Year 12 Term 2 - 2002 Ext I

Question 1: (10 marks)

(a) The rate at which an object warms in air is proportional to the difference between its temperature T° and the constant temperature S° of the surrounding air, i.e. $\frac{dT}{dt} = k(S - T)$ where t is the time measured in minutes and k is a constant.

(i) Show that $T = S + Ae^{-kt}$, where A is a constant, is a solution of $\frac{dT}{dt} = k(S - T)$. 2

For a particular object its initial temperature is 15° and after 40 minutes its temperature has risen to 30°. Given that the surrounding air temperature is 35°, find

(ii) the value of k, 2

(iii) the temperature of the object after one hour. 2

(b) Four girls (Alice, Betty, Carol and Dianne) and three boys (Ross, Steve and Terry) arrange themselves in a straight line at a supermarket checkout. If the arrangement is random, find the probability that:

(i) all the boys are together, 2

(ii) none of the boys are standing together, 1

(iii) Alice and Betty will be served before Ross. 1

Question 2: (10 marks) **START A NEW PAGE**

(a) In an experiment, water in a tank rises and falls with simple harmonic motion. The greatest depth of water is 9 metres and the least depth is 1 metre. At 7am the depth of water was 5 metres and increasing. Three hours later the depth has reached 9 metres for the first time. Given that the depth, x metres, of water at time t hours after 7am can be represented by the formula $x = b + a\sin nt$, find:

(i) the values of b, a and n, 3

(ii) the time after 7am when the depth of water first reaches 3 metres. 2

(b) A moving object has its velocity (v) defined by $v^2 = 16 + 6x - x^2$, where the velocity is in ms^{-1} and the displacement (x) is in metres.

(i) Show that the motion is simple harmonic. 1

(ii) Find the amplitude of the motion. 2

(iii) Find the greatest speed of the object. 2

Question 3: (10 marks) **START A NEW PAGE**

(a) An object moves in a straight line along a flat surface under the influence of a constant acceleration opposing the motion. The magnitude of the acceleration is k and the object start from the origin O with initial velocity V_o. During its motion the object passes through two points A and B which are both to the right of O. The points O, A and B are equally spaced d metres apart and the travelling times from O to A and A to B are t_1 and t_2 respectively.

(i) Starting with the equation $\ddot{x} = -k$, derive formulae for the velocity v and position x of the particle at time t. 3

(ii) Show that $k = \dfrac{2d(t_2 - t_1)}{t_1 t_2 (t_2 + t_1)}$. 3

(b) The letters from the word VOLUME are placed at random on the circumference of a circle. Each letter is used only once.

(i) Find the number of different arrangements that can be formed. 1

If one of the arrangements is chosen at random, find the probability that

(ii) all the vowels will be together, 2

(iii) the vowels and consonants will alternate. 1

Question 4: (10 marks) **START A NEW PAGE**

(a) The amount Q, measured in milligrams, of a substance present in a chemical reaction at time t minutes is given by $Q = 400(1+t)e^{-\frac{1}{4}t}$.

(i) Show that Q satisfies the differential equation $16\dfrac{d^2Q}{dt^2} + 8\dfrac{dQ}{dt} + Q = 0$. 3

(ii) Find the quantity of Q present at the start of the reaction. 1

(iii) Find the maximum value of Q and the time at which it occurs. 3

(b) The horizontal and vertical position, measured in metres, of an object at time t seconds after projection are given by $x = 30t$ and $y = 80 + 40t - 5t^2$. 3
Find the initial angle and speed of projection.

Question 5: (10 marks) **START A NEW PAGE**

(a) The acceleration of a particle is given by $\ddot{x} = x^3 - 3ax$ where $a > 0$ and with initial conditions $v = -\frac{1}{2}a\sqrt{6}$ when $x = \sqrt{a}$.

(i) Show that $v^2 = \frac{1}{2}x^4 - 3ax^2 + 4a^2$. 2

(ii) Find the position of the particle when it first comes to rest. 2

(b) On a shelf are fifteen English and ten Science books. If six books are selected at random, find the probability that

(i) they are all English books, 2

(ii) there is at least one Science books, 2

(iii) there is a majority of English books if it is known that at least one Science book has been chosen. 2

(Note: You may leave your answers in ${}^nc_r$ form)

Question 6: (10 marks) **START A NEW PAGE**

The position of an object P projected from ground level with initial velocity V at angle θ to the horizontal is given by the equations $x = Vt\cos\theta$ and $y = -\frac{1}{2}gt^2 + Vt\sin\theta$.

(a) Prove that for a given value of θ, the horizontal range R of object P is given by $R = \dfrac{V^2 \sin 2\theta}{g}$ and explain why its maximum range $R_{\max}$ equals $\dfrac{V^2}{g}$. 3

(b) Prove that for a given value of θ, the greatest height H of object P above the ground is given by $H = \dfrac{V^2 \sin^2\theta}{2g}$. 2

Two objects A and B are now projected with equal initial velocity V from the same ground position at angles α and $\frac{\pi}{2} - \alpha$ respectively.

(c) Show that they both have the same horizontal range. 1

(d) If they reach greatest heights of H_1 and H_2 respectively, show that their maximum range, $R_{\max}$, is equal to $2(H_1 + H_2)$. 2

(e) For the two objects above it is given that $\alpha = \tan^{-1}\frac{5}{12}$ and $V = 260\text{ms}^{-1}$. Find the difference in their projection times if they collide as they strike the horizontal plane. (use $g = 10$) 2