JAMES RUSE AGRICULTURAL HIGH SCHOOL

TERM TWO OPEN BOOK ASSESSMENT TASK

EXTENSION ONE 2001

INSTRUCTIONS:

1. Time allowed is 85 minutes
2. This is an open book test
3. Show all necessary formulae and working
4. Marks may be deducted for untidy or careless work
5. Start each section on a new sheet of paper
6. Put your number only on every page
7. Approved calculators only may be used

Question One (10 marks)

1. Find the period and amplitude of a particle moving in S.H.M. if its speed moving along a straight line is given by $v^2 = 84 - 16x - 4x^2$.

2. Tickets sold in the James Ruse Annual raffle are numbered from 2000 to 6999. What is the probability that the winning ticket is a number that is divisible by three?

3. A particle moving along a straight line such that its distance at t seconds is given by: $x = pt^2 + qt^3$, where p and q are constants. Find the values for p and q if the maximum velocity is 48m/s when t = 4 seconds.

Question Two (10 marks)

1. A cup of hot chocolate cooled from 90°C to 60°C after 10 minutes in a room whose temperature was 20°C. How much longer would it take the hot chocolate to cool to 35°C? (Assume Newton's law of cooling). (Answer to the nearest second).

2. A pan of warm water, 46°C, was put into a refrigerator. Ten minutes later the water temperature was 39°C, and ten minutes after that the water temperature was 33°C. Assuming Newton's law of cooling, how cold was the refrigerator?

3. a) The rate of increase in the Pokemon population is proportional to the amount present at time, t. Given that the population trebles in four years, write down an expression for P in terms of t and P_0.

   b) If the population reaches ten times the initial population there will be a Pokemon invasion, which we do not want. The army needs six months warning to get organised enough to successfully fight the invading Pokemon. If the initial measurement of Pokemon was done on the first of January 2001, when does the army get the warning? (Give your answer to the nearest month)

Question Three(10 marks)

1. a) How many different numbers using the digits 3, 4, 5, 6 and 7 can be formed, if the digits are used once only and the resulting number must be less than 7000?
   b) Of these numbers, one is selected at random, what is the probability that it ends in a 3?

2. Four men and four women are to be seated around a round table. How many arrangements are possible if:
   a) There are no restrictions?
   b) The men and women must alternate?
   c) Man A must not sit next to woman A, with the men and women still alternating?

3. From 26 soccer players, two teams of 13 are chosen:
   a) How many different pairs of teams are possible?
   b) If there are only two goalkeepers and each team must have a goalkeeper, how many different pairs of teams are possible?
   c) If players B and C refuse to play with keeper A, how many different pairs of teams are now possible?

Question Four(10 marks)

1. The acceleration of a particle is $(2x - 5)m/s^2$, where x is the distance in metres from the origin.
   i) Find an expression for the velocity of this particle in terms of x, given that the particle is at rest one metre to the left of the origin initially.
   ii) Describe the motion.

2. i) At time t, the displacement of a particle moving in a straight line is x. If the acceleration is given by $-3x$ and the particle starts from rest at $x = 2$, find its position in terms of t.
   ii) What is the period?

3. There are three identical green marbles and four identical yellow marbles arranged in a row. How many different arrangements of just five of these marbles are possible?

Question Five(10 marks)

1. A) For a boat to safely enter a harbour, it requires 12 metres of water. At low tide the harbour is 9m deep and 10 hours later at high tide the harbour is 13m deep. If low tide was at 4:30am, what is the earliest time the ship can safely enter the harbour?
   B) It takes three hours to unload the ship and reload it. What is the latest time the ship can safely leave the harbour?

2. A particle moving in a straight line has an acceleration of $(8t - 2)\ m/s^2$. The particle passes through the origin with a velocity of 2m/s in the negative direction two seconds after observations commenced:

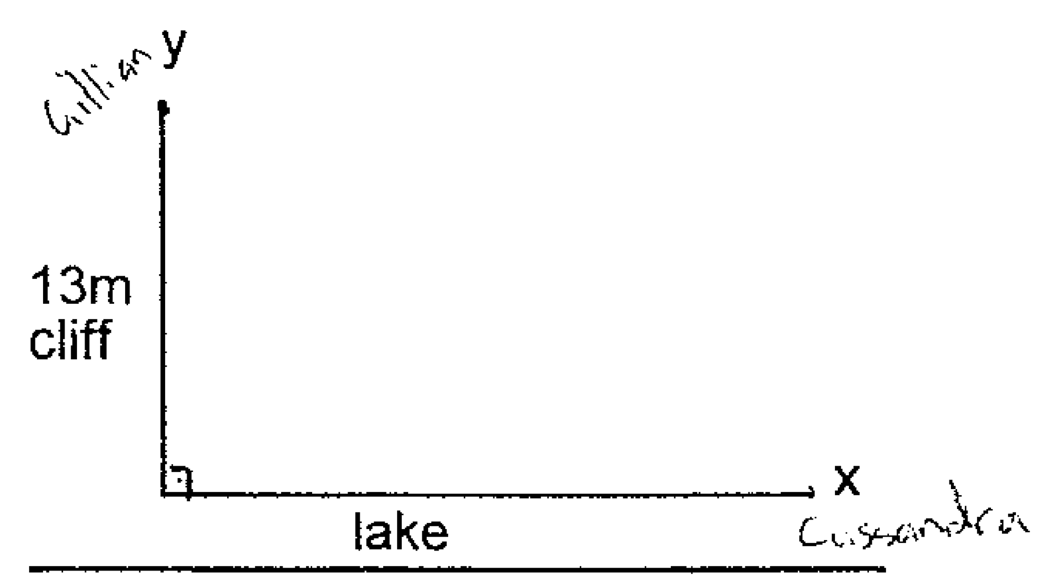
   i) Find expressions for its positions and velocity at time, t.

   ii) When will the particle next pass through the origin? Give your answer correct to two decimal places.

Question Six (10 marks)

Cassandra would like to play soccer, unfortunately Gillian has the soccer ball and she is standing on the top of a 13m vertical cliff. The cliff overlooks a lake, which Cassandra is standing on the edge of. It is known that Gillian will kick the ball with a velocity of 10m/s at an angle of 60° to the horizontal, towards Cassandra. Take g as $9.8m/s^2$.

a) Derive the equations of motion.
b) When does Cassandra get the ball? (Answer to 2 decimal places)
c) How far from the foot of the cliff is Cassandra standing? (Answer to 2 decimal places)
d) What speed and direction does the ball hit the ground at Cassandra's feet? (Answer to the nearest minute).



END OF PAPER!

$v^2 = 84 - 16x - 4x^2$

$v^2 = 4(21 - 4x - x^2)$

$v^2 = 4(3 - x)(7 + x)$ (1)

$v = 0$ at $x = 3$ or -7

∴ amplitude is 5 (1)

$a = \frac{d(\frac{1}{2}v^2)}{dx}$

$= \frac{d(42 - 8x - 2x^2)}{dx}$

$= -8 - 4x$

$= -4(+2 + x)$ (1)

∴ $n = 2$

∴ period is π (1)

2. sample space = 5000 (1)

2001, 2004, ... 6999

1667 terms

∴ Prob = $\frac{1667}{5000}$ (1)

3. $x = pt^2 + qt^3$

max. velocity is 48 m/s at $t = 4$

$v = 2pt + 3qt^2$

$a = 2p + 6qt$

$a = 0, t = 4, v = 48$

$0 = 2p + 24q$ (1)

$0 = p + 12q$ . . . . ①

$48 = 8p + 48q$

$6 = p + 6q$ . . . ② (1)

$6 = -12q + 6q$

$6 = -6q$

$q = -1$ (1)

∴ $p = 12$ (1)

QUESTION TWO

1. $T = B + Ae^{-kt}$

$90 = 20 + Ae^{0}$

$A = 70$

$60 = 20 + 70e^{-10k}$

$4/7 = e^{-10k}$

$\ln 4/7 = -10k$ (1)

$k = \frac{\ln 4/7}{-10}$

$35 = 20 + 70e^{\frac{\ln 4/7}{10}t}$

$\frac{15}{70} = e^{\frac{\ln 4/7}{10}t}$

$\ln(\frac{15}{70}) = \frac{\ln 4/7}{10}t$

$t = \frac{10\ln(\frac{15}{70})}{\ln(4/7)}$ (1)

t = 27 minutes 32 seconds

∴ It takes another 17 mins 32 seconds (1)

2. $T = B + Ae^{-kt}$

$46 = B + A$ at $t = 0$ . . . . ①

$39 = B + Ae^{-10k}$

sub in ①

∴ $39 = 46 - A + Ae^{-10k}$

$\frac{A-7}{A} = e^{-10k}$ . . . . ② (1)

$33 = B + Ae^{-20k}$

$33 = 46 - A + Ae^{-20k}$

$\frac{A-13}{A} = e^{-20k}$

$\frac{A-13}{A} = (e^{-10k})^2$ . . . ③

sub ② into ③

$\frac{A-13}{A} = (\frac{A-7}{A})^2$ (1)

$A(A-13) = (A-7)^2$

$A^2 - 13A = A^2 - 14A + 49$

∴ $A = 49$

∴ $B = -3$ (1)

∴ Temperature of the fridge is $-3°C$

a) $\frac{dP}{dt} \propto P$

$\frac{dP}{dt} = kP$

$3P_0 = P_0e^{4k}$ (1)

$\ln 3 = 4k$

$k = \frac{\ln 3}{4}$

∴ $P = P_0e^{\frac{\ln 3}{4}t}$ (1)

b) $10P_0 = P_0e^{\frac{\ln 3}{4}t}$ (½)

$\ln 10 = \frac{\ln 3}{4}t$ (½)

$t = 8.3836$

= 8 yrs 5 months (½)

∴ Pokemon warning given in November 2008! (½)

QUESTION THREE

1. a) $(5\times4\times3\times2) + (5\times4\times3) + (5\times4) + 5$

$= 120 + 60 + 20 + 5$ (2)

$= 205$

b) Prob(ends in 3) = $\frac{(4\times3\times2\times1) + (4\times3\times1) + (4\times1) + 1}{205}$

$= \frac{24 + 1 + 4 + 1}{205}$ (2)

$= \frac{41}{205}$

2. a) $7! = 5040$ (1)

b) $3!4! = 144$ (1)

3a) $\frac{^{26}C_{13}}{2!}$ (1)

c) $2!3!3! = 72$ (1)

3. b) $\frac{^{2}C_1 \times {}^{24}C_{12}}{2} = 2\ 704\ 156$ (1)

c) $^{22}C_{12} = 646\ 646$ (1)

1. i) $a = 2x - 5$

$v = 0$ when $x = -1, t = 0$

$\frac{1}{2}v^2 = \int(2x - 5)dx$

$\frac{1}{2}v^2 = x^2 - 5x + c$ (1)

$0 = 1 + 5 + c$

$\frac{1}{2}v^2 = x^2 - 5x - 6$

$v^2 = 2x^2 - 10x - 12$ (1)

$v = \pm\sqrt{2x^2 - 10x - 12}$

stat. at $x = -1$ and $x = 6$

$a = -7$, $v = 0$ (1), x, -1

∴ $v = -\sqrt{2x^2 - 10...}$

ii) starts from $x = -1$ from re[st], moves to the left with increa[sing] speed, never stopping again [or] changing direction. (1)

2. $a = -3x$

$v = 0, t = 0, x = 2$

$a = -n^2x$

∴ in S.H.M. where $n = \sqrt{3}$ (1)

∴ $x = a\cos(nt)$

$x = 2\cos(\sqrt{3}t)$ (1)

ii) period is $\frac{2\pi}{\sqrt{3}}$ (1)

3.

3a and 2y = $\frac{5!}{3!2!} = 10$ (½)

2a and 3y = $\frac{5!}{2!3!} = 10$ (½)

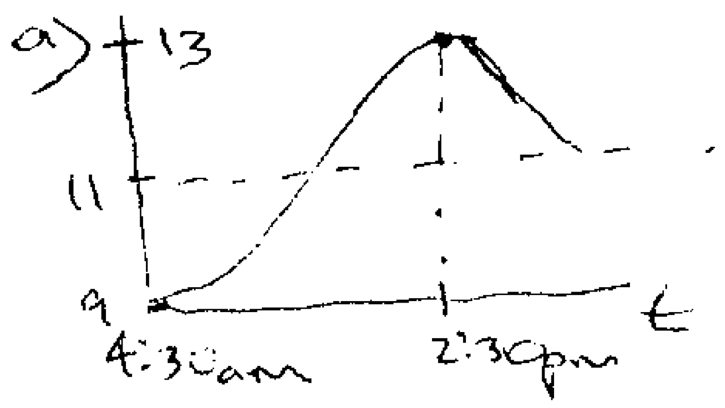
1a and 4y = $\frac{5!}{4!} = 5$ (½)

∴ Total = 10 + 10 + 5

= 25 (½)

QUESTION FIVE

a)



period = 20 hours $20 = \frac{2\pi}{n}$

$n = \frac{\pi}{10}$ (1)

$x = 11 - a\cos(nt)$

$x = 11 - 2\cos(\frac{\pi}{10}t)$ (1)

$12 = 11 - 2\cos(\frac{\pi}{10}t)$

$-\frac{1}{2} = \cos(\frac{\pi}{10}t)$

$\frac{\pi}{10}t = \frac{2\pi}{3}$ (1)

$t = \frac{20}{3} = 6$hrs 40mins

∴ Earliest time is 11:10am (1)

b) $\frac{4\pi}{3} = \frac{\pi}{10}t$

$t = \frac{40}{3} = 13$hrs 20mins. (1)

∴ latest time is 5:50pm

i) $a = 8t - 2$

$v = -2, x = 0, t = 2$

$v = 4t^2 - 2t + c$

$-2 = 16 - 4 + c$

$c = -14$

∴ $v = 4t^2 - 2t - 14$ (1)

∴ $x = \frac{4}{3}t^3 - t^2 - 14t + c_1$

$0 = \frac{32}{3} - 4 - 28 + c_1$

∴ $x = \frac{4}{3}t^3 - t^2 - 14t + 21\frac{1}{3}$ (1)

ii) passes thru $x = 0$ at $t = 2$

∴ $(t-2)$ is a factor of x. (1)

$$\begin{array}{r} \frac{4}{3}t^2 + \frac{5}{3}t - \frac{32}{3} \\ t-2\,\overline{)\,\frac{4}{3}t^3 - t^2 - 14t + 21\frac{1}{3}} \\ \frac{4}{3}t^3 \end{array}$$

∴ $x = (t-2)(\frac{4}{3}t^2 + \frac{5}{3}t - \frac{32}{3})$

passes next thru when

$\frac{4}{3}t^2 - \frac{5}{3}t - \frac{32}{3} = 0$ (1)

$4t^2 + 5t - 32 = 0$

$t = \frac{-5 \pm \sqrt{25 - 4(4)(-32)}}{8}$

$= \frac{-5 \pm \sqrt{537}}{8}$

but $t > 0$ $t = \frac{-5 + \sqrt{537}}{8}$

$= 2.27165$

(1) $= 2.27$ seconds

QUESTION SIX

a) $\ddot{x} = 0$

$\dot{x} = 10\cos 60°$

$\dot{x} = 5$ m/s (1)

$x = 5t$ (1)

$\ddot{y} = -9.8$

$\dot{y} = -9.8t + 10\sin 60°$ (1)

$\dot{y} = -9.8t + 5\sqrt{3}$

$y = -4.9t^2 + 5\sqrt{3}t + 13$ (1)

b) $y = 0$

∴ $0 = -4.9t^2 + 5\sqrt{3}t + 13$

$t = \frac{-5\sqrt{3} \pm \sqrt{75 - 4(13)(-4.9)}}{-9.8}$ (1)

$= \frac{-5\sqrt{3} \pm \sqrt{329.8}}{-9.8}$

$t = 2.7368$ or -0.9694

but $t > 0$ (1)

∴ $t = 2.74$ seconds (2 dec. pl.).

c) $x = 5 \times (2.7368)$ (1)

$x = 13.68$m

d) $\dot{x} = 5$

$\dot{y} = -9.8(2.7368) + 5\sqrt{3}$ (1)

$= -18.16$

$\tan\theta = \left|\frac{\dot{y}}{\dot{x}}\right| = \left|\frac{-18.16}{5}\right|$

$= 3.632077$

$\theta = 74°36'$ (1)

Speed $= \sqrt{(5)^2 + (-18.16)^2}$

$= \sqrt{354.7856}$

$= 18.8$ (1)

∴ hits the ground at 18.8 m/s at an angle of 74° 36'