}^9$$

$\therefore$ coefficient of x^4 is

$${}^4C_0\,{}^9C_4 + {}^4C_1\,{}^9C_3 + {}^4C_2\,{}^9C_2 + {}^4C_3\,{}^9C_1 + {}^4C_4\,{}^9C_0$$

since $(1+x)^4(1+x)^9 = (1+x)^{13}$

$${}^4C_0\,{}^9C_4 + {}^4C_1\,{}^9C_3 + {}^4C_2\,{}^9C_2 + {}^4C_3\,{}^9C_1 + {}^4C_4\,{}^9C_0 = {}^{13}C_4$$

b) $${}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_{n-1} x^{n-1} + {}^nC_n x^n = (1+x)^n$$

sub $x = 1$

$$1 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_{n-1} + 1 = 2^n$$

$${}^nC_1 + {}^nC_2 + \dots + {}^nC_{n-1} = 2^n - 2$$

c) $$1 + \binom{n}{1}x + \binom{n}{2}x^2 + \dots + \binom{n}{n}x^n = (1+x)^n$$

i) sub $x = -1$

$$1 - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \dots + (-1)^n\binom{n}{n} = 0$$

ii) differentiate both sides:

$$\binom{n}{1} + 2\binom{n}{2}x + 3\binom{n}{3}x^2 + \dots + n\binom{n}{n}x^{n-1} = n(1+x)^{n-1}$$

sub $x = 1$

$$\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \dots + n\binom{n}{n} = n.(2)^{n-1}$$

iii) Integrate both sides

$$x + \frac{1}{2}\binom{n}{1}x^2 + \frac{1}{3}\binom{n}{2}x^3 + \dots + \frac{1}{n+1}\binom{n}{n}x^{n+1} = \frac{(1+x)^{n+1}}{n+1} + C$$

to find C, let $x = 0$

$$0 = \frac{1}{n+1} + C$$

$$\therefore C = \frac{-1}{n+1}$$

$\therefore\ x+\frac{1}{2}\binom{n}{1}x^2+\frac{1}{3}\binom{n}{2}x^3+\cdots+\frac{1}{n+1}\binom{n}{n}x^{n+1}=\frac{(1+x)^{n+1}}{n+1}-\frac{1}{n+1}$

when $x=-1$

$-1+\frac{1}{2}\binom{n}{1}-\frac{1}{3}\binom{n}{2}+\cdots+(-1)^{n+1}\cdot\frac{1}{n+1}\binom{n}{n}=\frac{-1}{n+1}$

$-1+\frac{1}{2}\binom{n}{1}-\frac{1}{3}\binom{n}{2}+\cdots+(-1)^n\cdot(-1)^1\cdot\frac{1}{n+1}\binom{n}{n}=\frac{-1}{n+1}$

$1-\frac{1}{2}\binom{n}{1}+\frac{1}{3}\binom{n}{2}-\cdots\cdots+(-1)^n\frac{1}{n+1}\binom{n}{n}=\frac{1}{n+1}$