JRAHS Ext1 Term2 2009

QUESTION 1 (9 Marks) **Marks**

(a) The velocity of a particle, v metres per second, moving in a straight line is given as $v = 8t^2 + 8t - 24$, where t is the time in seconds. The particle is initially 7 metres from the origin.

(i) Find the displacement as a function of t. **2**

(ii) Find the acceleration as a function of t. **2**

(iii) What is the magnitude of the acceleration when $t = 2$. **1**

(iv) When does the particle change direction? **1**

(b) Twelve people are to be seated around a table.

(i) In how many ways can they be seated? **1**

(ii) In how many ways can they be seated if two particular people are not to be put together? **1**

(iii) Find the probability that 2 friends will be seated together. **1**

QUESTION 2 (9 Marks) START A NEW PAGE

(a) A particle is moving in a straight line with its acceleration as a function of x given by $\ddot{x} = -e^{-2x}$. It is initially at the origin and is travelling with a velocity of 1 metre per second.

(i) Show that $\dot{x} = \dfrac{1}{e^x}$. **2**

(ii) Hence derive an expression for the displacement of the particle as a function of t. **2**

(b) Consider a particle undergoing SHM, with its displacement, in metres, is given by

$$x = 2\cos\left(t + \frac{\pi}{4}\right) \text{ at time } t \text{ seconds.}$$

(i) Find the time at which the particle will first be at the origin. **1**

(ii) Calculate the velocity of the particle when it passes through the origin for a second time. **2**

(iii) What is the magnitude of the greatest acceleration for this particle and when does it first occur? **2**

QUESTION 3 (9 Marks) START A NEW PAGE **Marks**

(a) The letters A, E, I, O and U are vowels.

(i) How many arrangements of the letters in the word *MATHEMATICS* are possible? **1**

(ii) How many arrangements of the letters in the word *MATHEMATICS* are possible if the vowels must occupy the 3rd, 5th , 7th and 10th positions? **2**

(b) In SHM, the acceleration of a particle at any time is proportional to its displacement from the origin and is directed towards the origin.
Show that a particle with displacement $x = a \tan nt$ is not moving in SMH. **3**

(c) Carbon-14 is a radioactive isotope of carbon that has a half life of 5600 years. It is used extensively in dating organic material that is tens of thousands of years old. What percentage of the original amount of Carbon-14 in a sample would be present after 10,000 years? [Assume $N = N_0 e^{kt}$] **3**

QUESTION 4 (9 Marks) START A NEW PAGE

(a) A freshly caught fish, initially at 18^0 C, is placed in a freezer that has a constant unknown temperature of x^0 C. The cooling rate of the fish is proportional to the difference between the temperature of the freezer & the temperature T^0C, of the fish.

It is known that T satisfies the equation $$\frac{dT}{dt} = -k(T - x),$$

where t is the number of minutes after the fish is placed in the freezer.

(i) Show that $T = x + Ae^{-kt}$ satisfies this equation. **2**

(ii) If the temperature of the fish is 10^0 C after 7½ minutes, show that the fish's temperature after t minutes is given by
$$T = x + (18 - x)e^{\frac{2}{15}\log_e\left[\frac{10-x}{18-x}\right] t}.$$ **3**

(iii) Find the temperature of the fish after 15 minutes when the initial freezer temperature is 5^0 C. Answer to the nearest degree. **1**

(b) The velocity of a particle is given by $v = 4\sqrt{x+1}$. If the particle's displacement after 2 seconds is 3 metres, find its displacement after 1 second. **3**

QUESTION 5 (9 Marks) START A NEW PAGE **Marks**

(a) Four digit numbers are to be formed from the digits 4, 5, 6, 7, 8, 9. Find how many 4-digit numbers can be formed if no digit to appear more than once in the number. **2**

(b) A rocket is fired at 30 metres per second at an angle of 30^0 to the horizontal at a fireworks display. After 2 seconds, it explodes into two equal particles. One part falls vertically downwards, while the other part gets projected at 60 metres per second at an angle of 60^0 to the horizontal.

Assume acceleration due to gravity is 10 metre per second squared and that both parts fall back on the same level ground.

(i) Through what distance does the vertically falling particle travel when it hits the ground? **2**

(ii) What is the time taken for the second particle to fall back to the ground? (correct to the nearest second). **3**

(iii) What is the distance travelled by the second particle land from its launching site, to one decimal place? **2**

QUESTION 6 (9 Marks) START A NEW PAGE

(a) 5 cards are dealt out from a well-shuffled standard 52 card pack. Find the probability that this hand will contain:

(i) the 4 queens and another card. **1**

(ii) 2 jacks and 3 kings. **2**

(iii) a 3, 4, 5, 6 and 7. **1**

(b) A particle moves in a straight line and its position, x in metres at time t seconds is given by $x = 4 + \sin 2t + \sqrt{3}\cos 2t$

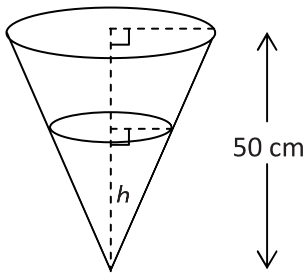
(i) Prove that the particle is moving in simple harmonic motion about $x = 4$. **2**

(ii) Find the period and amplitude of the motion. **2**

(iii) What is the speed of the particle as it travels through the equilibrium position? **1**

END of PAPER

(c) An inverted conical vessel, as shown below, is 50 centimetres in radius and 50 centimetres in depth. The vessel is being filled with water at a constant rate of 25 cm^3/s. The depth of the water at any time t seconds is h centimetres.



Not to Scale

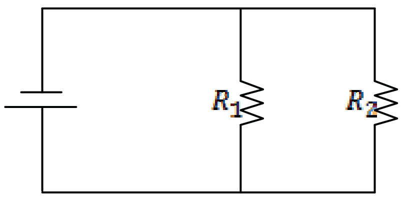
(i) Show that the surface area of the cone can be expressed as $A = \pi r^2(1+\sqrt{2})$, where r is the radius of the cone. 2

(ii) Hence, or otherwise determine the rate of increase of the surface area of the water when the depth is 20 centimetres. 3

(c) A plane flying horizontally at an altitude of one kilometre and at a constant speed of 800 kilometres per hour passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing at the instant when the plane is four kilometres away from the station. 2

(a) If two resistors with resistances R_1 and R_2 are connected in parallel, as shown in the figure below, then the total resistance R, measured in ohms, is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$



If R_1 and R_2 are increasing at rates of 0.3 ohm per second and 0.2 ohm per second respectively, how fast is R changing when $R_1 = 80$ ohms and $R_2 = 100$ ohms ? Express answer to 3 significant figures. 3

Y. 12 T2 2u

MATHS EXT 1

MATHEMATICS: Question...1...

Suggested Solutions	Marks	Marker's Comments
a i) $x(t) = \frac{8}{3}t^3 + \frac{8}{2}t^2 - 24t + c$	1	
$x = 7$ when $t = 0$ $\therefore$ $c = 7$		
$\therefore x(t) = \frac{8}{3}t^3 + 4t^2 - 24t + 7$	1	
ii) $\ddot{x} = 16t + 8$	1+1	
iii) $\ddot{x}(2) = 40$ m/s²	1	forgot m/s² $-\frac{1}{2}$m
iv) $8t^2 + 8t - 24 = 0$		
$t^2 + t - 3 = 0$		
$t = \frac{-1 \pm \sqrt{13}}{2}$		no marks if stops here.
$t \geq 0$ $\therefore t = \frac{-1+\sqrt{13}}{2}$ sec	1	forgot sec $-\frac{1}{2}$m
b i) 39916800	1	11! only $\frac{1}{2}$m
ii) ~~10! ×~~ 32659200	1	10! × 9 only $\frac{1}{2}$m
iii) $\frac{2}{11}$	1	$\frac{10! \times 2}{11!}$ only $\frac{1}{2}$m

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MATHEMATICS Extension 1 : Question 2

a) Suggested Solutions	Marks	Marker's Comments
(i) $\ddot{x} = -e^{-2x}$ $t=0$ $x=0$ $\dot{x}=1$	2	$\frac{1}{2}$ using $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$
$\frac{d}{dx}\left[\frac{1}{2}\dot{x}^2\right] = -e^{-2x}$		
$\frac{1}{2}\dot{x}^2 = \int -e^{-2x}\,dx$		
$\frac{1}{2}\dot{x}^2 = \frac{1}{2}e^{-2x} + c$		$\frac{1}{2}$ correct integration
$x=0$ $\dot{x}=1$		
$\frac{1}{2} = \frac{1}{2} + c$ $\therefore c=0$		$\frac{1}{2}$ "c" value
$\dot{x}^2 = e^{-2x}$		
$\lvert\dot{x}\rvert = e^{-x}$		
as $e^{-2x} \neq 0$ $\therefore \dot{x} \neq 0$		
$\therefore$ object never stops and cannot change direction		$\frac{1}{2}$ full explanation that $\dot{x}>0$
when $t=0$ $\dot{x}=1$		
$\therefore \dot{x}>0$ for all motion		
$\therefore \dot{x} = e^{-x} = \frac{1}{e^{x}}$		
(ii) $\dot{x} = \frac{1}{e^{x}} = e^{-x}$	2	$\frac{1}{2}$ $\frac{dt}{dx} = e^{x}$
$\frac{dx}{dt} = e^{-x}$		
$\frac{dt}{dx} = e^{x}$		$\frac{1}{2}$ correct integral including "c"
$t = \int e^{x}\,dx$		
$= e^{x} + c$		
$t=0$ $x=0$		
$0 = 1 + c$ $\therefore c=-1$		
$t = e^{x} - 1$		1 $x = \ln(t+1)$
$e^{x} = t+1$		
$x = \ln(t+1)$		

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MATHEMATICS Extension 1 : Question 2

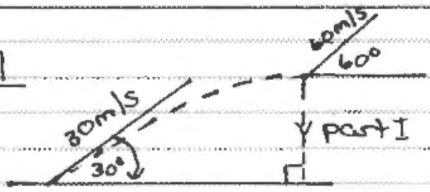
Suggested Solutions	Marks	Marker's Comments
(b) (i) $x = 2\cos\left(t + \frac{\pi}{4}\right)$	1	
origin $x = 0$		
$0 = 2\cos(t + \pi/4)$		
$t + \frac{\pi}{4} = \frac{\pi}{2}, \frac{3\pi}{2} \ldots$		1 correct answer.
$t = \frac{\pi}{4}$ (first time) (time is $\pi/4$ sec)		
(ii) at origin second time $\frac{3\pi}{2} - \frac{\pi}{4} = \frac{5\pi}{4}$ sec.		1 $\frac{5\pi}{4}$ 2nd time at origin
$x = 2\cos(t + \pi/4)$		
$\dot{x} = -2\sin(t + \pi/4)$		
when $t = \frac{5\pi}{4}$		
$\dot{x} = -2\sin\left(\frac{3\pi}{2}\right)$		1 correct answer.
$= +2$		
velocity is 2 m/s.		
(iii) $\ddot{x} = -2\cos(t + \pi/4)$		1 correct accel. ± 2 accepted. (-2) not accepted.
max acceleration is +2 m/s. occurs when		
$\ddot{x} = 2 = -2\cos(t + \pi/4)$		
$-1 = \cos(t + \pi/4)$		
$t + \frac{\pi}{4} = \pi, 3\pi$		
$\therefore$ First time is $\pi - \pi/4 = \frac{3\pi}{4}$.		1 correct time
time is $3\pi/4$ sec.		

Ext 1 MATHEMATICS: Question 3

Suggested Solutions	Marks	Marker's Comments
(a) (i) $\dfrac{11!}{2!2!2!}$	1 mark	If they forget the 2!2!2! — 0 marks
$= 4\,989\,600$		
(ii) $\dfrac{4!}{2!} \times \dfrac{7!}{2!2!}$	2 marks	1 mk off for each error
$= 15\,120$		
(b) $x = a\tan(nt)$		
$\dot{x} = an\sec^2(nt) = an(\cos(nt))^{-2}$	1 mk.	
$\ddot{x} = -2an(\cos(nt))^{-3} \cdot -n\sin(nt)$		
$= \dfrac{2an^2(\sin nt)}{\cos^3(nt)}$		
$= +2an^2\tan(nt)\cdot\sec^2(nt)$	1 mk	
$= 2n^2\sec^2(nt)\cdot x$ (as $x = a\tan(nt)$)	½	
$\neq -n^2 x$		
as n must be a positive constant and $\sqrt{2}\sec(nt)$ is not a constant $\therefore x = a\tan(nt)$ is not in SHM	½	
(c) $N = N_0 e^{5600k}$		
$\frac{1}{2}N_0 = N_0 e^{5600k}$		
$k = \dfrac{\ln 0.5}{5600}$		
$= -1.24 \times 10^{-4}$	1 mk	
$N = N_0 e^{\frac{\ln ½}{5600} t}$		
when $t = 10\,000$, $N = ?$		
$N = N_0 e^{\frac{\ln ½}{5600} \times 10000}$	1 mk	
$= N_0 \times 0.2900 \ldots$		
$\therefore$ Percentage left is 29%	1 mk.	

2009 TERM 2 EXT 1. **MATHEMATICS: Question** 4

Suggested Solutions	Marks	Marker's Comments
a) i) Substitute $T = x + Ae^{-kt}$ into $\frac{dT}{dt} = k(T-x)$		½ off for no conclusion
LHS: $\frac{dT}{dt} = -kAe^{-kt}$ (x constant) RHS: $-k(T-x) = -k(Ae^{-kt})$ $= -kAe^{-kt}$		
LHS = RHS $\therefore$ $\underline{\underline{T = x + Ae^{-kt} \text{ satisfies } \frac{dT}{dt} = k(T-x)}}$	2.	
ii) When $t=0$, $T=18$		
$\therefore 18 = x + A$ (by substitution)		
$\underline{A = 18 - x}$	1	
When $t = 15/2$, $T = 10$		
$\therefore 10 = x + Ae^{-15k/2}$		
$10 = x + (18-x)e^{-15k/2}$		
$\therefore \frac{10-x}{18-x} = e^{-15k/2} \Rightarrow \frac{18-x}{10-x} = e^{15k/2}$		
$\therefore \frac{15k}{2} = \ln\left(\frac{18-x}{10-x}\right) \Rightarrow \underline{\underline{k = \frac{2}{15}\ln\left(\frac{18-x}{10-x}\right)}}$	1	
$\therefore T = x + (18-x)e^{-\frac{2}{15}t\ln\left(\frac{18-x}{10-x}\right)}$		Some leniency here because of question ambiguity.
$= \underline{\underline{x + (18-x)e^{2t/15}\ln\left(\frac{10-x}{18-x}\right)}}$	1	
When $t=15$ and $x=5$		
$T = 5 + 13e^{2\ln(5/13)} = 5 + \frac{25}{13}$		
$\underline{\underline{= 7°C}}$ (to nearest degree C)	1	
b) $v = \frac{dx}{dt} = 4(x+1)^{1/2}$		
$\int \frac{dx}{(x+1)^{1/2}} = 4\int dt$		
$\underline{2(x+1)^{1/2} = 4t + k}$	1	
When $t=2$, $x=3$ $\therefore k=-4$		
$\underline{(x+1)^{1/2} = 2t - 2}$	1	
When $t=1$, $(x+1)^2 = 0$		
$\therefore \underline{\underline{x = -1}}$ (1 m to left of $x=0$)	1	

Suggested Solutions	**Marks**	**Marker's Comments**
Question 5:		
(a) O O O O $6\times5\times4\times3$		
No. of ~~4~~ digit no.'s $= 6\times5\times4\times3$	(2)	
$= 360$		
OR ${}^6C_4 \times 4! = 360$		
OR ${}^6P_4 = 360$		
OR $\frac{6!}{2!} = 360$		
Question 7		
(a)(i) 		
$\ddot{y} = -10$		
$\dot{y} = -10t + C$		
but $\dot{y} = 30\sin 30°$ when $t=0$		
$\therefore \dot{y} = 15 - gt$		
$y = -\frac{1}{2}gt^2 + 15t + C$		
When $t=0$, $y=0$, $C=0$		
$\therefore y = -5t^2 + 15t$	(1)	
When $t=2$		
$y = -20+30$		
$y = 10$		
$\therefore$ Part I falls vertically for 10m when it hits the ground.	(1)	
(ii) $\ddot{y} = -10$		
$\dot{y} = -10t + 60\sin 60°$		
$\dot{y} = -10t + 30\sqrt{3}$		
$y = -5t^2 + 30\sqrt{3}t + 10$ from (i)		
When $y=0$ $t^2 - 6\sqrt{3}t - 2 = 0$		
$t = \frac{6\sqrt{3} \pm \sqrt{36\times3 - 4\times-1\times2}}{2}$		
$= \frac{6\sqrt{3} \pm \sqrt{108+8}}{2}$		
$= 3\sqrt{3} \pm \sqrt{29}$		½ mark lost for
$\therefore t = -0.189$ or 10.58	(1)	not eliminating
as $t>0$ $t = 10.58$		$t = -0.189$
Nearest second is 11 sec		
Total time $= 2+11$		
$= 13$ seconds	(1)	11 seconds scored
		Full marks

TERM 2 MATHEMATICS Extension 1 : Question 5 and 7 2009		
Suggested Solutions	**Marks**	**Marker's Comments**
Question 7 (cont.) (iii) When $t=2$, $x = 30\cos 30° \times 2$ $= 60 \times \frac{\sqrt{3}}{2}$ $x = 30\sqrt{3}$ Now $x_1 = 60\cos 60° t$ $x_1 = 30t$ $= 30(3\sqrt{3}+\sqrt{79})$ $= 317\cdot 4\,m$ $\therefore$ Total distance $= 317\cdot 4\,m + 30\sqrt{3}\,m$ $= 369.4\,m$	1 1 /8	1 mark for each part

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Q6

Year 12 Ext 1 2009 Term 2 - Question 6 Marking scheme

		mark	comment
6a(i)	$\Pr ob. = \dfrac{{}^4C_4 \times {}^{48}C_1}{{}^{52}C_5} = \dfrac{1}{54145}$ Or $\Pr ob. = \left(\dfrac{4}{52}\times\dfrac{3}{51}\times\dfrac{2}{50}\times\dfrac{1}{49}\times\dfrac{48}{48}\right)\times\left(\dfrac{5!}{4!}\right)$	1	No half marks
6a(ii)	$\Pr ob. = \dfrac{{}^4C_2 \times {}^4C_3}{{}^{52}C_5} = \dfrac{1}{108290}$ Or $\Pr ob. = \left(\dfrac{4}{52}\times\dfrac{3}{51}\times\dfrac{4}{50}\times\dfrac{3}{49}\times\dfrac{2}{48}\right)\times\left(\dfrac{5!}{3!\times 2!}\right)$	2	1 for numerator containing ${}^4C_2 \times {}^4C_3$ with any other term
6a(iii)	$\Pr ob. = \dfrac{{}^4C_1 \times {}^4C_1 \times {}^4C_1 \times {}^4C_1 \times {}^4C_1}{{}^{52}C_5} = \dfrac{64}{162435}$ Or $\Pr ob. = \left(\dfrac{4}{52}\times\dfrac{4}{51}\times\dfrac{4}{50}\times\dfrac{4}{49}\times\dfrac{4}{48}\right)\times(5!)$	1	
			Repeated incorrect expressions (i) $\Pr ob. = \left(\dfrac{4}{52}\times\dfrac{3}{51}\times\dfrac{2}{50}\times\dfrac{1}{49}\times\dfrac{48}{48}\right)$ (ii) $\Pr ob. = \left(\dfrac{4}{52}\times\dfrac{3}{51}\times\dfrac{4}{50}\times\dfrac{3}{49}\times\dfrac{2}{48}\right)$ (iii) $\Pr ob. = \left(\dfrac{4}{52}\times\dfrac{4}{51}\times\dfrac{4}{50}\times\dfrac{4}{49}\times\dfrac{4}{48}\right)$ Max 2/4

6b(i)	$x = 4 + \sin 2t + \sqrt{3}\cos 2t$ $\dot{x} = 2\cos 2t - 2\sqrt{3}\sin 2t$ $\ddot{x} = -4\sin 2t - 4\sqrt{3}\cos 2t$ $= -4\left(\sin 2t + \sqrt{3}\cos 2t\right)$ $= -4(x-4)$ since $\sin 2t + \sqrt{3}\cos 2t = x - 4$	2	1 for $\dot{x}$ ½ for $\ddot{x}$ ½ for showing $\ddot{x} = -4(x-4)$ Could also use $x = 4 + 2\sin\left(2t + \frac{\pi}{3}\right)$ Or $x = 4 + 2\cos\left(2t - \frac{\pi}{6}\right)$
6b(ii)	Period = π sec Amplitude = 2 m	2	1 mark for each answer
6b(iii)	Speed = 4 m/s	1	1 for correct speed ½ if speed negative