JAMES RUSE AGRICULTURAL HIGH SCHOOL

TERM TWO ASSESSMENT TASK

3 UNIT 1998

STRUCTIONS:
Answer each section on a new piece of paper.
Each section is worth 10 marks.
Show all necessary working and formulae.
Marks may be deducted for careless or untidy working.
Textbooks may be used for this exam.
The exam is 85 minutes duration.
Put your name and class on every page.

CTION A (10 marks)

The position of a particle on a line at time t, is given by $x = 5 + e^{3t}$ where x is its distance from a fixed int O.Find: a) Its initial position and initial velocity.

b) An expression for its acceleration in terms of x.

a) Show that the displacement of a particle moving in a straight line about a fixed point O is in the form : $\ddot{x} = -n^2x$, if $x = \sqrt{3}\cos 3t - \sin 3t$.

b) When is this particle first at the point O ?

CTION B (10 marks)

a) Show that $N = 80 + 45e^{-0.08t}$ is a solution to $\frac{dN}{dt} = 0.08(80 - N)$.

b) If $N = 110$ find t to the nearest whole number.

c) Graph $N = 80 + 45e^{-0.08t}$ for $0. \leq t \leq 50.$

A particle moving in a straight line begins its journey from rest at a fixed point O. If its acceleration at me t, is given by $a = (t + 1)^2$, where $t \geq 0$, find an expression for its displacement in terms of t.

Jenny, Alicia, Nicky, Lauren and Sarah attend a BBQ,the only round table left is one for six people.If aniel is already seated at the table, how many different arrangements are possible if Nicky refuses to sit xt to neither Daniel nor Sarah ?

SECTION C (10 marks):

1. A particle is moving in a straight path and its velocity at time t is given by $v = -3\sin 2t$.
   a) Calculate its velocity when t= 0.9seconds correct to two decimal places.
   b) Write down an expression for the displacement at time t, if initially the particle is 3m to the right of O.

2.a) A committee of five is to be chosen from a group of four boys and four girls.How many different committees are possible if there is to be a majority of girls on the committee ?

b) What is the probability of a particular girl being on the committee and a particular boy is not ? (Assuming there still is a majority of girls on the committee.)

3. A particle moves in a straight line so that at time t, its displacement from a fixed point is x metres and its velocity is v m/s.If its acceleration is $3 - x$ and $v = 2$ when $x = 0$, find v when $x = 1$ giving reasons.

SECTION D (10 marks):

1. A particle is projected from a window 9 metres above the horizontal ground at an angle of $\tan^{-1}\left(\frac{3}{4}\right)$ at 20 m/s. a) Write down the expressions for the x and y components of the displacement in terms of t, take g to be $10m/s^2$, and hence show that the equation of the flight path is given by $y = -\frac{5}{256}x^2 + \frac{3}{4}x + 9$

b) Find the maximum height of the particle above the ground.

c) How long will the particle take to reach the ground ?

d) Find the velocity and angle of impact with the horizontal.

SECTION E (10 marks):

1. A particle is moving in Simple Harmonic Motion in a straight line with an amplitude of 4m. If its speed is 6m/s when the particle is 2m from the centre of the path find the period of the motion and its maximum speed.

2. From the word " FORMULA " five letters are selected at random and arranged in a straight line. What is the probability that if one such arrangement is selected at random that the three vowels are together ?

3. On a certain day, the depth of water in a bay at high tide is 11metres. At low tide,6.25 hours later, the depth of water is 7metres. If high tide is due at 2:50am, what is the earliest time after midday that a ship requiring a depth of at least 10metres of water can enter the bay ?

SECTION F (10 marks):

1. If $\frac{dx}{dt} = (5 - x)^2$ and x= 4 when t = 0 find :

a) x as a function of t.

b) $\frac{d^2x}{dt^2}$ as a function of x.

2. Inspector Smith is called to a murder scene at 1:16 am.The victim's body temperature is measured at that time to be 36.4°C , half an hour later it has dropped to 35°C.The victim's body temperature is cooling in accordance with Newton's Law of Cooling.Assuming the victim's body temperature was 37.2°C when he was alive and that the night's temperature was constant at 8°C, what was the murder time ? (assume he died instantly !)

END OF PAPER

1998 3U T2

SECTION A

Q1. a) $x = 5 + e^{3t}$

$v = 3e^{3t}$ (1)

when $t = 0$; $x = 5 + 1 = 6$m (1)

$v = 3$m/s (1)

b) $a = 9e^{3t}$ (1)

$= 9(x - 5)$ (1)

Q2. a) $x = \sqrt{3}\cos 3t - \sin 3t$

(1) $\dot{x} = -3\sqrt{3}\sin 3t - 3\cos 3t$

(1) $\ddot{x} = -9\sqrt{3}\cos 3t + 9\sin 3t$

$= -9(\sqrt{3}\cos 3t - \sin 3t)$

(1) $= -9x$

which is in the form $-n^2x$.

b) $0 = x$, $t = ?$

$0 = \sqrt{3}\cos 3t - \sin 3t$

$\frac{\sin 3t}{\cos 3t} = \sqrt{3}$ (1)

$\tan 3t = \sqrt{3}$

$3t = \pi/3$ (1)

$\therefore t = \pi/9$ seconds

SECTION B

Q1. a) $N = 80 + 45e^{-0.08t}$

$\frac{dN}{dt} = -0.08 \times 45e^{-0.08t}$

(1) $= -0.08(N - 80)$ as $N - 80 = 45e^{\dots}$

$= 0.08(80 - N)$

QED

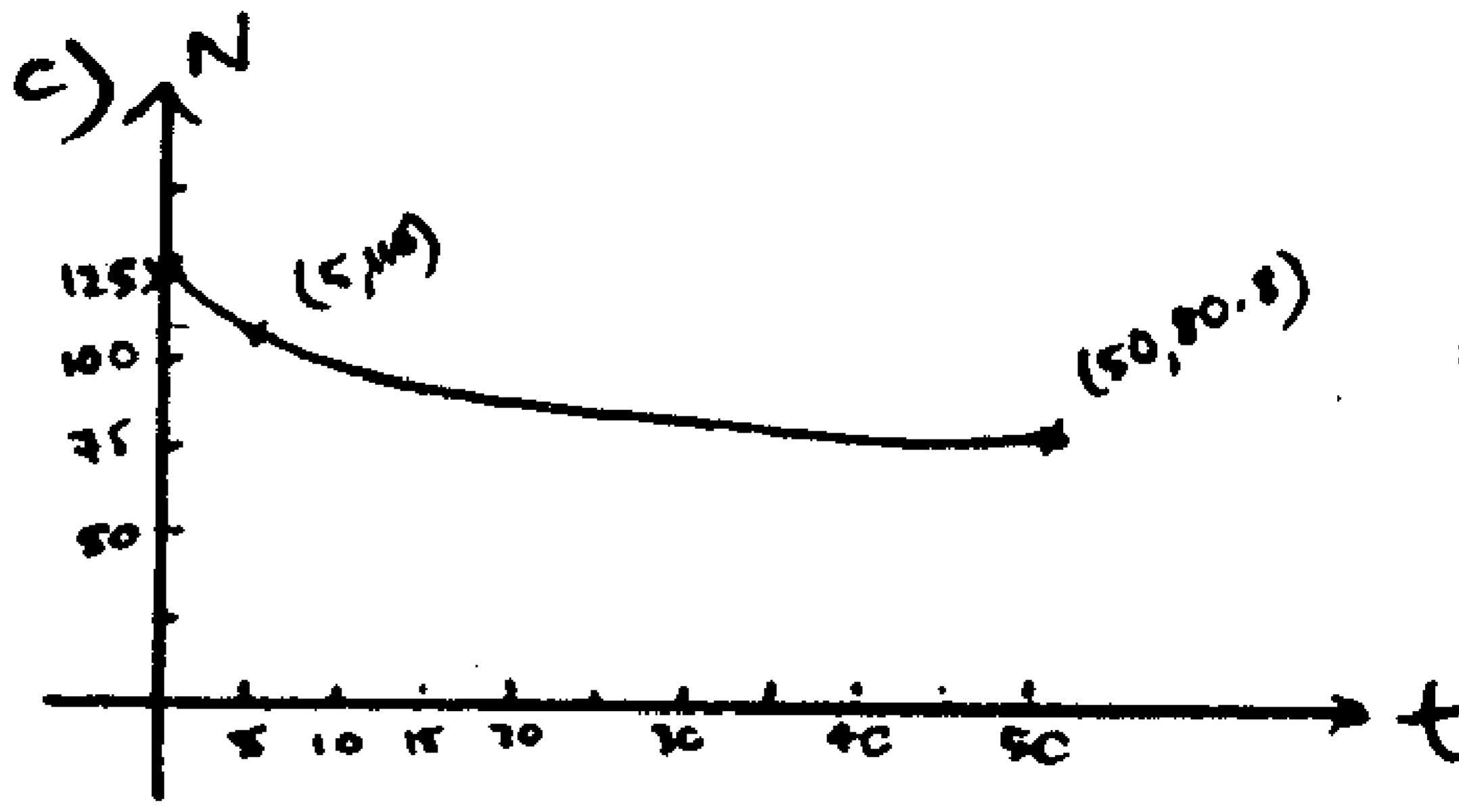
b) $110 = 80 + 45e^{-0.08t}$

$2/3 = e^{-0.08t}$ (1)

$t = \frac{\ln 2/3}{-0.08}$ (1)

$t \doteqdot 5$ seconds (nearest whole n...

c)



(1)

Q2 $a = (t+1)^2$

$v = \frac{1}{3}(t+1)^3 + c$ (1)

$t = 0, v = 0 \quad \therefore c = -1/3$ (1)

$\therefore v = \frac{1}{3}(t+1)^3 - 1/3$

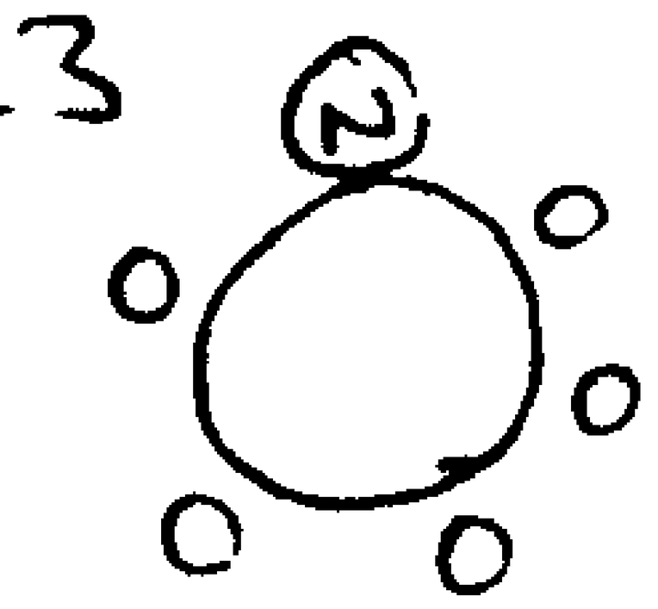
$x = \frac{1}{12}(t+1)^4 - \frac{1}{3}t + k$ (1)

$t = 0, x = 0 \quad \therefore 0 = \frac{1}{12} + k$

$k = -1/12$ (1)

$\therefore x = \frac{1}{12}(t+1)^4 - \frac{1}{3}t - \frac{1}{12}$

Q3



Seat Nicky

Daniel has 3 choices

Sarah has 2 choices (1)

Then 3! ways for rest

no. of arrangements $= 3 \times 2 \times 3!$

$= 36$ (1)

1. a) $v = -3\sin 2t$

$= -3\sin 1.8$

$= -2.92$ units/s (1)

b) $x = 3/2\cos 2t + c$

$3 = 3/2 \times 1 + c$ (1)

$\therefore c = 1\frac{1}{2}$

$\therefore x = \frac{3}{2}\cos 2t + 1\frac{1}{2}$ (1)

2. a) no. $= {}^4C_4 \cdot {}^4C_1 + {}^4C_3 \, {}^4C_2$

$= 4 + 24$ (1)

$= 28$

b) Prob $= \frac{{}^3C_1{}^1C_1 + {}^3C_3{}^3C_2}{28} = \frac{12}{28} = \frac{3}{7}$ (2)

3. $a = 3 - x$

$a = \frac{d(\frac{1}{2}v^2)}{dx} = 3 - x$

$\frac{1}{2}v^2 = 3x - \frac{1}{2}x^2 + c$ (1)

$v = 2, x = 0 \quad \therefore 2 = c$

$\frac{1}{2}v^2 = 3x - \frac{1}{2}x^2 + 2$ (1)

$v^2 = 6x - x^2 + 4$

$v = \pm\sqrt{6x - x^2 + 4}$ (1)

when $x = 1 \quad v = \pm 3$m/s (1/2)

It is moving in S.H.M. (1/2)

The particle has an acceleration of $\ddot{x} = 3 - x$ which is in the form of $\ddot{x} = -(x-3)$. It executes SHM and oscillates about $x = 3$

$v^2 = 6x - x^2 + 4$

Roots $x^2 - 6x - 4 = 0$

$x = \frac{6 \pm \sqrt{36 + 16}}{2}$

$= \frac{6 \pm \sqrt{52}}{2}$

$= 3 \pm \sqrt{13}$

a) $x = 16t$ (1) $y = -5t^2 + 12t + \dots$

$\therefore t = x/16$

$\therefore y = -\frac{5x^2}{256} + 12\frac{x}{16} + 9$

$y = \frac{-5x^2}{256} + \frac{3x}{4} + 9$

b) $x = -b/2a = \frac{-3/4}{2 \times -\frac{5}{256}} = 19\frac{1}{5}$

$\therefore y = -\frac{5}{256}(19\frac{1}{5})^2 + (\frac{3}{4} \times 19\frac{1}{5}) \dots$

$= -7\frac{1}{5} + 14\frac{2}{5} + 9$

$= 16\frac{1}{5}$m (1)

c) hits ground when $y = 0, t = ?$

$0 = 5t^2 - 12t - 9$

$0 = 5t^2 - 15t + 3t - 9$ (1)

$0 = 5t(t-3) + 3(t-3)$

$0 = (5t+3)(t-3)$

$\therefore t = -3/5$ or $t = 3$ (1...

$\therefore$ hits ground when $t = 3$ sec... as $t \geq 0$. (1/2)

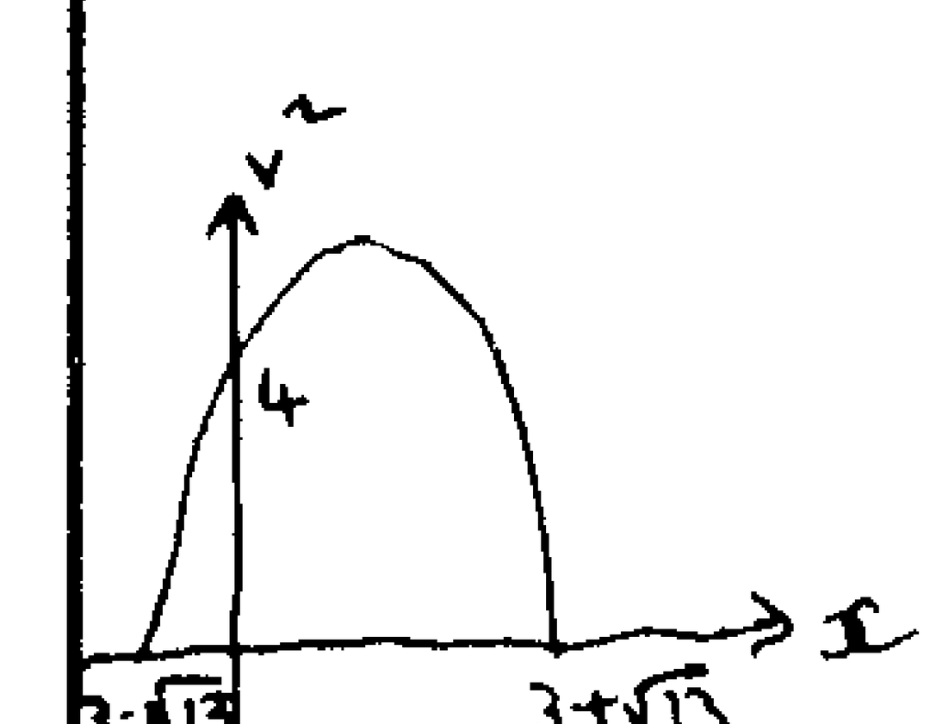
d) when $t = 3$, $\dot{x} = 16$ (1) $\dot{y} = -1\dots$

$\therefore \tan\theta = \frac{-18}{16}$

(1) $\theta = 131°38'$ with... direction o...

velocity $= \sqrt{16^2 + 18^2}$

(1) $\doteqdot 24$m/s (neares... numb...



1. $v^2 = n^2(a^2 - x^2)$

$36 = n^2(16-4)$

$3 = n^2$

$n = \sqrt{3}$ (1)

∴ period is $\frac{2\pi}{\sqrt{3}}$ seconds (1)

∴ max. speed when $x=0$, (1)

$v^2 = 3(16-0)$

$= 48$

$v = \sqrt{48}$

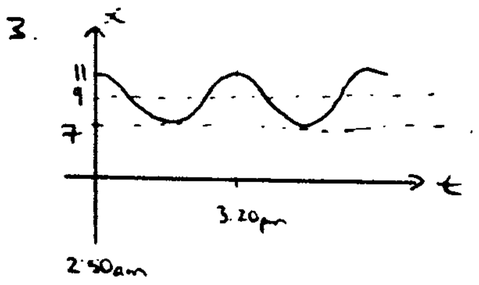
$= 4\sqrt{3}$ m/s (1)

2.

$\text{Prob.} = \frac{3 \times 3! \times 4 \times 3}{{}^7P_5} = \frac{216}{2520} = \frac{3}{35}$

(1) (1)

3.



$x = 2\cos nt + 9$

$7 = 2\cos(6\frac{1}{4}n) + 9$

$-1 = \cos(6\frac{1}{4}n)$

$6\frac{1}{4}n = \pi$ (1)

$\therefore n = \frac{4\pi}{25}$

∴ $x = 2\cos(\frac{4\pi}{25}t) + 9$

$10 = 2\cos(\frac{4\pi}{25}t) + 9$ (1)

$\frac{1}{2} = \cos(\frac{4\pi}{25}t)$

$\frac{\pi}{3} = \frac{4\pi}{25}t$

$t = 2\frac{1}{2}$ hours (1)

∴ Earliest time is 1:15pm. (1)

1.a) $v = (5-x)^2$

$\frac{dt}{dx} = (5-x)^{-2}$ (1)

$t = (5-x)^{-1} + c$

$t = 0, x = 4 \therefore c = -1$ (1)

$\therefore t = \frac{1}{5-x} - 1$

$t + 1 = \frac{1}{5-x}$ (1)

$\frac{1}{t+1} = 5 - x$

$x = 5 - \frac{1}{t+1}$

$x = \frac{5t+4}{1+t}$ (1)

b) $a = \frac{d(\frac{1}{2}v^2)}{dx} = \frac{d[\frac{1}{2}(5-x)^2]}{dx}$ (1)

$a = \frac{1}{2} \times 4(5-x)^3 \times -1$

$= -2(5-x)^3$ (1)

2. $N = B + Ae^{-kt}$

$26.9 = 8 + Ae^0$

$A = 28.4$ (1)

∴ $N = 8 + 28.4e^{-kt}$

$35 = 8 + 28.4e^{-\frac{1}{2}k}$

$0.95070 = e^{-\frac{1}{2}k}$

$\therefore k = 0.101104558$ (1)

now; $37.2 = 8 + 28.4e^{-0.101104t}$

$1.028169 = e^{-0.101104t}$

$0.0277796 = -0.101104t$ (1)

$t = -0.27476$

$t = 16$ mins 29 seconds earlier

∴ Murdered at 12:59 31am (1)

THE END