James Ruse Agricultural High School
Year 12 Assessment Term 2 1996
Mathematics 3/4 Unit

Time Allowed: 85 Minutes.

Instructions:

- All questions to be attempted.
- Approved calculators may be used.
- All necessary working must be shown.
- Each section is to be handed in separately.

SECTION A: (12 Marks) START A NEW PAGE.

1. The rate of change of a population is given by:

$$\frac{dP}{dt} = 0{\cdot}3\,(P - 50)$$

(a) Show that $P = 50 + Ae^{0.3t}$ is a solution of the equation:

$$\frac{dP}{dt} = 0{\cdot}3\,(P - 50).$$

(b) If $t = 0$ when $P = 70$, find P when $t = 15$.

2. (a) From a regular pack of 52 cards, how many different ways are there of selecting four cards?

(b) How many of these will involve exactly 2 aces?

3. In how many ways can the letters of the word EQUATION be arranged if:

(a) The new arrangement begins with N and ends with E.

(b) The vowels are together at the beginning of the new arrangement.

SECTION B: (12 Marks) START A NEW PAGE.

1. Susan and Richard are invited to Veronica's dinner party of eight. Find the number of ways of seating the eight people around a circular table if:

(a) There is no restrictions on where a person sits.

(b) Richard wishes to sit between Susan and Veronica.

2. During a chemical reaction the amount, n grams, of a substance unconverted after t hours is given by the formula $n = 5 + e^{-2kt}$

(a) Show that $\frac{dn}{dt} = -2k(n - 5)$.

(b) If initially n decreases at the rate of 0·05 gms/hour, find the value of k.

(c) Taking the value of k found in part (a), sketch the graph of $n = 5 + e^{-2kt}$ and indicate the values that n can take.

SECTION C: (12 Marks) START A NEW PAGE.

1. In an aerobics competition there are 8 contestants, 3 of which are men and 5 are women. If there is to be a team of four chosen, what is the probability that at least 3 of them are women?

2. A projectile is fired from ground level at an angle α to the horizontal and with a velocity vm/s. If the projectile returns to the ground after 5 seconds and its range is 150 metres, assuming that acceleration due to gravity is 10m/s^2 and the ground is level, find:

(a) The value of v and α.

(b) The maximum height attained.

3. Andrew and Katherine throw a die for a prize which is won by the player who first throws a six. If Katherine was first throw, find her chance of winning.

SECTION D: (12 Marks) START A NEW PAGE.

1. A particle vibrates in Simple Harmonic Motion, making 100 oscillations per second.

   (a) Show that the acceleration is given by $\ddot{x} = -40\,000\pi^2 x$.

   (b) If the amplitude of the motion is 20cm, calculate the speed of the particle at:

   (i) The centre of its motion.

   (ii) The extremities of the motion.

   (iii) A distance 10cm from the centre of the motion.

2. A particle moving on a straight line has acceleration $\frac{-4}{x^2}$. If v = 3 when x = 1, then:

   (a) Find v^2 as a function of x.

   (b) Describe the motion.

SECTION E: (12 Marks) START A NEW PAGE.

1. A man standing on level ground throws a cricket ball with a speed of $15ms^{-1}$ at an angle of $\tan^{-1}\frac{3}{4}$. If the ball leaves the man's hand at a height of 2 metres above the ground, then:-

   (a) Write down the equations x = x(t) and y = y(t) which represent the motion taking the origin at ground level, vertically below the man's hand and $g = 10m/s^2$. Find the position of the ball after 1 second.

   (b) Prove that the equation of the path of the ball is $y = \frac{1}{144}(288 + 108x - 5x^2)$.

   (c) Find the point where the ball reaches the ground.

   (d) Find the greatest height reached by the ball.

   (e) At what angle should the ball be thrown if it is to hit the top of a 1 metre tall stump, whose horizontal distance from the man is 20 metres. Give your answer(s) to the nearest degree.

SECTION F: (12 Marks) START A NEW PAGE.

1. The acceleration of a particle is given by $\ddot{x} = -e^{-2x}$. If initially x = 0 and $v = 1ms^{-1}$, find x as a function of t.

2. The rise and fall of the tide at the mouth of the Ord River in Western Australia is assumed to be Simple Harmonic Motion. The depth of water at low tide on a particular day was 0·7 metres and the depth of water at high tide was 3·7 metres. If low tide occurred at 8:55am and high tide at 3:05pm, find the earliest possible time after low tide at which a fishing boat could enter the mouth of the river if it required the water to be at least 2 metres deep.

END OF PAPER

SECTION A

1996 3 UNIT

TERM 2

① (i) $P = 50 + Ae^{0.3t}$

$$\frac{dP}{dt} = 0.3Ae^{0.3t}$$

$$= 0.3(P-50) \quad \text{since } Ae^{0.3t} = P-50$$

(ii) $t=0,\ P=70$

$70 = 50 + A$

$A = 20$

$\therefore P = 50 + 20e^{0.3t}$

when $t = 15$

$P = 50 + 20e^{0.3\times15}$

$= 1850$ (to nearest integer)

② (a) ${}^{52}C_4$

(b) ${}^{4}C_2 \cdot {}^{4}C_2$.

③ (i) N ⓒ ⓢ ⓨ ⓢ ⓞ ⓓ E

$N^{\circ} = 6!$

(ii) Vowels together $= 5!$

(vowels) ③ ② ①

Total $= 5!3!$

SECTION B

① (a) $7!$

(b). (SRV) $= 2!$

(SRV) $+5$ other in $\bigcirc = 5!$

Total $= 2!5!$

$= 240$

② (a) $\frac{dn}{dt} = -2Ke^{-2Kt}$

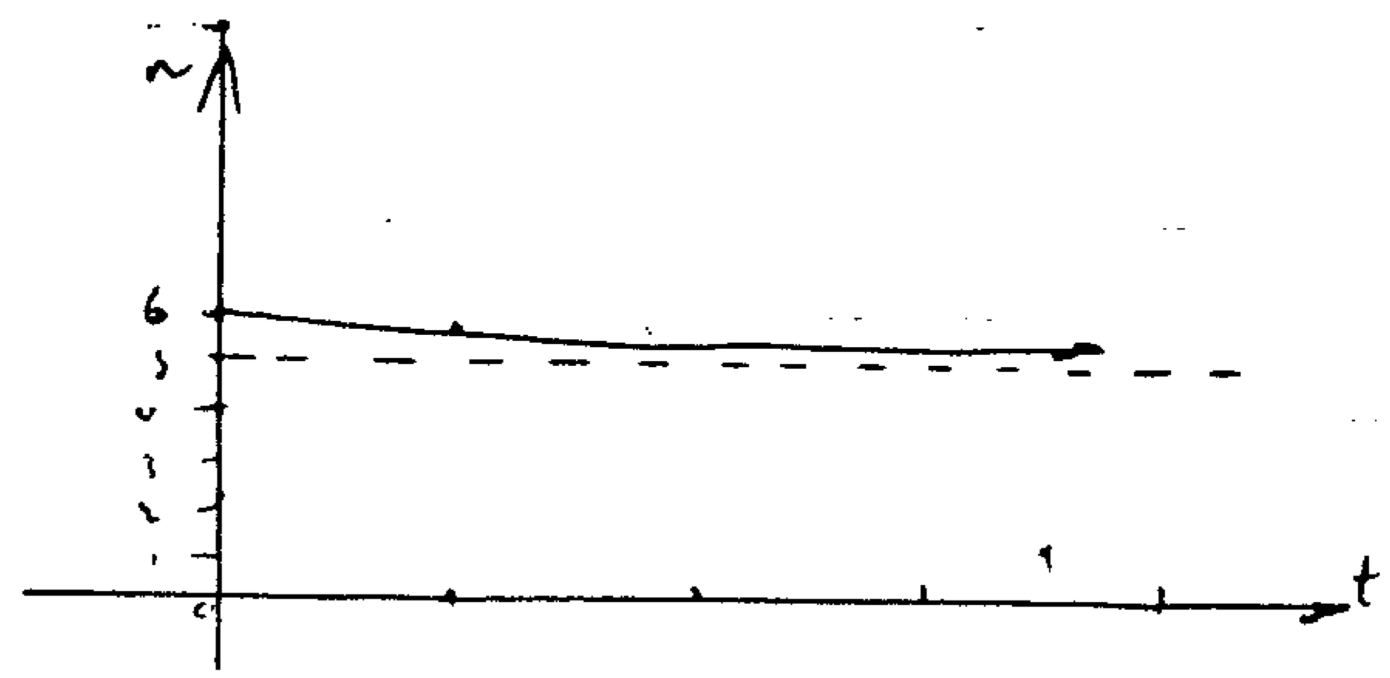
$= -2K(n-5)$ since $e^{-2Kt} = n-5$

(b) $t=0,\ \frac{dn}{dt} = -0.05$

$-0.05 = -2K$.

$K = 0.025$

(c)



$5 < n \leq 6$

SECTION C

① (i) 3M, 5W

$$\text{Fav. evt} = {}^5C_3 \cdot {}^3C_1 + {}^5C_4 \quad (3W,1M \text{ or } 4W)$$

$$S/\text{space} = {}^8C_5$$

$$\text{Prob} = \frac{{}^5C_3 \cdot {}^3C_1 + {}^5C_4}{{}^8C_4}$$

② $x = V\cos\alpha\, t \qquad y = -\tfrac{1}{2}gt^2 + V\sin\alpha\, t$

(a) $t=5,\ y=0$

$$0 = -5(5)^2 + 5V\sin\alpha$$
$$V\sin\alpha = 25$$

$t=5,\ x=150$

$$150 = 5V\cos\alpha$$
$$V\cos\alpha = 30$$

$$\tan\alpha = 5/6$$
$$\alpha \doteq 39°48'$$

$$V^2 = 25^2\ 30^2$$
$$= 1525$$
$$V = 5\sqrt{61}\ \text{m/s}$$

(b) $\text{max ht.} = \dfrac{V^2\sin^2\alpha}{2g} = \dfrac{(V\sin\alpha)^2}{2g}$

$$= \frac{25^2}{20}$$

$$= 31.25\ \text{m}$$

③ $P(\text{Kath. wins}) = \tfrac{1}{6} + (\tfrac{5}{6})^2\cdot\tfrac{1}{6} + (\tfrac{5}{6})^4\cdot\tfrac{1}{6} + \cdots$

$$= \tfrac{1}{6}\left[1 + \tfrac{25}{36} + (\tfrac{25}{36})^2 + \cdots\right]$$

$$= \tfrac{1}{6}\left[\frac{1}{1-\frac{25}{36}}\right] = \tfrac{1}{6} \times \frac{36}{11}$$

SECTION D

(1) (a) $T = 2\pi/n$

$$2\pi/n = \frac{1}{100}$$
$$n = 200\pi$$

$$\ddot{x} = -n^2 x$$
$$\ddot{x} = -40000\pi^2 x$$

(b) (i) centre is at $x=0$

$\cancel{x=20}$ $\quad v^2 = n^2(c^2 - x^2)$

(iii) $V^2 = 40000\pi^2(400 - x^2)$

$x=0,\quad V^2 = 40000\pi^2(400)$

$$V = 200\pi \times 20$$
$$= 4000\pi\ \text{cm/s}$$

(ii) $x=\pm 20,\quad v = 0\ \text{cm/s}$

(iii) $V^2 = 40000\pi^2(400-100)$

$$= 40000\pi^2 \times 300$$
$$V = 200\pi \times 10\sqrt{3}$$
$$= 2000\sqrt{3}\,\pi\ \text{cm/s}$$

(2) (a) $\ddot{x} = -4/x^2$

$$\frac{d}{dx}\left(\tfrac{1}{2}v^2\right) = -4/x^2$$
$$\tfrac{1}{2}v^2 = 4/x + c$$

$x=1,\ v=3$

$$9/2 = 4 + c$$
$$c = 1/2$$
$$V^2 = 8/x + 1.$$

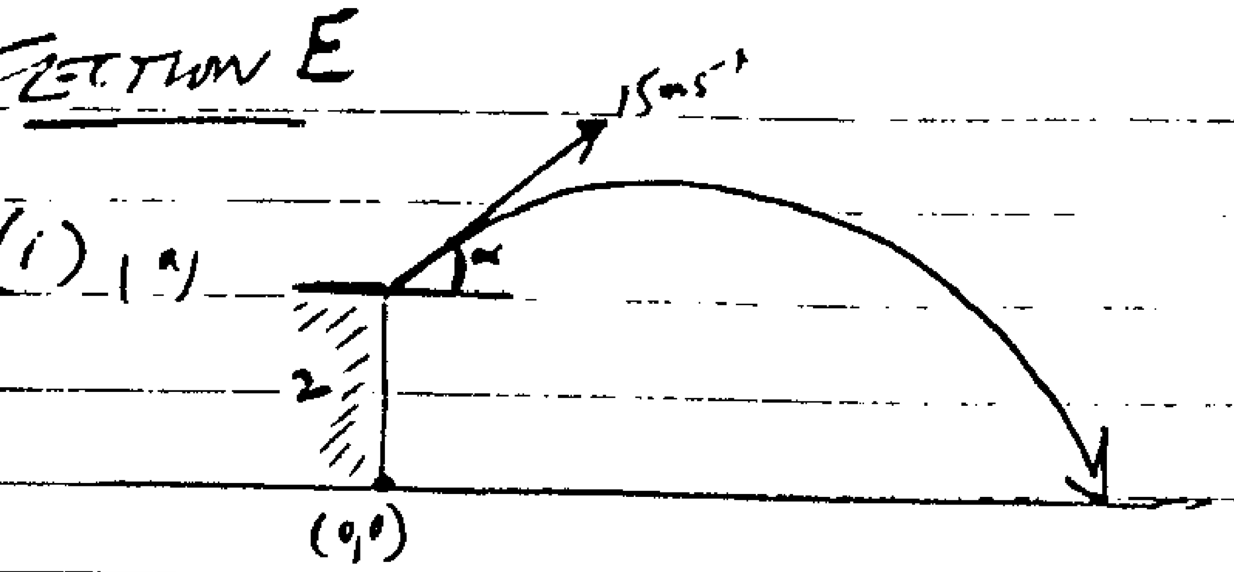
(b) $V=0$ when $8/x + 1 = 0$

$$8/x = -1$$
$$x = -8$$

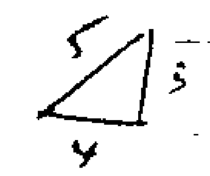
particle moves to right from $x=1$ ∴ never comes to rest since $v=0$ when $x=-8$, however $v \to 1$ as $x \to \infty$ ∴ particle moves to right with decreasing speed

SECTION E

(1) (a)



$\tan\alpha = 3/4$



$x = 15\cos\alpha\, t$

$= 15 \cdot 4/5\, t$

$= 12t$

$y = -5t^2 + 15 \cdot 3/5\, t + 2$

$y = -5t^2 + 9t + 2$

when $t = 1$, $x = 12$, $y = 6$

(b) $y = -5\left(\frac{x}{12}\right)^2 + 9\left(\frac{x}{12}\right) + 2$

$= \frac{-5x^2}{144} + \frac{3}{4}x + 2$

$y = \frac{1}{144}\left(288 + 108x - 5x^2\right)$

(c) $y = 0$, $288 + 108x - 5x^2 = 0$

$x = \frac{-108 \pm \sqrt{108^2 - 4(288)(-5)}}{-10}$

$= \frac{-108 \pm 132}{-10}$

$= 24, -12/5$

but $x \geq 0$

$\therefore$ dist $= 24$ m

(d) max ht at vertex

ie. $x = \frac{-108}{-10}$

$= +10.8$

$y = \frac{1}{144}\left[288 + 108(10.8) - 5(10.8)^2\right]$

$= 6.05$ m

(e) $y = x\tan\alpha - \frac{gx^2}{2V^2}(1 + \tan^2\alpha) + 2$

$-1 = 20\tan\alpha - \frac{80}{9}(1 + \tan^2\alpha)$

$-9 = 180\tan\alpha - 80 - 80\tan^2\alpha$

$80\tan^2\alpha - 180\tan\alpha + 71 = 0$

$\tan\alpha = \frac{180 \pm \sqrt{180^2 - 4(80)(71)}}{160}$

$= 1.74, 0.51$

$\alpha = 60°, 27°$

SECTION F

① $\ddot{x} = -e^{-2x}$

$\frac{d}{dx}\left(\tfrac{1}{2}v^2\right) = -e^{-2x}$

$\tfrac{1}{2}v^2 = \tfrac{1}{2}e^{-2x} + c$

$t = 0$, $x = 0$, $v = 1$

$\tfrac{1}{2} = \tfrac{1}{2} + c$

$c = 0$

$v^2 = e^{-2x}$

$v \neq 0$ $\therefore$ never stops

$\therefore$ moves in original direction $\rightarrow$.

$v = e^{-x}$ $(v > 0)$

$\frac{dx}{dt} = e^{-x}$

$\frac{dt}{dx} = e^{x}$

$t = e^{x} + d$

$t = 0$, $x = 0$

$0 = 1 + d$

$d = -1$

$t = e^{x} - 1$

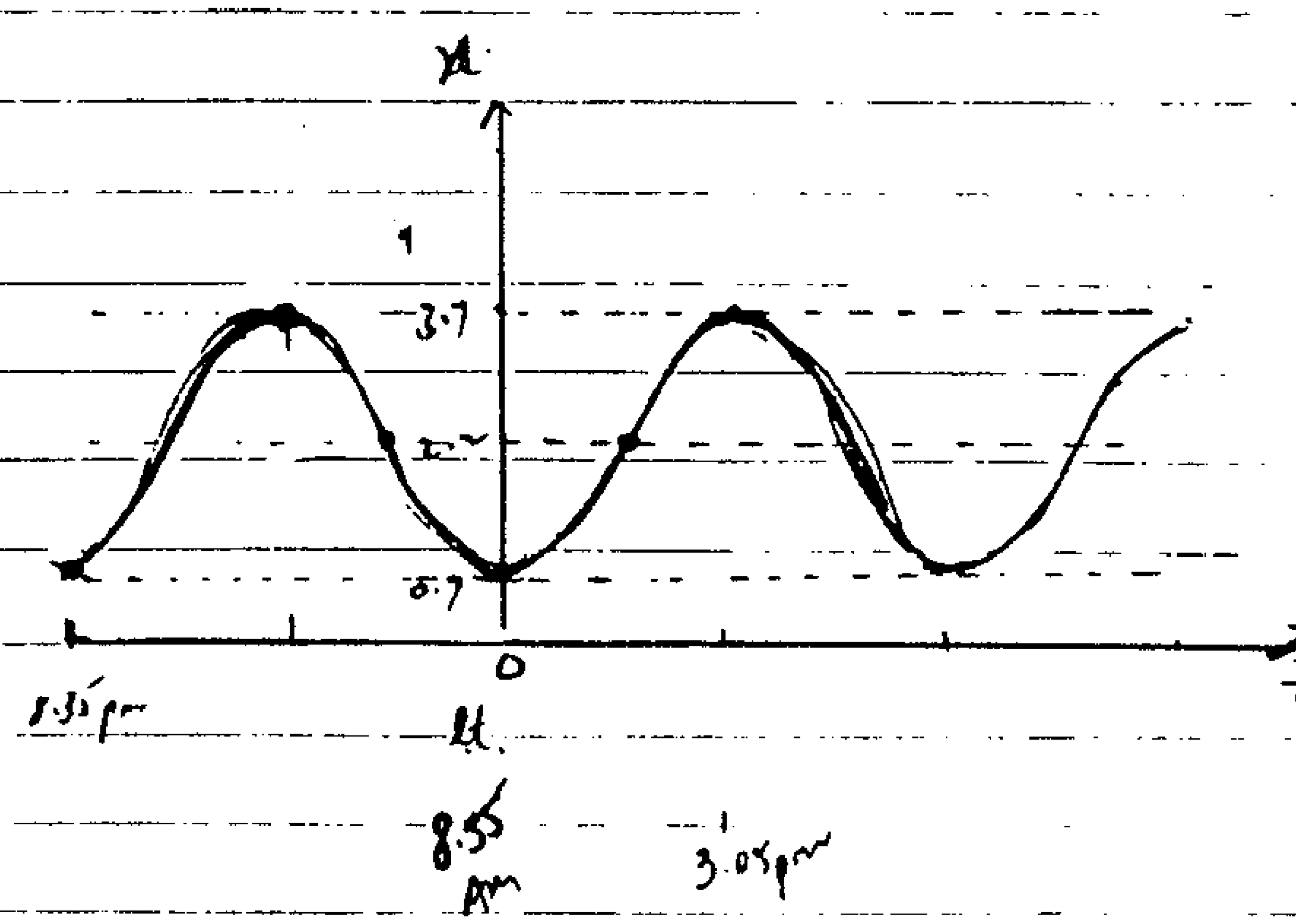
$t + 1 = e^{x}$

$x = \ln(t + 1)$

② $T = 12\tfrac{1}{3}$

$\frac{2\pi}{n} = \frac{37}{3}$

$n = \frac{6\pi}{37}$



$x = 2.2 - 1.5\cos\frac{6\pi}{37}t$

when $x = 2$

$2 = 2.2 - 1.5\cos\frac{6\pi}{37}t$

$\cos\frac{6\pi}{37}t = \frac{0.2}{1.5}$

$t = \frac{37}{6\pi}\left\{2n\pi \pm \cos^{-1}\left(\frac{2}{15}\right)\right\}$

[illegible]