QUESTION 1 (9 Marks) **Marks**

(a) A particle is moving in a straight line so that at t seconds it is x metres to the right of the origin O. Its velocity v ms^{-1} is described by the equation

$$v^2 = 24x - 20 - 4x^2$$

(i) Show that the particle is moving with Simple Harmonic Motion. **2**

(ii) Find the centre, period and amplitude of the motion. **3**

(b) A boy had 10 coins and 4 identical envelopes. He put 1 coin in the first envelope, 2 coins in the second, 3 coins in the third and 4 coins in the fourth envelope. As he put the envelopes into his bag, one coin fell out of one of the envelopes.

(i) What is the probability that the coin fell out of the fourth envelope? (assume all the coins had an equal chance of falling out) **1**

(ii) He then chose one of the envelopes at random. What is the probability that this envelope had an odd number of coins in it? **3**

QUESTION 2 (9 Marks) Start a new page

(a) A committee of 4 people is to be chosen from a group of 6 men and 5 women. At least one man and one woman must be on the committee.

What is the probability that a committee chosen at random will consist of a majority of men? **2**

(b) The output voltages of two electric circuits are varying according to the differential equations

$$\frac{dV_1}{dt} = -\,0.12(V_1 - 10) \text{ and } \frac{dV_2}{dt} = 0.08(V_2 - 5)$$

where t is the time in minutes after they are switched on and V_1 and V_2 are the output voltages of the circuits respectively in Volts.

The circuits are turned on at the same time.

(i) Show that $V_1 = 10 + 15e^{-0.12t}$ is a solution to the first differential equation. **2**

(ii) Initially $V_2 = 10$. Write a formula for V_2 as a function of t. **2**

(iii) The two circuits are connected so that their output voltages are added together when they are switched on. When will the minimum total output voltage occur? **3**

QUESTION 3 (9 Marks) Start a new page **Marks**

(a) The James Ruse Knitwits decide to knit scarves. They have 10 different coloured wools from which to choose. Each scarf may consist of any number of the 10 colours.

(i) How many different wool colour combinations are possible? **1**

(ii) One student chooses a colour combination at random. **1**
What is the probability that the student uses 6 colours for his scarf?

(b) The number of hours of daylight during the year at a particular location can be approximately modelled by the Simple Harmonic Motion equations. At this location the longest number of hours of day light is 16 hours and 'shortest' day has 10 hours of daylight.

A particular species of plant will only produce flowers when it receives 14 or more hours of daylight.

(i) Sketch a graph showing the number of Hours of Daylight against the number of days after the 'shortest' day, for 1 year. (1 year = 365 days) **2**

Let H = Hours of Daylight and t = number of days after the 'shortest' day.

(ii) Write the equation for the graph in the form $H = A - B\cos\left(\dfrac{2\pi t}{365}\right)$, **1**

where A and B are constants.

(iii) How many days of the year is the plant expected to have flowers on it? **4**

QUESTION 4 (9 Marks) Start a new page

A particle is projected upwards with a velocity of 40 ms^{-1} at an angle of 60^{o} to the horizontal from a point 100 metres above ground level.

You many assume the projectile motion equations.

$$x = Vt\cos\alpha \text{ and } y = Vt\sin\alpha - \frac{1}{2}\text{gt}^2$$

(use $g = 10\ \text{ms}^{-2}$ as the acceleration due to gravity)

(i) Calculate the time taken to reach the highest point of the particle's trajectory. **2**

(ii) What is the maximum height above the ground reached by the particle? **1**

(iii) Find the exact time that the particle is in the air. **3**

(iv) How fast is the particle travelling as it hits the ground? **3**

QUESTION 5 (9 Marks) Start a new page **Marks**

A particle is released from rest at a position $\frac{5\pi}{4}$ metres to the right of the origin and travels in a straight line. Its acceleration is described by the equation

$$\frac{d^2x}{dt^2} = 2\cos x$$

where x is the displacement in metres from the origin O and t is the time in seconds.

(i) In which direction will the particle first move? Justify your answer. **2**

(ii) Show that its velocity is given by $v^2 = 4\sin x + 2\sqrt{2}$. **3**

(iii) Where is the particle stationary? Explain your answer. **3**

(iv) Is the particle's motion Simple Harmonic? Justify your answer. **1**

QUESTION 6 (9 Marks) Start a new page

(a) The numbers 1, 2, 3, 4, 5, 6, 7, 8 are arranged in a circle. **4**
What is the probability that at least 3 odd numbers are together?

(b) A new species of bird was introduced onto an island. To study the spread of the species across the island, scientists counted the number of nests (N) each year and determined that the number of nests could be calculated by the using the equation

$$N = A + (N_0 - A)e^{-kt}$$

where N_0, A and k are constants and t is the number of years after 1 January 2009.

The following table shows the scientists' results for three years.

Date	1 January 2009	1 January 2010	1 January 2011
Number of Nests	1100	1500	1800

(i) Calculate the values of N_0, A and k. **4**

(ii) What is the predicted maximum number of nests? **1**

QUESTION 7 (9 Marks) Start a new page **Marks**

The acceleration of a particle moving in a straight line is given by

$$\frac{d^2x}{dt^2} = -0.05v^3$$

where x is the displacement in metres from the origin O and v metres per second is the velocity of the particle at time t seconds.

When $t = 0$ the particle passes the origin with a velocity of 10 ms^{-1}.

(i) Show that $v^2 = \dfrac{100}{10t+1}$ **4**

(ii) Find the time for particle to travel 20 metres? **4**

(iii) Briefly describe the motion of the particle. **1**

END OF EXAMINATION

Yr12 3U - 2011 T2

Q1	marks	Comments
a i) $\ddot{x} = \frac{d(\frac{1}{2}v^2)}{dx} = \frac{1}{2}[24-8x] = 12-4x$	1	Lots of students write $\ddot{x} = -4(x-3)$ $-\frac{1}{2}$m
$\ddot{x} = -2^2(x-3)$ which is in the form of $\ddot{x} = -n^2(x-b)$ where $n=2$ $\therefore b = 3$	1	
ii) C.O.M $x = 3$	1	forget unit $-\frac{1}{2}$m
Period $= \frac{2\pi}{2} = \pi$ sec	1	must define T, a $-\frac{1}{2}$m
amplitude = ? $4x^2 - 24x + 20 = 0$ $x^2 - 6x + 5 = 0$ $(x-5)(x-1) = 0$ $\therefore x = 5$ or 1		
amplitude $= \frac{5-1}{2} = 2$m	1	
b(i) $P(4^{th}$ envelop$) = \frac{4}{10} = \frac{2}{5}$	1	
ii) P(odd envelop) $= \frac{1}{4} \times \frac{9}{10} +$ (envelop 1 not chosen) $\frac{1}{4} \times \frac{2}{10} +$ E2 chosen $\frac{1}{4} \times \frac{7}{10} +$ E3 not chosen $\frac{1}{4} \times \frac{4}{10}$ E4 chosen $= \frac{11}{20}$	3	Show $\frac{1}{4}$ throughout $\frac{1}{2}$m other fraction $\frac{1}{2}$m each final answer $\frac{11}{20}$ $\frac{1}{2}$m
or $\frac{1}{10} \times \frac{1}{4} = \frac{1}{40}$ (If E1 falls out, only E3 odd) $+ \frac{2}{10} \times \frac{3}{4} = \frac{6}{40}$ (E2 falls out E1, E2 E3 odd) $+ \frac{3}{10} \times \frac{1}{4} = \frac{3}{40}$ (E3 falls out E1 odd) $+ \frac{4}{10} \times \frac{3}{4} = \frac{12}{40}$ (E4 falls out E1 E3 E4 odd) $= \frac{11}{20}$	3	Show $\frac{1}{4}$, $\frac{3}{4}$ throughout $\frac{1}{2}$m $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$ 1m Some misinterpret question answer $\frac{1}{2}$ $\frac{1}{2}$m

Y12 MATH EXT 1. ASSESSMENT TASK 3
TERM 2, 2011

MATHEMATICS Extension 1 : Question 2

Suggested Solutions | **Marks** | **Marker's Comments**

(a) A committee of 4 from 6M and 5W with at least 1M and 1W on the committee. [∴ conditional]

Cases for COMM.		
M(6)	W(5)	No of ways
3	1	${}^6C_3 \times {}^5C_1 = 100$
2	2	${}^6C_2 \times {}^5C_2 = 150$
1	3	${}^6C_1 \times {}^5C_3 = 60$
		$\therefore n(S) = 310$

Marks: $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

$$n(E = \text{maj } M) = n(E=(3,1)) = {}^6C_3 \times {}^5C_1 = 100$$

$$\therefore P(E) = \frac{100}{310} = \frac{10}{31}$$

($\frac{1}{2}$) For ${}^6C_1 \times {}^5C_1 \times {}^9C_2$ although F.

$n(S) = {}^{11}C_4 - {}^6C_4 - {}^5C_4 = 310$

$\frac{1}{2}$ For $\frac{100}{\cdots}$

${}^{11}C_4$ worth ϕ

$\boxed{2}$

(b) $\frac{dV_1}{dt} = -0.12(V_1 - 10)$; $\frac{dV_2}{dt} = 0.08(V_2 - 5)$

(i) RTS $V_1 = 10 + 15e^{-0.12t}$ is the solution.

LHS

$$\frac{dV_1}{dt} = 15 \times (-0.12)e^{-0.12t}$$
$$= -1.8e^{-0.12t}$$

RHS

$$-0.12(V_1 - 10) = -0.12(10 + 15e^{-0.12t} - 10)$$
$$= -0.12(15e^{-0.12t})$$
$$= -1.8e^{-0.12t}$$

∴ LHS = RHS

∴ $V_1 = 10 + 15e^{-0.12t}$ is the (general) solution to the 1st DE.

1 For LHS

$\frac{1}{2}$ For RHS

$\frac{1}{2}$ For conclusion

$\boxed{2}$

Note: $\frac{dV_1}{dt} = 15 \times (-0.12)e^{-0.12t} = -0.12 \times 15e^{-0.12t}$

as $V_1 = 10 + 15e^{-0.12t}$

so $15e^{-0.12t} = V_1 - 10$

ie $\frac{dV_1}{dt} = -0.12(V_1 - 10)$

this is actually the converse of the Q. asked! but 2 Mks was given!

(ii) $\frac{dV_2}{dt} = 0.08(V_2 - 5)$

∴ $V_2 = 5 + Be^{0.08t}$ as DE is of the same form as (i) reason

but $t = 0$ $V_2 = 10$

so $10 = 5 + B$

∴ $B = 5$

∴ $V_2 = 5 + 5e^{0.08t}$ ✓ $\frac{1}{2}$

$\frac{1}{2}$ For $V_2 = 5 + \ldots$

$\frac{1}{2}$ For $e^{0.08t}$

$\frac{1}{2}$ For $B = 5$

$\boxed{2}$

Note: If "done" by integration (not in program/syllabus) and a lack of absolute brackets and reason to remove to get to $V_2 = 5 + 5e^{0.08t}$ $-\frac{1}{2}$ Mk.

MATHEMATICS Extension 1 : Question 2.

Suggested Solutions | **Marks** | **Marker's Comments**

(b) (iii) Let $V = V_1 + V_2$

i.e. $V = 15 + 5e^{0.08t} + 15e^{-0.12t}$; $t \geqslant 0$

$\frac{dV}{dt} = 0.4e^{0.08t} - 1.8e^{-0.12t}$ ½ For $\frac{dV}{dt}$

* For possible max/min values of V to occur $\frac{dV}{dt} = 0$ ½ For 'explaining'

∴ $0.4e^{0.08t} - 1.8e^{-0.12t} = 0$

so $0.4e^{0.08t} = 1.8e^{-0.12t} = \frac{1.8}{e^{0.12t}}$

∴ $e^{0.08t} \times e^{0.12t} = \frac{1.8}{0.4} = \frac{9}{2} = 4.5$

$e^{0.20t} = 4.5$

$0.2t = \ln 4.5$ 1 For getting $t = 5\ln 4.5$

$t = \frac{\ln 4.5}{0.2} = 5\ln 4.5 = 7.52\ldots$

$V = 30.209\ldots$

TEST nature

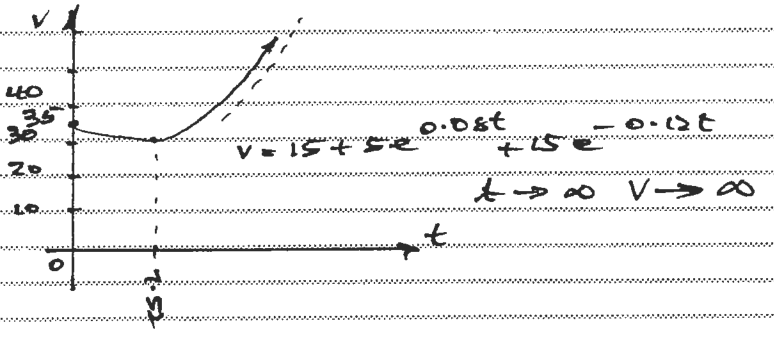
$\frac{d^2V}{dt^2} = 0.032e^{0.08t} + 0.216e^{-0.12t}$ $= 5\times(0.8)^2 e^{0.08t} + 15\times(0.12)^2 e^{-0.12t}$

> 0 as $e^{0.08t}$ and $e^{-0.12t} > 0 \ \forall t \geqslant 0$

or $\frac{d^2V}{dt^2} = 0.146\ldots > 0$ ½ For Test

∴ concave upwards and since a SP.

∴ a Relative min TP at $t = 7.52\ldots$



$\boxed{3}$

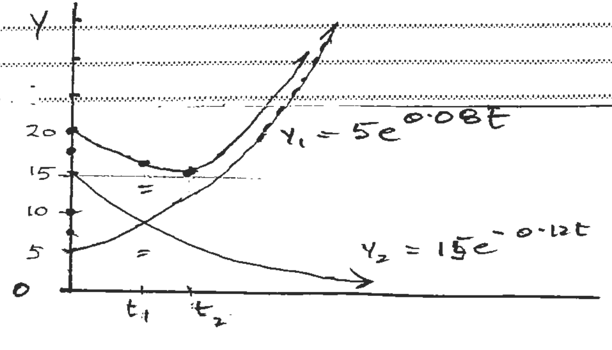
½ For justifying abs min at $t = 5\ln 4.5$

As V is continuous and there are no other TPs for $t \geqslant 0$

∴ the absolute min occurs when $t = 5\ln 4.5$

i.e. the time is 7.52 minutes (2dp)

Note : If ADD 2 curves say $Y_1 = 5e^{0.08t}$, $Y_2 = 15e^{-0.12t}$

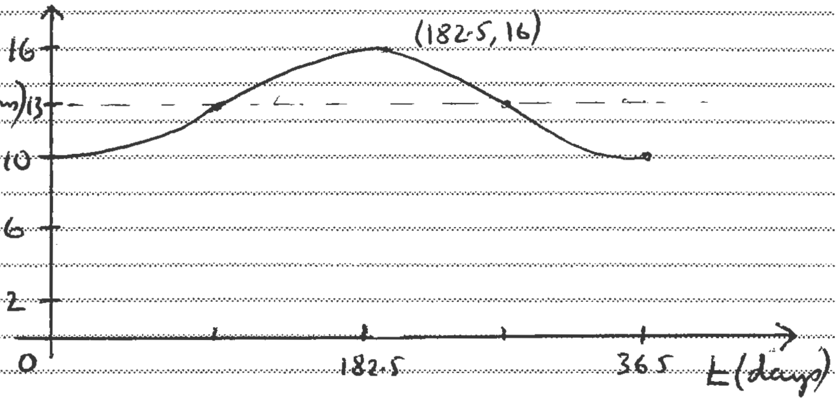


$t_1 = 5\ln 3$ $Y = Y_1 + Y_2 = 15.518\ldots$

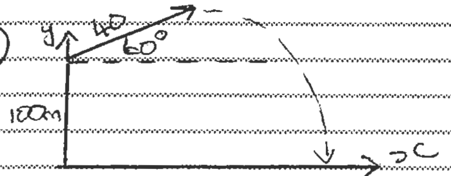
$t_2 = 5\ln 4.5$ $Y = Y_1 + Y_2 = 15.209\ldots$ (min)

2.

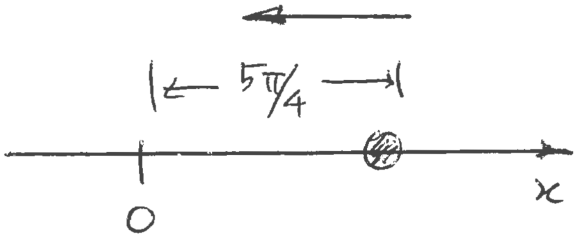
MATHEMATICS Extension 1 : Question 3

Suggested Solutions	Marks	Marker's Comments
a) Each colour either used or not used 2^{10} Rule out case when no colour chosen 1 $\therefore 2^{10}-1 = 1023$	1	
ii) Different ways of choosing 6 is ${}^{10}C_6$ $\therefore p(6 \text{ colours}) = \frac{{}^{10}C_6}{1023} = \frac{210}{1023} = \frac{70}{341}$	1	
b) i) 	2	½ for shape ½ for period ½ for amplitude ½ for centre (implied)
ii) Given $H = A - B\cos\frac{2\pi t}{365}$ When $t=0$, $H=10 \Rightarrow 10 = A - B$ When $t=182\cdot5$, $H=16 \Rightarrow 16 = A + B$ These solve to $A=13$, $B=3$ $\therefore H = 13 - 3\cos\frac{2\pi t}{365}$	1	(½ each)
iii) Find range of t so that $H \geq 14$ $13 - 3\cos\frac{2\pi t}{365} \geq 1$ $\cos\frac{2\pi t}{365} \leq -\frac{1}{3}$ First two solutions of $\cos\theta = -\frac{1}{3}$ are 1·9106 and $2\pi - 1\cdot9106$ $\therefore 111 \leq t \leq 254$ Since both end days flower, it flowers on 144 days	4	½ for equation 1 for general form or implied equivalent 1,1 for each time ½ for adding endpoint

Extension 1 2011 **MATHEMATICS Extension 1 : Question** 4 TERM 2

Suggested Solutions	**Marks**	**Marker's Comments**
④ 		
(i) highest point is when $\dot{y}=0$		
$y = vt\sin\alpha - \frac{1}{2}gt^2$		
$\dot{y} = v\sin\alpha - gt$		
$0 = 40\sin 60^\circ - 10t$	1	
$t = \frac{40}{10} \times \frac{\sqrt{3}}{2}$		
$t = 2\sqrt{3}$ seconds		
∴ Time taken is $2\sqrt{3}$ seconds	1	½ mark deducted for no units
(ii) $y = vt\sin\alpha - \frac{1}{2}gt^2 + 100$		
$= 40 \times 2\sqrt{3} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times 10 \times (2\sqrt{3})^2$		
$= 120 - 5 \times 4 \times 3 + 100$		
$y = 160$	1	½ mark only for 60m
∴ Highest point is 160m above the ground		
(iii) Particle strikes the ground when $y = 0$		
$0 = vt\sin\alpha - \frac{1}{2}gt^2 + 100$		
$= 40 \times \frac{\sqrt{3}}{2}t - \frac{1}{2} \times 10 \times t^2 + 100$		
$0 = -5t^2 + \frac{40}{2}\sqrt{3}t + 100$		
∴ $5t^2 - 20\sqrt{3}t - 100 = 0$	1	
$t = \frac{20\sqrt{3} \pm \sqrt{1200 + 4(5)(100)}}{10}$		An answer of $4\sqrt{3}$s scored a max. of 1 mark as qu. was simplified.
$= \frac{20\sqrt{3} \pm \sqrt{3200}}{10}$	1	
$= \frac{20\sqrt{3} \pm 40\sqrt{2}}{10}$		
time $= (2\sqrt{3} + 4\sqrt{2})$ seconds as $t > 0$	1	9.12... Decimal approx. lost ½ mark
(iv) $\dot{x} = 40\cos\alpha = 40 \times \frac{1}{2} = 20$	½	$\dot{x} = 20$ ½ mark
$\dot{y} = v\sin\alpha - gt = 40 \times \frac{\sqrt{3}}{2} - 10 \times (2\sqrt{3} + 4\sqrt{2})$	½	$\dot{y} = -40\sqrt{2}$ ½ mark
$= 20\sqrt{3} - 20\sqrt{3} - 40\sqrt{2}$		
Speed $= \|v\| = \sqrt{\dot{x}^2 + \dot{y}^2}$ as speed > 0		
$\|v\| = \sqrt{400 + 1600 \times 2}$	1	
$= \sqrt{3600}$		$-\frac{1}{2}$ for not qualifying $\|v\|$
$= 60$		
∴ Speed is 60 m/s	1	

Question (5)



(i) $\ddot{x} = 2\cos x$

When $x = 5\pi/4$, $\ddot{x} = 2\cos 5\pi/4$

i.e. $\ddot{x} = -2/\sqrt{2} = -\sqrt{2}$

< 0

Also, when $x = 5\pi/4$

$\dot{x} = 0$

∴ particle moves to the left

$\ddot{x} < 0$ $(\ddot{x} = -\sqrt{2})$	[1]
$\dot{x} = 0$ when $x = \frac{5\pi}{4}$	[½]
moves to the left	[½]

(ii) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 2\cos x$

$\frac{1}{2}v^2 = 2\sin x + c$

When $x = \frac{5\pi}{4}$, $v = 0$

$0 = 2\left(-\frac{1}{\sqrt{2}}\right) + c$

$\Longrightarrow c = \sqrt{2}$.

∴ $\frac{1}{2}v^2 = 2\sin x + \sqrt{2}$

$v^2 = 4\sin x + 2\sqrt{2}$.

Correct form of D.E. and integration	[1]
$c = \sqrt{2}$	[1]
Correct expression for v^2	[1]

[1]

(iii) $v = 0$

$4\sin x + 2\sqrt{2} = 0$

$\therefore \sin x = -\frac{1}{\sqrt{2}}$.

$\therefore x = \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{13\pi}{4}, \ldots$

- But $x \neq \frac{7\pi}{4}$ since the particle initially moves from $x = \frac{5\pi}{4}$ continuing moving to the left. i.e $x = -\frac{\pi}{4}$.

$x = n\pi + (-1)^n \left(\frac{5\pi}{4}\right)^n$ *

or $x = n\pi + (-1)^n \left(-\frac{\pi}{4}\right)^n$

$(n \in \mathbb{Z}^+)$

- $v = 0$ when $x = -\frac{\pi}{4}$.

When $x = -\frac{\pi}{4}$, $\ddot{x} > 0$.

$\Longrightarrow$ the particle moves to the right and stops again at $x = \frac{5\pi}{4}$.

- Motion continues between $x = -\frac{\pi}{4}$ and $x = \frac{5\pi}{4}$ ONLY.

(iv) Motion is not SHM

as $\ddot{x} = \begin{cases} n^2(x-b) \\ n^2 x \end{cases}$

$\therefore \ddot{x} = k\cos x$, $k = 2$.

"Correct" solution $x = -\frac{\pi}{4}$ with explanation

$x = \frac{7\pi}{4}$ [$\frac{1}{2}$]

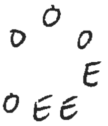
$x = -\frac{\pi}{4} \begin{cases} \dot{x} = 0 \\ \ddot{x} > 0 \end{cases}$

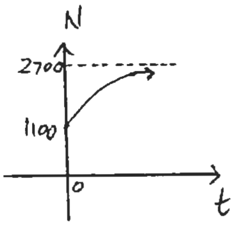
[1]

Explanation of the motion continues between 2 places

[1]

No with Reason [1]

Expected answers	**Marks**	**Comments**
6 (a)		
Sample Space: 8 Objects in a circle Number of ways = $\frac{8!}{8} = 5040$	$\frac{1}{2}$	Sample space
Event Space: Case (i): 3 Odd numbers together 		
4 odds, choose 3 $\Longrightarrow 4_{C_3} = 4$ ways, and internally arranged in 3! ways i.e. 24 ways	$\frac{1}{2}$	For 4 choose 3 and arranged in 3! ways
2 evens on either side of the 3 grouped odds gives: 4 choose 2 and arranged in 2! ways i.e. $4_{C_2} \times 2! = 12$ ways	$\frac{1}{2}$	
Now 4 objects (EOOOE and O and E and E) in a circle are arranged in $3! = 6$ ways.		
Hence number of arrangements = $24 \times 12 \times 6 = 1728$ ways	$\frac{1}{2}$	
Case (ii): (4 odd numbers together) 		
4 odds arranged in $4_{C_4} \times 4! = 24$ ways	$\frac{1}{2}$	
Now the 5 objects (odd group and 4 evens) arranged in a circle in 4! ways = 24 ways	$\frac{1}{2}$	
Hence the number of ways = 24 x 24 = 576	$\frac{1}{2}$	
Probability (of at least 3 odds), $P = P(3\ odds) + P(4\ odds) = \frac{1728+576}{5040} = \frac{16}{35}$	$\frac{1}{2}$	$\frac{24}{35}$ max $2\frac{1}{2}$

Expected answers	**Marks**	**Comments**
(b) $N = A + (N_0 - A)e^{-kt}$		
(i) At $t = 0,\ N = 1100$ $\therefore 1100 = A + (N_0 - A)e^0$ $\Rightarrow N_0 = 1100$	$\frac{1}{2}$	For N_0 value (incl. Working)
Now when $t = 1, N = 1500$ $\Rightarrow 1500 = A + (1100 - A)e^{-k}$ $\Rightarrow e^{-k} = \frac{1500-A}{1100-A}$ ...(1)	1	For equation in terms of A and k
Now when $t = 2, N = 1800$ $\Rightarrow 1800 = A + (1100 - A)e^{-2k}$ $\Rightarrow e^{-2k} = \frac{1800-A}{1100-A}$ $\Rightarrow (e^{-k})^2 = \frac{1800-A}{1100-A}$ ...(2)	1	For equation in terms of A and k
Sub (1) into (2): $(\frac{1500 - A}{1100 - A})^2 = \frac{1800 - A}{1100 - A}$	$\frac{1}{2}$	For algebra
$\Rightarrow (1500 - A)^2 = (1800 - A)(1100 - A)$ $\Rightarrow 11 \times 18 \times 10^4 - 2900A = 225 \times 10^4 - 3000A$ $\Rightarrow A = 2700$	$\frac{1}{2}$	For value of A
Sub $A = 2700$ into (1): $e^{-k} = \frac{1500 - 2700}{1100 - 2700} = \frac{3}{4}$ $\Rightarrow k = \ln(\frac{4}{3})$	$\frac{1}{2}$	For value of k
(ii) Now $N = 2700 - 1600e^{-\ln(\frac{4}{3})t}$ Maximum will be approached when $t \rightarrow \infty \Rightarrow e^{-\ln(\frac{4}{3})t} \longrightarrow 0 \Rightarrow$ maximum approaches 2700  	1	For maximum predicted value

1/5

MATHEMATICS Extension 1 : Question ..7..

Q7 Suggested Solutions	Marks	Marker's Comments
(i) $\ddot{x} = -0.05v^3$		
$\frac{dv}{dt} = -0.05v^3$		
$\frac{dt}{dv} = \frac{-1}{0.05v^3}$		
$= \frac{-20}{v^3}$		
$t = -20\int \frac{1}{v^3}\,dv$	①	
$= -20\int v^{-3}\,dv$		
$\therefore t = -20\left[\frac{1}{-2v^2} + c\right]$	½	
$t = \frac{10}{v^2} + -20c$		
when $t=0, v=10$	½	-1 if they forget "c".
$\therefore 0 = \frac{10}{100} - 20c$		
$20c = \frac{1}{10}$		
$c = \frac{1}{200}$	½	
$\therefore t = \frac{10}{v^2} - \frac{1}{10}$		
$t + \frac{1}{10} = \frac{10}{v^2}$	½	
$\frac{10t+1}{10} = \frac{10}{v^2}$		
$\frac{10}{10t+1} = \frac{v^2}{10}$	½	
$\therefore v^2 = \frac{100}{10t+1}$	½	
or method 2		
$\frac{dV}{dt} = -0.05V^3$		
$\frac{dV}{V^3} = -0.05dt$		
$\int_{10}^{v} \frac{dv}{v^3} = -0.05\int_0^t dt$	①	
$\left[\frac{-1}{2v^2}\right]_{10}^{V} = -0.05(t-0)$	½	

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MATHEMATICS Extension 1 : Question 7 cont

Suggested Solutions	Marks	Marker's Comments
$-\frac{1}{2v^2} + \frac{1}{200} = -0.05t$	½	
$0.05t + \frac{1}{200} = \frac{1}{2v^2}$	½	
$\frac{10t+1}{200} = \frac{1}{2v^2}$	½	
$\frac{200}{10t+1} = 2v^2$	½	
$v^2 = \frac{100}{10t+1}$	½	
or method 3		
$\ddot{x} \quad v\frac{dv}{dx}$		
$-0.05v^3 = v\frac{dv}{dx}$	½	
$\therefore \frac{dv}{dx} = -0.05v^2$		
$\frac{dx}{dv} = \frac{1}{-0.05v^2}$		
$= \frac{-20}{v^2}$	½	
$x = \int \frac{-20}{v^2}\,dv$		
$x = \frac{20}{v} + c$		
when $x=0, v=10$		
$\therefore 0 = \frac{20}{10} + c$	½	
$\therefore c = -2$		
$\therefore x = \frac{20}{v} - 2$		
$x+2 = \frac{20}{v}$		
$\therefore v = \frac{20}{x+2}$	½	
$\frac{dx}{dt} = \frac{20}{x+2}$		
$\frac{dt}{dx} = \frac{x+2}{20}$		
$t = \int \frac{x}{20} + \frac{1}{10}\,dx$	½	
$t = \frac{1}{40}x^2 + \frac{1}{10}x + C_2$	½	

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MATHEMATICS Extension 1 : Question 7 continued.

Suggested Solutions	Marks	Marker's Comments
when $t=0, x=0$		
$\therefore 0 = 0+0+C_2$		
$c_2=0$		
$\therefore t = \frac{1}{40}x^2 + \frac{1}{10}x$	$\frac{1}{2}$	
$40t = x^2+4x$		
$4+40t = x^2+4x+4$		
$4+40t = (x+2)^2$		
now $v = \frac{20}{x+2}$		
$\therefore v^2 = \frac{20^2}{(x+2)^2}$	$\frac{1}{2}$	
$v^2 = \frac{400}{4+40t}$		
$v^2 = \frac{100}{1+10t}$		
QED.		
(ii) $v = \pm\sqrt{\frac{100}{1+10t}}$		
$v = \frac{\pm 10}{\sqrt{1+10t}}$		
but when $t>0$, $v>0$	$\left.\right\}$ $\frac{1}{2}$	
$\therefore v = \frac{10}{\sqrt{1+10t}}$		
$\therefore \frac{dx}{dt} = \frac{10}{\sqrt{1+10t}}$	$\frac{1}{2}$	
$x = 10\int \frac{1}{\sqrt{10t+1}}\,dt = 10\int (10t+1)^{-\frac{1}{2}}dt$	$\frac{1}{2}$	
$x = 10.\frac{1}{10}.2\left[(10t+1)^{\frac{1}{2}}\right]+c$	$\frac{1}{2}$	
$x = 2\sqrt{10t+1}+c$	$\frac{1}{2}$	

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MATHEMATICS Extension 1 : Question…….

Suggested Solutions	Marks	Marker's Comments
when $t=0$, $x=0$		
$0 = 2\sqrt{0+1} + c$	1/2	
$c = -2$		
$\therefore x = 2\sqrt{10t+1} - 2$		
when $x=20$, $t=?$		−1 off if they forgot "c" when they integrated.
$\therefore 20 = 2\sqrt{10t+1} - 2$	1/2	
$22 = 2\sqrt{10t+1}$		
$11 = \sqrt{10t+1}$		
$121 = 10t+1$		
$120 = 10t$		
$t = 12$	1/2	
$\therefore$ It takes 12 seconds		
or method 2		
$v\frac{dv}{dx} = -0.05v^3$		
$\frac{dv}{dx} = -0.05v^2$		
$\frac{dx}{dv} = \frac{-1}{0.05v^2}$	1	
$x = \frac{-1}{0.05}\int \frac{1}{v^2}\,dx$		
$x = -20\left[\frac{-1}{v} + c\right]$		
when $x=0$, $v=10$		
$0 = -20\left[\frac{-1}{10} + c\right]$	1	
$\therefore c = \frac{1}{10}$		
$\therefore x = -20\left[\frac{-1}{v} + \frac{1}{10}\right]$		
now when $x = 20$		
$20 = -20\left[\frac{-1}{v} + \frac{1}{10}\right]$		
$-1 = \frac{-1}{v} + \frac{1}{10}$	1	
$\frac{1}{v} = \frac{11}{10}$ $\quad \therefore v = \frac{10}{11}$		

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MATHEMATICS Extension 1 : Question …7…

Suggested Solutions	Marks	Marker's Comments
now $v^2 = \frac{100}{10t+1}$ when $v = \frac{10}{11}$, $t = ??$ $\frac{100}{121} = \frac{100}{10t+1}$ $121 = 10t + 1$ $120 = 10t$ $t = 12$ ∴ time is 12 seconds.	①	
(iii) The particle passes through the origin at 10m/s when t=0. It is travelling to the right. The particle continues to move to the right, slowing down but never stopping or changing direction		looking for - moving to right - slowing down - never stops - never changes direction. *If the student's left off one item they only scored ½ a mark. *Discussing the initial conditions did not score marks as that was the original data. *Writing was awful. If the writing couldn't be read, marks were deducted.