Question One: **Marks**

(a) A particle is travelling with an acceleration of $\sin t + e^{-2t}$. m/s^2 **3**
Initially the particle is 4.25 metres to the right of the origin and moves from rest.
Find its displacement function in terms of *t*.

(b) The equation of motion of a particle moving along the *x*-axis is given by **2**

$$x = 5\sin(t + \frac{\pi}{3}).$$

(i) Draw the displacement-time graph for $0 \le t \le \frac{5\pi}{3}$. **2**

(ii) When does the particle first change directions and where is it at this time? **2**

(iii) Find the distance travelled by the particle in the first $\frac{7\pi}{6}$ seconds. **2**

Question Two: (Start a new page):

(a) The acceleration of a particle P moving in a straight line is given by:
$\frac{d^2x}{dt^2} = \frac{1}{2}x^3 + 2x$, where *x* is the distance from the origin *O* at time *t*.
Initially the particle is at the origin *O* with a velocity of 2m/s.

(i) Explain why the velocity of the particle is $v = \frac{1}{2}(x^2 + 4)$ m/s **4**

(ii) Prove that the displacement *x*, at any time *t*, is given by $x = 2\tan t$. **2**

(iii) Calculate the exact displacement, velocity and acceleration **3**
when $t = \frac{\pi}{4}$ seconds.

Question Three (Start a new page):

(a) A particle moves along the *x*-axis in Simple Harmonic Motion according to the equation: $v^2 = 144 - 54x - 9x^2$.

(i) Prove that $\frac{d^2x}{dt^2} = -27 - 9x$. **2**

(ii) Find the amplitude and centre of motion. **3**

(iii) Find the maximum speed in metres per second. **1**

(iv) State the period of the motion. **1**

Q 3 is continued over the page.

Q3 continued **Marks**

(b) The rise and fall of the tide at the "James Ruse" Port is in SHM. **2**
The time interval between high tides is 10 hours.
The port entrance has a depth of 8m at high tide and 2m at low tide.
On Monday morning low tide is at noon.
What is the earliest time, on Monday afternoon, that a ship needing 6m of water can pass through the entrance?

Question Four (Start a new page):

(a) A tray of scones is placed in an oven, which is heated at a constant temperature of 180^0 C
Initially the scones have a temperature of 18^0 C and after two minutes the scones have a temperature of 24^0 C.

(i) Show that: $T = 180 - 162e^{\frac{1}{2}\ln(\frac{26}{27})\,t}$, where T is the temperature of the scones at time t minutes. **2**

(ii) Neatly sketch the graph of: $T = 180 - 162e^{\frac{1}{2}\ln(\frac{26}{27})\,t}$. **2**

(b) A pendulum is moving in SHM. Find the time taken to reach one quarter of the distance from an endpoint to the centre of motion if it starts from an endpoint and its period is π seconds. (answer correct to one decimal place). **2**

(c) Find the probability that if 5 couples are arranged about a round table that Mrs Bostik is not sitting next to Mr Bostik, if it is known that the men and women alternate. **3**

Question Five (Start a new page):

(a) A freshly made pot of tea cools from 100^oC to 85^oC in 90 seconds in a room with temperature 27^oC. **4**
If the pot of tea is cooling according to Newton's law of cooling, find how long it would take for the pot of tea to cool to 50^oC.

(b) On planet X initially the temperature of an object being cooked, in a special oven, is twice that of the oven's temperature. **3**
Ten minutes later the object's temperature has trebled.
How long will it take for the object's temperature to double its initial temperature? (Answer to the nearest second).

(c) Find all values of n if ${}^nP_2 = 210$. **2**

Question Six (Start a new page): Marks

(a) A gardener watering the James Ruse Rose bushes on level ground observes that the water flowing from the garden hose on the ground forms a parabolic arc. **4**
Assuming the parametric equations of motion for a projectile and if the maximum height of the water is two fifths of the range,
find the possible angles of projection.

(b) A soccer ball is kicked with an initial velocity of v m/s at an angle of projection of θ from a point O on horizontal ground.
After two seconds it just passes over a 10m high wall, which is 20m from the point of projection. (Take g = 10m/s^2).
Assuming the parametric equations of motion for a projectile

(i) Find θ, to the nearest minute. **3**

(ii) Find v, correct to one decimal place. **2**

Question Seven (start a new page):

(a) Lotto is won by selecting 6 correct numbers from numbers 1 to 45, (inclusive). **2**
Joe selects 6 random numbers.
Find the probability that he wins lotto (Answer as a fraction).

(b) The area between the x-axis, the curve: $y = \dfrac{3}{x}$, the lines x = 1 and x = 4 is reflected in the x-axis forming a shape like a trumpet that is used as a target for throwing darts. **3**
Assuming that all dart throwers will hit some part of the trumpet, find the probability that the dart will hit the area between x = 2 to x = 3,
(answer as a percentage correct to one decimal place).

(c) (i) How many different arrangements of the letters of the word "***PHOTOGRAPHS***" in a straight line are there? **2**

(ii) What is the probability that there are exactly four letters between the two ***H***'s when the letters of the word "***PHOTOGRAPHS***" are arranged in a straight line? **2**

End of Paper

Term 2 – 2006 – Ext. I

Question 1

(a) $a = \sin t + e^{3t}$

$v = -\cos t + \frac{1}{3}e^{3t} + c$

$t = 0,\ v = 0 \quad \therefore 0 = -1 + \frac{1}{3} + c$

$c = 2/3$

$\therefore v = -\cos t + \frac{1}{3}e^{3t} + 2/3$

$x = -\sin t + \frac{1}{9}e^{3t} + \frac{2}{3}t + K$

$x = \frac{10}{9},\ t = 0$

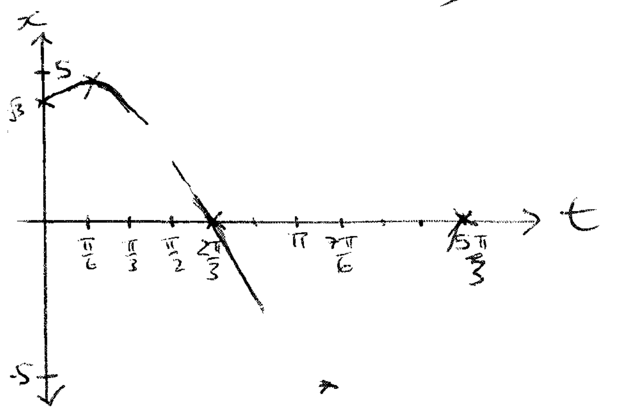
3

$\therefore \frac{10}{9} = 0 + \frac{1}{9} + 0 + K$

$\therefore K = 1$

$\therefore x = -\sin t + \frac{1}{9}e^{3t} + \frac{2}{3}t + 1$

(b) (i) $x = 5\sin(t + \frac{\pi}{3})$



2

(ii) First changes direction at $t = \frac{\pi}{6}$ seconds when $x = 5$m 2

(iii) $\text{dist} = (5 - \frac{5}{2}\sqrt{3}) + 5 + 5$ 2

$= 10.67$m

Question 2

(a) (i) $\frac{d^2x}{dt^2} = \frac{1}{2}x^3 + 2x$

$\frac{1}{2}v^2 = \int(\frac{1}{2}x^3 + 2x)\,dx$

$\frac{1}{2}v^2 = \frac{1}{8}x^4 + x^2 + c$

$v^2 = \frac{1}{4}x^4 + 2x^2 + 2c$

at $t = 0,\ x = 0,\ v = 2$

$\therefore 4 = 0 + 0 + 2c$

$\therefore c = 2$

$\therefore v^2 = \frac{1}{4}x^4 + 2x^2 + 4$

$v^2 = \frac{1}{4}(x^4 + 8x^2 + 16)$

$v^2 = \frac{1}{4}(x^2 + 4)^2$ 4

·1= $v = \pm\frac{1}{2}(x^2 + 4)$

but initially $v = 2$, so it's moving to the right and $a > 0$ for all $x > 0$ $\therefore$ it continues to move to the right with increa[sing] speed

$\therefore v = \frac{1}{2}(x^2 + 4)$ only.

(ii) $\frac{dx}{dt} = \frac{1}{2}(x^2 + 4)$

$\frac{dt}{dx} = \frac{2}{x^2 + 4}$ 2

$t = \tan^{-1}(\frac{x}{2}) + c_1$

$t = 0,\ x = 0 \quad \therefore c_1 = 0$

$t = \tan^{-1}(\frac{x}{2})$

$\tan t = \frac{x}{2}$

$\therefore x = 2\tan t$ 2

(iii) at $t = \frac{\pi}{4}$ seconds

$x = 2\tan\frac{\pi}{4} = 2$

$v = \frac{8}{2} = 4$

$a = \frac{8}{2} + 4 = 8$ 3

$\therefore$ At $t = \frac{\pi}{4}$ seconds, the particle is 2m to the right of the origin travelling with a velocity of 4m/s and acceleration of 8m/s².

Question 3

(a) (i) $v^2 = 108 - 36x - 9x^2$

$\frac{1}{2}v^2 = 54 - 18x - \frac{9}{2}x^2$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -18 - 9x$

$\therefore \ddot{x} = -18 - 9x$

$= -9(x+2)$ 2

(ii) amp when $v = 0$

ie $0 = 108 - 36x - 9x^2$

$0 = x^2 + 4x - 12$

$0 = (x+6)(x-2)$

$\therefore x = -6$ or $x = 2$

3 $\therefore$ amplitude is 4 units

c. of m. is at $x = -2$.

(iii) $n = 3$, $a = 4$

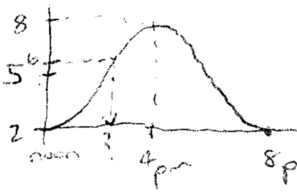
1 max. speed $= 12^2 = 144$ m/s

(iv) $T = \frac{2\pi}{n}$

$= \frac{2\pi}{3}$ 1

(b) $T = \frac{2\pi}{n} = 8$

$n = \frac{\pi}{4}$



$x = b + a\cos(nt)$

$x = 5 - 3\cos\left(\frac{\pi}{4}t\right)$

$6 = 5 - 3\cos\left(\frac{\pi}{4}t\right)$

$-\frac{1}{3} = \cos\left(\frac{\pi}{4}t\right)$

$t = 2.43\,269\,3792$

$t = 2$hrs 26mins

$\therefore$ Earliest time is 2:26pm.

Question 4

(a) $T = B + Ae^{kt}$

$15 = 180 + Ae^{0}$

$\therefore A = -165$

$\therefore T = 180 - 165e^{kt}$

$25 = 180 - 165e^{2k}$

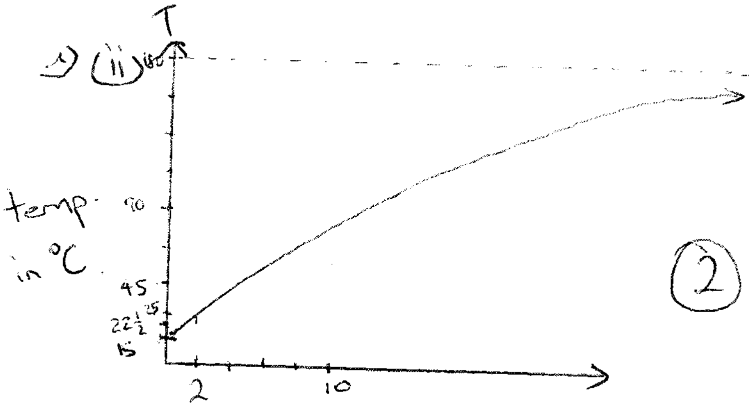
$\frac{-155}{-165} = e^{2k}$

$\ln\left(\frac{155}{165}\right) = 2k$ 2

$\therefore k = \frac{1}{2}\ln\left(\frac{31}{33}\right)$

$\therefore T = 180 - 165e^{\frac{1}{2}\ln\left(\frac{31}{33}\right)t}$

(ii)



(2)

(b)

$-a \quad 0 \quad \frac{2}{3}a \quad a$

period $= \pi$ $\quad \therefore \pi = \frac{2\pi}{n}$

$n = 2$

$x = a\cos(2t)$

$\frac{2}{3}a = a\cos(2t)$

$2/3 = \cos 2t$

$t = \frac{1}{2}\cos^{-1}(2/3)$

$t = 0.420534335$

$t = 0.4$ seconds

(2)

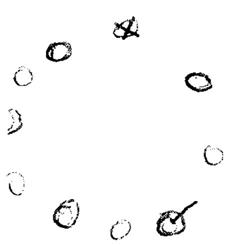
(c) P(Mrs W not next to Mr W)

seat Mr W first $- 1$

seat Mrs W 2nd $- 3$

fill other men $- 4!$

fill other women $- 4!$



total $= 3 \times 4! \times 4!$

$= 1728$

(3)

$P = \frac{1728}{4!5!} = \frac{1728}{2880} = \frac{3}{5}$

Question 5

(a) $T = B + Ae^{-kt}$

$100° = 27° + A$

$\therefore A = 73°$

$T = 27 + 73e^{-kt}$ (1)

$t = 1\frac{1}{2}, \ T = 85°C$

$\therefore 85 = 27 + 73e^{-1\frac{1}{2}k}$

$\ln\left(\frac{58}{73}\right) = -\frac{3}{2}k$

$k = -\frac{2}{3}\ln\left(\frac{58}{73}\right)$ (1)

$50 = 27 + 73e^{2/3\ln(\frac{58}{73})t}$

$\frac{23}{73} = e^{2/3\ln(\frac{58}{73})t}$

$\ln\left(\frac{23}{73}\right) = \frac{2}{3}\ln\left(\frac{58}{73}\right)t$ (1)

$t = \frac{3\ln(23/73)}{2\ln(\frac{58}{73})}$

$t = 7.53$ (1)

$\therefore t = 7$ mins 32 seconds

(b) $T = B + Ae^{-kt}$

$2B = B + Ae^{0}$

$\therefore A = B$

$T = B + Be^{-kt}$

$t = 10, \ T = 6B$ $\quad \therefore 6B = B + Be^{10k}$

$5 = e^{10k}$

$k = -\frac{\ln 5}{10}$

$T = 4B, \ t = ??$ $\quad \therefore 4B = B + Be^{t}$

$3 = e^{t\frac{\ln 5}{10}}$

$\ln 3 = \frac{\ln 5}{10}t$

$t = \frac{10\ln 3}{\ln 5}$

$t = 6$ mins 5

(3)

5(c) $^nP_2 = 132$

$$\frac{n!}{(n-2)!} = 132$$

$n(n-1) = 132$

$n^2 - n - 132 = 0$

$(n-12)(n+11) = 0$ 2

$\therefore n = 12$ or -11

but $n > 0$ $\therefore n = 12$ only

Question 6

i) $\ddot{x} = 0$ $\quad\quad$ $\ddot{y} = -g$

$\dot{x} = v\cos\theta$ $\quad\quad$ $\dot{y} = -gt + v\sin\theta$

$x = vt\cos\theta$ $\quad\quad$ $y = -\frac{g}{2}t^2 + vt\sin\theta$

max. height when $\dot{y} = 0$

ie $0 = -gt + v\sin\theta$

$$t = \frac{v\sin\theta}{g}$$

sub. into y

$$y = -\frac{g}{2}\frac{v^2\sin^2\theta}{g^2} + \frac{v^2\sin^2\theta}{g}$$

$$= \frac{-v^2\sin^2\theta + 2v^2\sin^2\theta}{2g}$$

$$= \frac{v^2\sin^2\theta}{2g}$$

now max height $= \frac{1}{3}$ of range

range is when $y = 0$

ie $0 = -\frac{g}{2}t^2 + vt\sin\theta$

$0 = t(-\frac{g}{2}t + v\sin\theta)$

$t = 0$ or $t = \frac{-2v\sin\theta}{g}$

$\therefore$ range $= \frac{2\cos\theta\sin\theta v^2}{g}$

$$\therefore \frac{v^2\sin^2\theta}{2g} = \frac{2}{3}\frac{\cos\theta\sin\theta v^2}{g}$$

$3\sin^2\theta = 4\sin\theta\cos\theta$

$0 = 4\sin\theta\cos\theta - 3\sin^2\theta$

4 $\quad 0 = \sin\theta(4\cos\theta - 3\sin\theta)$

$\sin\theta = 0$ or $4\cos\theta = 3\sin\theta$

$\theta = 0°, 180°, 360°$ $\quad\quad$ $\tan\theta = 4/3$

$\theta = 53°8'$

$\therefore$ angle of projection is $53°8'$

(b) (i) $\ddot{x} = 0$ $\quad\quad$ $\ddot{y} = -g$

$\dot{x} = v\cos\theta$ $\quad\quad$ $\dot{y} = v\sin\theta - gt$

$x = vt\cos\theta$ $\quad\quad$ $y = vt\sin\theta - \frac{gt^2}{2}$

when $t = 2, x = 20, y = 10$

$20 = 2v\cos\theta$

$10 = v\cos\theta$ ... (1)

$10 = 2v\sin\theta - 5(4)$ 3

$15 = v\sin\theta$ ... (2)

(2) ÷ (1) $\tan\theta = 1.5$

$\theta = 56°19'$

(ii) $(1)^2 + (2)^2$ $\quad 100 = v^2\cos^2\theta$

$225 = v^2\sin^2\theta$ 2

$\therefore v^2 = 325$

$v = 18.0$ m/s

Question 7

(a) $\text{Prob(winning)} = \dfrac{{}^{6}C_{6}}{{}^{45}C_{6}}$ (2)

$= \dfrac{1}{8145060}$

(b) $\text{Prob.} = \dfrac{2\int_2^3 \frac{1}{x}\,dx}{2\int_1^4 \frac{1}{x}\,dx}$

$= \dfrac{2(\ln 3 - \ln 2)}{2(\ln 4 - \ln 1)}$

$= \dfrac{2\ln(3/2)}{2\ln 4}$

$= \dfrac{0.81093}{2.772589}$ (3)

$= 0.29248125$

$= 29.2\%$

(c) (i) $\text{arrangements} = \dfrac{11!}{2!2!2!}$ (2)

$= 4989600$

(ii) $\text{Prob.} = \dfrac{x}{4989600}$

$x \Rightarrow$ ⊗ ○ ○ ○ ⊗ ○ ○ ○ ○ ○ ○

6 ways of placing "group"

$\dfrac{9!}{2!2!}$ ways of arranging rest.

(1)

$x = 6 \times \dfrac{9!}{2!2!} = 544320$

$\therefore \text{Prob.} = \dfrac{544320}{4989600}$

$= \dfrac{6}{55}$ (1)