JAMES RUSE AGRICULTURAL HIGH SCHOOL

YEAR 12

1997 Mathematics 3u Term 2 Open Book Assessment

Time Allowed: 85 minutes
- Start each question on a new page
- Marks may not be awarded for poorly arranged work
- No equipment may be borrowed during the exam.

Question 1 (Start a new page)

(a) The rate of change in the number of birds (B) in a colony is given by $\frac{dB}{dt} = k(B - 1000)$ where t is in years and k is a constant. The number of birds at the start of 1990 was 1500.

(i) Show that $B = 500e^{kt} + 1000$

(ii) At the start of 1992 the number was 1800. Find the number at the start of 1997 to the nearest 100.

(b) A particle moves in a straight line so that at time t its displacement (x) from the origin is given by $x = t^3 - 2t^2 + 3t - 1$.

(i) By finding v (in terms of t) show that the particle never stops
(ii) Find when it is travelling at its slowest speed (Justify your answer)

Question 2 (Start a new page)

(a) The letters of the word DEPOSITOR are to be arranged in a row. Find the number of arrangements if :-

(i) the "word" starts and ends with a consonant
(ii) the vowels occupy the even places.

(b) The acceleration of a particle moving in a straight line is given by $\frac{d^2x}{dt^2} = 2x - 4$ where x is the displacement, in metres, from the origin O and t is the time in seconds. Initially the particle is at rest at x = 4

(i) If the velocity of the particle is v m/s, show that $v^2 = 2(x^2 - 4x)$
(ii) Show that the particle does not pass through the origin.
(iii) Find the position of the particle when $v = \sqrt{90}$. Justify your answer.

Question 3 (Start a new page)

A particle P is moving in SHM about a fixed point O. The displacement x metres at time t seconds is given by $x = 4\sqrt{2}\sin(3t + \frac{\pi}{4})$

(i) Find the equation for velocity in terms of x
(ii) Show $\ddot{x} = -9x$
(iii) Find the initial displacement and velocity of P
(iv) Find the time taken to reach the maximum displacement from O, for the first time.

Question 4 (Start a new page)

(a) The letters A, B, C, D, E, F, G are arranged around a circle. Find the probability that A and B are not to together.

(b) A daredevil on a motorbike is planning to jump a gorge 200m wide. At the gorge's edge the speed of the bike is 50m/s and the angle of projection is 30° above the horizontal.

(i) Write down without proof the cartesian equation of the trajectory.
(ii) Show that the rider succeeds in jumping the river.
(iii) Find the angle that the bike makes with the horizontal on the other side of the river.

Question 5 (Start a new page)

(a) The speed V (m/s) of a point moving aong the x axis is given by

$$v^2 = 144x - 36x^2 + 180$$

(i) Prove the motion is SHM
(ii) What is the maximum speed of the motion.

(b)A particle is projected with an initial speed of 64m/s (at an angle θ)towards a wall which is 100 m horizontally from the point of projection and 10 m high. (Air resistance is to be neglected and the acceleration due to gravity is taken as $10m/s^2$.)

(i) Show that the particle just clears the wall when

$$50000\tan^2\theta - 409600\tan\theta + 90960 = 0$$

(ii) In what range must the angle of projection lie for the wall to be cleared.

Question 6 (Start a new page)

(a) A total of 6 players is selected at random to form a volleyball team from a group of 10 girls and 5 boys. 6 of the girls and 3 of the boys have played the game before. Find the probability that of the 6 selected :-

(i) 4 are girls and 2 are boys.
(ii) at least 4 of the players are girls that have played the game before.

(b) To lift a weight a boy uses a rope 20m long thrown over a tree branch which is 6m off the ground. He ties the rope around his waist (1m off the ground) and walks off at constant speed of 1m/s.
At time t, let the boy be distant x metres from the vertical below the weight and let the weight be y metres above the ground.

(i) Draw a diagram illustrating the above information
(ii)Find how fast the weight is rising when it is 3m above the ground.

THIS IS THE END OF THE TEST

Q1 (a)

(i) $B = 500e^{kt} + 1000$

$\frac{dB}{dt} = 500ke^{kt}$

$= k(B - 1000)$ 2

(ii) $1800 = 500e^{2k} + 1000$

$k = 0.235$ $\left(\frac{1}{2}\ln 8/5\right)$

when $t = 7$

$B = 500e^{0.235 \times 7} + 1000$

$= 3590$

$= 3600$ (Nearest ...) 2

(b) $x = t^3 - 2t^2 + 3t - 1$

(i) $V = 3t^2 - 4t + 3$ 2

which is pos def $\left(\begin{matrix} a>0 \\ \Delta<0 \end{matrix}\right)$

$\therefore$ always > 0

(ii) $a = 6t - 4$

Slowest speed when $t = 2/3$

Up to that point acceleration is acting against motion, thus slowing it & after $t = 2/3$ acceleration acting in same direction as motion thus accelerating it 2

Q2 (a)

(i) $\frac{5 \times 4 \times 7!}{2!} = 50400$ 2

(ii) $\frac{5! \times 4!}{2} = 1440$

(b) (i) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 2x - 4$

$\frac{1}{2}v^2 = x^2 - 4x + c$

$c = 0$

$v^2 = 2(x^2 - 4x)$ 2

(ii) Particle starts from rest at $x = 4$ & proceeds right $(a > 0)$ after $x = 4$ velocity is never 0 $\therefore$ it continues right $\therefore$ Never reaches 0. 2

(iii) when $v = \sqrt{90}$

$90 = 2(x^2 - 4x)$

$45 = x^2 - 4x$

$x^2 - 4x - 45 = 0$ 2

$(x - 9)(x + 5) = 0$

$x = 9$ or -5.

but it cannot be $x = -5$ see previous comment

$\therefore x = 9$

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TERM 2

Q3

(i) $\dot{x} = 12\sqrt{2}\cos\left(3t + \frac{\pi}{4}\right)$

$= \pm 12\sqrt{2}\sqrt{\cos^2(3t + \pi/4)}$

$= \pm 12\sqrt{2}\sqrt{1 - \sin^2(3t + \pi/4)}$

$= \pm 12\sqrt{2}\sqrt{1 - \frac{x^2}{32}}$

$= \pm 3\sqrt{32 - x^2}$ 3

(ii) $v^2 = 9(32 - x^2)$ 2

$\frac{1}{2}v^2 = 9/2\,(32 - x^2)$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \ddot{x} = -9x$

(iii) $t = 0$ $x = 4$ m 1

$V = 12$ m/s 1

(iv) Displacement$_{MAX} = \sqrt{32} = 4\sqrt{2}$

Max displacement when $v = 0$

$\sqrt{32} = 4\sqrt{2}\sin\left(3t + \pi/4\right)$ 3

$1 = \sin\left(3t + \pi/4\right)$

$t = \frac{\pi}{12}$ sec

Qu3

(i) $\dot{x} = 12\sqrt{2}\cos\left(3t + \frac{\pi}{4}\right)$

Square both sides

$v^2 = 144 \times 2\cos^2\left(3t + \frac{\pi}{4}\right)$

$v^2 = 288\left(1 - \sin^2\left(3t + \frac{\pi}{4}\right)\right)$ 1 3

$v^2 = 288\left(1 - \frac{x^2}{32}\right)$ as $x^2 = 32\sin^2\left(3t + \frac{\pi}{4}\right)$

$v^2 = ~~288~~ \ldots x^2$ $\frac{288(32 - x^2)}{32}$ 1

$v = \pm ~~\ldots~~ x^2$ $9(32 - x^2)$

$v = \pm 3\sqrt{32 - x^2}$ 1

(ii) $x = 4\sqrt{2}\sin\left(3t + \frac{\pi}{4}\right)$

$v = 12\sqrt{2}\cos\left(3t + \frac{\pi}{4}\right)$ 1 2

$\ddot{x} = -12\sqrt{2}\sin\left(3t + \frac{\pi}{4}\right) \times 3$

$= -36\sqrt{2}\sin\left(3t + \frac{\pi}{4}\right)$

$= -9\left[4\sqrt{2}\sin\left(3t + \frac{\pi}{4}\right)\right]$ 1

$= -9x$

Qu3 (iii) $x = 4\sqrt{2}\sin\left(3t + \frac{\pi}{4}\right)$

when $t = 0$

$x = 4\sqrt{2} \times \frac{1}{\sqrt{2}}$ 1

$x = 4$

$v = 12\sqrt{2}\cos\left(3t + \frac{\pi}{4}\right)$

$= 12\sqrt{2} \times \frac{1}{\sqrt{2}}$ 1

$v = 12$

(iv) Max displacement when

$v = \sqrt{32 - x^2}$

$\therefore x = \sqrt{32}$

$\sqrt{32} = 4\sqrt{2}\sin\left(3t + \frac{\pi}{4}\right)$

$1 = \sin\left(3t + \frac{\pi}{4}\right)$

$\sin\frac{\pi}{2} = \sin\left(3t + \frac{\pi}{4}\right)$

$3t = \frac{\pi}{4}$

$t = \frac{\pi}{12}$ sec.

4

(a) $\frac{4! \times 5 \times 4}{6!} = \frac{2}{3}$

(b)(i) $y = x\tan\alpha - \frac{gx^2}{2V^2}(1 + \ldots$

(ii) $0 = \frac{x}{\sqrt{3}} - \frac{10x^2}{5000} \cdot 4/3$

$\therefore x\left(\frac{1}{\sqrt{3}} - \frac{4x}{1500}\right) = \frac{x}{\sqrt{3}}$

$x = \frac{1500\sqrt{3}}{12}$

$= 216.5$ m

$\therefore$ successful

(iii) $y = \frac{\sqrt{3}x}{3} - \frac{4x^2}{1500}$

$y' = \frac{\sqrt{3}}{3} - \frac{8x}{1500}$ $\left(= -\frac{1}{\sqrt{3}}\right)$

$= -0.5773$ when $x = 216.5$

$\tan\theta = -0.5773$ 1

$\theta = 150°$ (accept 30° as an answer)

Q5

(a) (i) $v^2 = 144x - 36x^2 + 180$

$\frac{1}{2}v^2 = 72x - 18x^2 + 90$

3 $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 72 - 36x$

$= -36(x-2)$

which is of the form $-n^2(x-b)$

∴ SHM

(ii) Max speed occurs at centre of motion:

2 $v^2 = 144x - 36x^2 + 180$ at $x = 2$

$= 288 - 144 + 180$

$= 324$

$v = 18$

Max speed = 18 m/s

(b) (i) $y = x\tan\alpha - \frac{gx^2}{2v^2}(1+\tan^2\alpha)$

$y = 10$, $x = 100$, $g = 10$

$10 = 100\tan\alpha - \frac{10 \cdot 10000}{2.(64)^2}(1+\tan^2\alpha)$

$10 = 100\tan\alpha - \frac{50000}{4096}(1+\tan^2\alpha)$

$50000\tan^2\alpha - 409600\tan\alpha + 90960 = 0$

(ii) $\tan\theta = \frac{40960 \pm \sqrt{(40960)^2 - (4.5000.9096)}}{10000}$

3 $= \frac{40960 \pm 38675.94}{10000}$

$\theta = 82°50'$ or $12°52'$

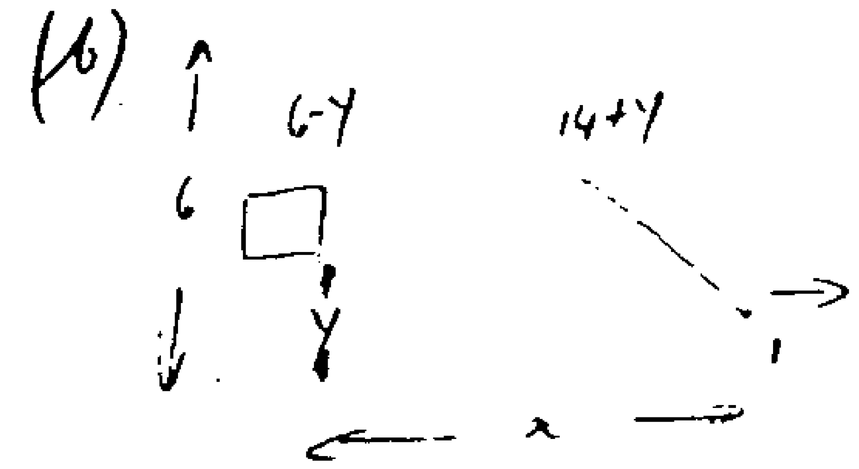
∴ to clear wall $12°52' < \theta < 82°50'$

Q6

(a) (i) $\frac{\binom{10}{4}\binom{5}{2}}{\binom{15}{6}} = \frac{60}{143}$ 2

(ii) $\frac{\binom{6}{4}\binom{9}{2} + \binom{6}{5}\binom{9}{1} + \binom{6}{6}}{\binom{15}{6}}$ 2 $= \frac{17}{143}$

(b)



1

$(14+y)^2 = x^2 + 25$ 2

$y = (x^2+25)^{\frac{1}{2}} - 14$

$y' = \frac{1}{2}(x^2+25)^{-\frac{1}{2}} \cdot 2x$

$= \frac{x}{\sqrt{x^2+25}}$ 1

when $y = 3$ $(17)^2 = x^2 + 25$

$x = \sqrt{264}$

$\frac{dy}{dx} = \frac{\sqrt{264}}{\sqrt{289}}$ 1

$= \frac{2\sqrt{66}}{17}$

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

$= \frac{2\sqrt{66}}{17} \times 1$ 1

$= \frac{2\sqrt{66}}{17}$ m/s